

Electric Machines and their Controllers

Electric motors are a key component of an electric vehicle, and in this chapter we consider the main types of motors used. The more advanced modern electric machines need fairly complex controllers, so these are also described in this chapter.

We start, in Section 7.1, with an explanation of the simplest types of DC motors that can run with hardly any electronic control. In Section 7.2 we cover the basics of the power electronics that are needed to operate the more advanced motors frequently used nowadays. Then, in Section 7.3 we consider the different types of these more sophisticated motors, such as the ‘switched reluctance motor’ (SRM), the ‘brushless’ motor and the tried and tested induction motor. There are issues that apply to all motors when it comes to their selection and use, and these aspects, such as cooling, efficiency, size and mass, are considered in Section 7.4. Finally, in Section 7.5, we consider the special factors that apply to electric machines in hybrid electric vehicles (HEVs).

7.1 The ‘Brushed’ DC Electric Motor

7.1.1 *Operation of the Basic DC Motor*

Electric vehicles use what can seem a bewildering range of different types of electric motors. However, the simplest form of electric motor, at least to understand, is the ‘brushed’ DC motor. This type of motor is very widely used in applications such as portable tools, toys, electrically operated windows in cars, and small domestic appliances such as hair dryers, even if they are AC mains powered.¹ However, it is also still used as a traction motor, although the other types of motors considered later in this chapter are becoming more common for this application. The brushed DC motor is a good starting point because, as well as being widely used, most of the important issues in electric motor control can be more easily explained with reference to this type of motor.

The classical DC electric motor is shown in Figure 7.1. It is a DC motor, equipped with permanent magnets and brushes. This simplified motor has one coil, and the current

¹ In this case the appliance will also have a small rectifier.

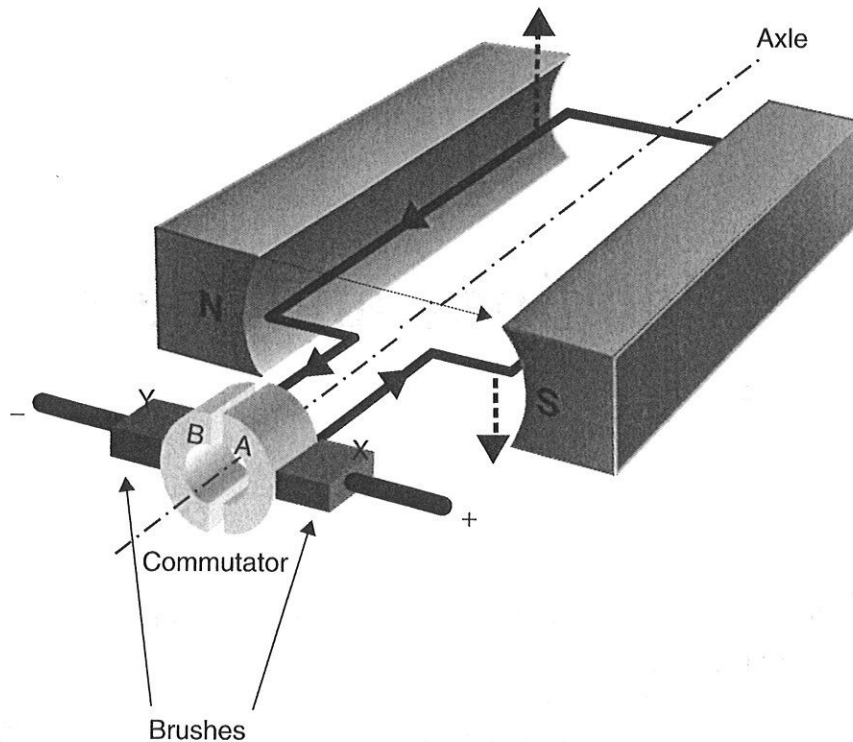


Figure 7.1 Diagram to explain the operation of the simple permanent magnet DC motor

passing through the wire near the magnet causes a force to be generated in the coil. The current flows through brush X, commutator half ring A, round the coil, and out through the other commutator half ring B and brush Y (XABY). On one side (as shown in the diagram) the force is upwards, and on the other it is downwards, because the current is flowing back towards the brushes and commutator. The two forces cause the coil to turn. The coil turns with the commutator, and once the wires are clear of the magnet the momentum carries it on round until the half rings of the commutator connect with the brushes again. When this happens the current is flowing in the same direction relative to the magnets, and hence the forces are in the same direction, continuing to turn the motor as before. However, the current will now be flowing through brush X, half ring B, round the coil to A and out through Y, so the current will be flowing in the opposite direction through the coil (XBAY).

The commutator action ensures that the current in the coil keeps changing direction, so that the force is in the same direction, even though the coil has moved.

Clearly, in a real DC motor there are many refinements over the arrangement of Figure 7.1. The most important of these are as follows:

- The rotating wire coil, often called the armature, is wound round a piece of iron, so that the magnetic field of the magnets does not have to cross a large air gap, which would weaken the magnetic field.
- More than one coil will be used, so that a current-carrying wire is near the magnets for a higher proportion of the time. This means that the commutator does not consist of two half rings (as in Figure 7.1) but several segments, two segments for each coil.

- Each coil will consist of several wires, so that the torque is increased (more wires, more force).
- More than one pair of magnets may be used, to increase the turning force further.

Figure 7.2a is the cross-section diagram of a DC motor several steps nearer reality than that of Figure 7.1. Since we are in cross-section, the electric current is flowing in the wires either up out of the page, or down into the page. Figure 7.2b shows the convention used when using such diagrams. It can be seen that most of the wires are both carrying a current and in a magnetic field. Furthermore, all the wires are turning the motor in the same direction.

7.1.2 Torque Speed Characteristics

If a wire in an electric motor has a length l metres, carries a current I amperes and is in a magnetic field of strength B webers per square metre, then the force on the wire is

$$F = BIl \quad (7.1)$$

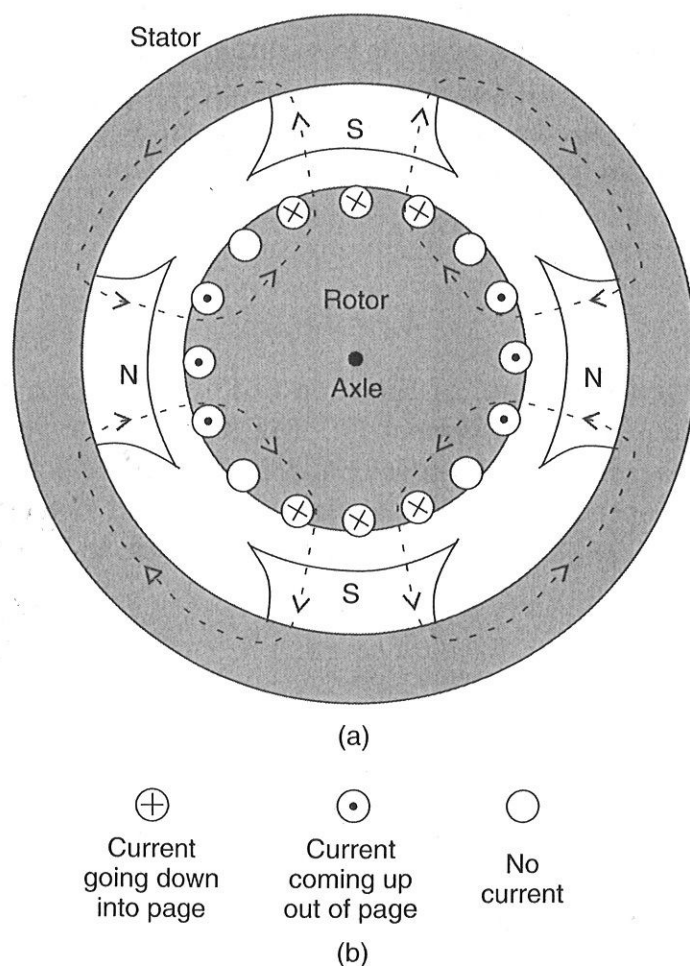


Figure 7.2 (a) Cross-section through a four-pole DC motor. The dashed lines show the magnetic flux. The motor torque is clockwise. (b) Shows the convention used to indicate the direction of current flow in wires drawn in cross-section

If the radius of the coil is r and the armature consists of n turns, then the motor torque T is given by the equation

$$T = 2nrBIl \quad (7.2)$$

The term $2Blr = B \times \text{area}$ can be replaced by Φ , the total flux passing through the coil. This gives

$$T = n\Phi I \quad (7.3)$$

However, this is the peak torque, when the coil is fully in the flux, which is perfectly radial. In practice this will not always be so. Also, it does not take into account the fact that there may be more than one pair of magnetic poles, as in Figure 7.2. So, we use a constant K_m , known as the motor constant, to connect the average torque with the current and the magnetic flux. The value of K_m clearly depends on the number of turns in each coil, but also on the number of pole pairs and other aspects of motor design. Thus we have

$$T = K_m \Phi I \quad (7.4)$$

We thus see that the motor torque is directly proportional to the rotor (also called armature) current I . However, what controls this current? Clearly it depends on the supply voltage to the motor, E_s . It will also depend on the electrical resistance of the armature coil R_a . But that is not all. As the motor turns, the armature will be moving in a magnetic field. This means it will be working as a generator or dynamo. If we consider the basic machine of Figure 7.1, and consider one side of the coil, the voltage generated is expressed by the basic equation

$$E_b = Blv \quad (7.5)$$

This equation is the generator form of Equation (7.1). The voltage generated is usually called the 'back EMF', hence the symbol E_b . It depends on the velocity v of the wire moving through the magnetic field. To develop this further, the velocity of the wire moving in the magnetic field depends on ω , the angular velocity, and r , the radius according to the simple equation $v = r\omega$. Also, the armature has two sides, so Equation (7.5) becomes

$$E_b = 2Blr\omega$$

But as there are n turns, we have

$$E_b = 2nrBl\omega$$

This equation should be compared with Equation (7.2). By similar reasoning we simplify it to an equation like Equation (7.4). Since it is the same motor, the constant K_m can be used again, and it obviously has the same value. The equation gives the voltage or 'back EMF' generated by the dynamo effect of the motor as it turns:

$$E_b = K_m \Phi \omega \quad (7.6)$$

This voltage opposes the supply voltage E_s and acts to reduce the current in the motor. The net voltage across the armature is the difference between the supply voltage E_s and

the back EMF E_b . The armature current is thus

$$I = \frac{V}{R_a} = \frac{E_s - E_b}{R_a} = \frac{E_s}{R_a} - \frac{K_m \Phi}{R_a} \omega$$

This equation shows that the current falls with increasing angular speed. We can substitute it into Equation (7.4) to get the equation connecting the torque and the rotational speed:

$$T = \frac{K_m \Phi E_s}{R_a} - \frac{(K_m \Phi)^2}{R_a} \omega \quad (7.7)$$

This important equation shows that the torque from this type of motor has a maximum value at zero speed, when stalled, and it then falls steadily with increasing speed. In this analysis we have ignored the losses in the form of torque needed to overcome friction in bearings, and at the commutator, and windage losses. This torque is generally assumed to be constant, which means the general form of Equation (7.9) still holds true, and gives the characteristic graph of Figure 7.3.

The simple linear relationship between speed and torque, implied by Equation (7.9), is replicated in practice for this type of constant magnetic flux DC motor. However, except in the case of very small motors, the low-speed torque is reduced, either by the electronic controller, or by the internal resistance of the battery supplying the motor. Otherwise, the currents would be extremely high and would damage the motor. Let us take an example. A popular motor used on small electric vehicles is the 'Lynch'-type machine, an example of which is shown in Figure 7.4. A typical motor of this type² might have the following data given in its specification:

- Motor speed = 70 rpm V⁻¹
- Armature resistance = 0.016 Ω.

The motor speed information connects with Equation (7.6), and refers to the no-load speed. Equation (7.6) can be rearranged to

$$\omega = \frac{E}{K_m \Phi} \text{ rad s}^{-1} = \frac{60}{2\pi K_m \Phi} E \text{ rpm}$$

So in this case we can say that

$$\frac{60}{2\pi K_m \Phi} = 70 \Rightarrow K_m \Phi = \frac{60}{2\pi \times 70} = 0.136$$

If this motor were to be run off a fixed 24 V supply, Equation (7.7) for this motor would be

$$T = 205 - 1.16\omega \quad (7.8)$$

since R_a is given as 0.016 Ω. However, this would mean an initial zero-speed torque of 205 N m. This is a huge figure, but may not seem impossibly large till the current is

² The data given is for a 1998 model of a Lynch disc armature 'type 200' DC motor.

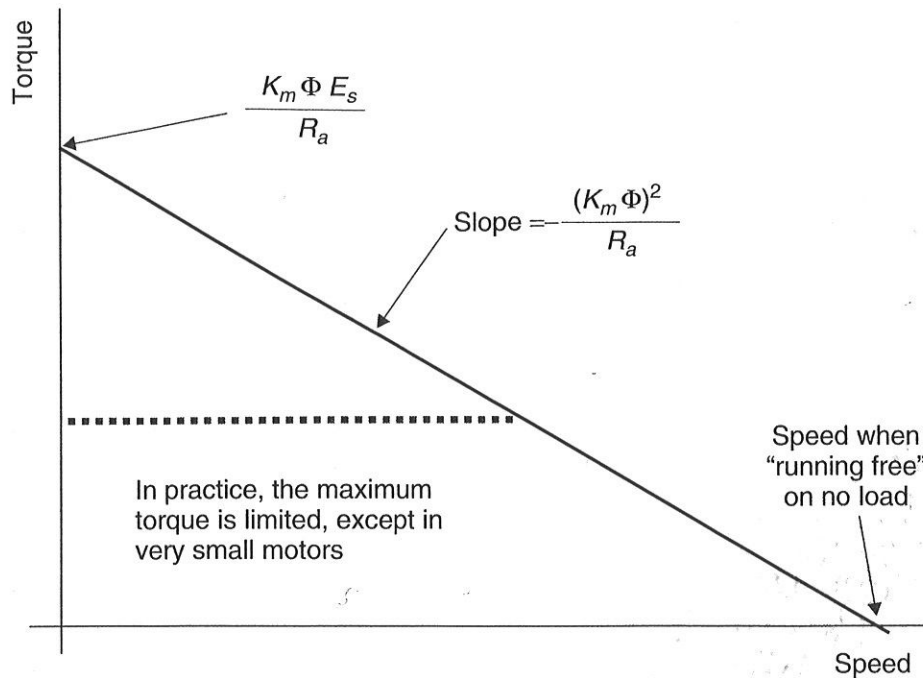


Figure 7.3 Torque/speed graph for a brushed DC motor

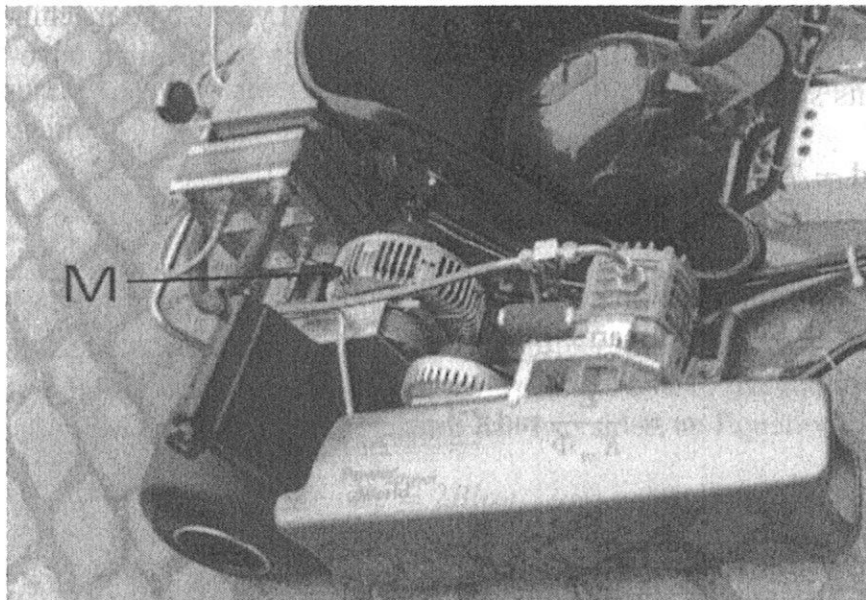


Figure 7.4 Small (10kW) DC 'Lynch'-type DC motor, which is labelled M. This go-kart is fuel cell powered. The unit in front of the motor is the air pump, which is driven by its own motor – a smaller version of the traction motor

calculated. At zero speed there is no back EMF, and thus only this armature resistance R_a opposes the 24 V supply, so the current would be

$$I \frac{V}{R} = \frac{E_s}{R_a} = \frac{24}{0.016} = 1500 \text{ A}$$

This is clearly far too large a current. The stated limit on current is 250 or 350 A for up to 5 s. We can use this information, and Equation (7.4), to establish the maximum torque as

$$T = K_m \Phi I = 0.136 \times 250 = 34 \text{ N m} \quad (7.9)$$

Equation (7.8), modified by Equation (7.9) giving a maximum torque, is typical of the characteristic equations of this type of motor. The maximum power is about 5 kW.

7.1.3 Controlling the Brushed DC Motor

Figure 7.5 and Equation (7.7) show us that the brushed DC motor can be very easily controlled. If the supply voltage E_s is reduced, then the maximum torque falls in proportion, and the slope of the torque/speed graph is unchanged. In other words, any torque and speed can be achieved below the maximum values. We will see in Section 7.2 that the supply voltage can be controlled simply and efficiently, so this is a good way of controlling this type of motor.

However, reducing the supply voltage is not the only way of controlling this type of motor. In some cases we can also achieve control by changing the magnetic flux Φ . This is possible if coils rather than permanent magnets provide the magnetic field. If the magnetic flux is reduced then the maximum torque falls, but the slope of the torque/speed graph becomes flatter. Figure 7.5 illustrates this. Thus the motor can be made to work

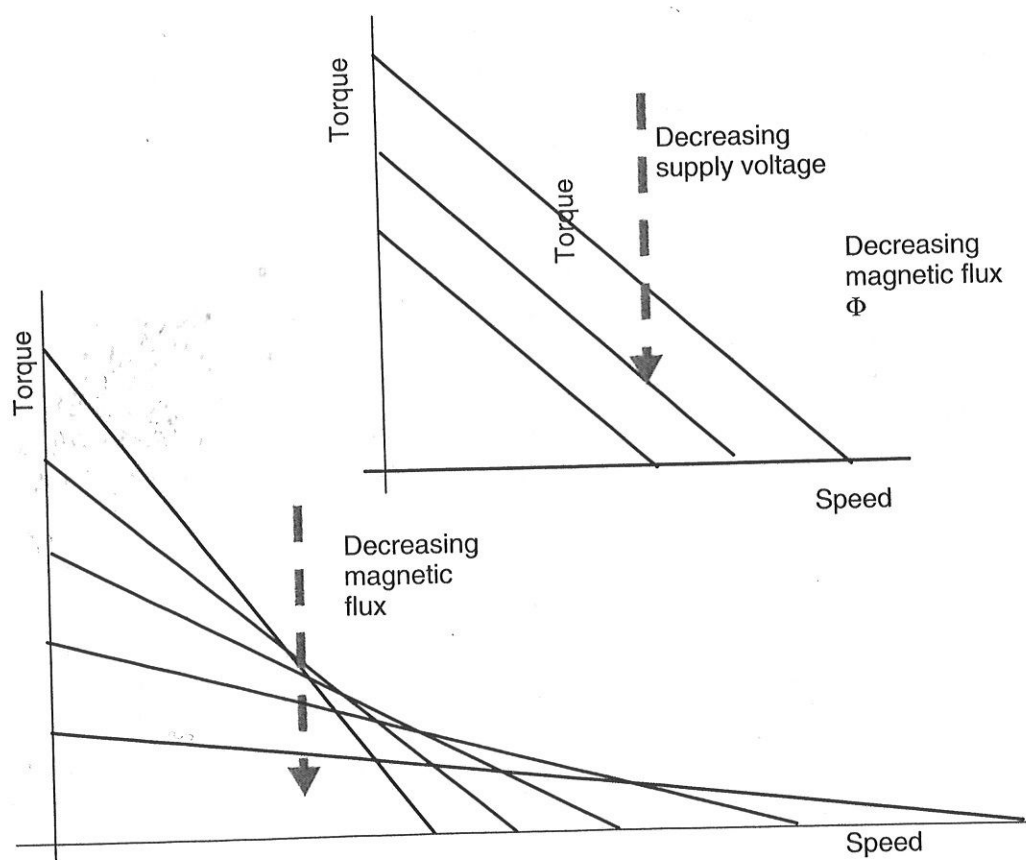


Figure 7.5 How changing the supply voltage and the magnetic field strength affects the torque/speed characteristic of the DC motor

at a wide range of torque and speed. This method is sometimes better than simply using voltage control, especially at high-speed/low-torque operation, which is quite common in electric vehicles cruising near their maximum speed. The reason for this is that the iron losses to be discussed in Section 7.1.5 below, and which are associated with high speeds and strong magnetic fields, can be substantially reduced.

So, the brushed DC motor is very flexible as to control method, especially if the magnetic flux Φ can be varied. This leads us to the next section, where the provision of the magnetic flux is described.

7.1.4 Providing the Magnetic Field for DC Motors

In Figures 7.1 and 7.2 the magnetic field needed to make the motor turn is provided by permanent magnets. However, this is not the only way this can be done. It is possible to use coils, through which a current is passed, to produce the magnetic field. These *field windings* are placed in the stator of the electric motor.

An advantage of using electromagnets to provide the magnetic field is that the magnetic field strength Φ can be changed, by changing the current. A further advantage is that it is a cheaper way of producing a strong magnetic field – though this is becoming less and less of a factor as the production of permanent magnets improves. The main disadvantage is that the field windings consume electric current and generate heat – thus it seems that the motor is almost bound to be less efficient. In practice the extra control of magnetic field can often result in *more* efficient operation of the motor, as the iron losses to be discussed in the next section can be reduced. The result is that brushed DC motors with field windings are still often used in electric vehicles.

There are three classical types of brushed DC motor with field windings, as shown in Figure 7.6. However, only one need concern us here. The behaviour of the ‘series’ and ‘shunt’ motors is considered in books on basic electrical engineering. However, they do allow the control of speed and torque that is required in an electric vehicle; the only serious contender is the ‘separately excited’ motor, as in Figure 7.6c.

The shunt (or parallel) wound motor of Figure 7.6a is particularly difficult to control, as reducing the supply voltage also results in a weakened magnetic field, thus reducing the back EMF and tending to increase the speed. A reduction in supply voltage can in some circumstances have very little effect on the speed. The particular advantage of the series motor of Figure 7.6b is that the torque is very high at low speeds and falls off

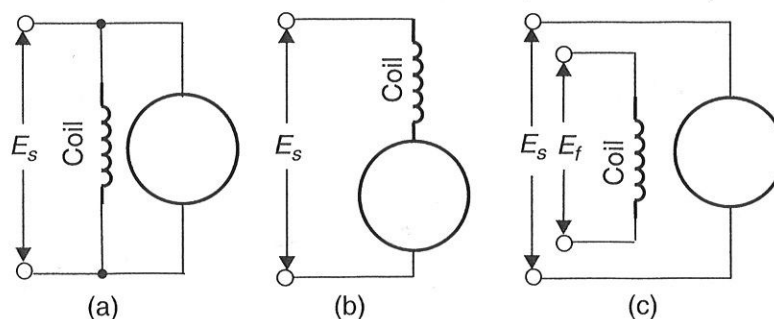


Figure 7.6 (a–c) Three standard methods of supplying current to a coil providing the magnetic field for brushed DC motors

rapidly as the speed rises. This is useful in certain applications, for example the starter motor of IC engines, but it is not what is usually required in traction applications.

The separately excited motor of Figure 7.6c allows us to have independent control of both the magnetic flux Φ (by controlling the voltage on the field winding E_f) and also the supply voltage E_s . This allows the required torque at any required angular speed to be set with great flexibility. It allows both the control methods of Figure 7.5 to be used – reducing armature supply voltage E_s or reducing the magnetic flux Φ .

For these reasons the separately excited brushed DC motor is quite widely used as the traction motor in electric vehicles. In the case of the many smaller motors that are found on any vehicle, the magnetic field is nearly always provided by permanent magnets. This makes for a motor that is simpler and cheaper to manufacture. Such permanent magnet (PM) motors are also sometimes used as traction motors.

7.1.5 DC Motor Efficiency

The major sources of loss in the brushed DC electric motor are the same as for all types of electric motor, and can be divided into four main types.

Firstly, there are the **copper losses**. These are caused by the electrical resistance of the wires (and brushes) of the motor. This causes heating, and some of the electrical energy supplied is turned into heat energy rather than electrical work. The heating effect of an electric current is proportional to the square of the current:

$$P = I^2 R$$

However, we know from Equations (7.3) and (7.4) that the current is proportional to the torque T provided by the motor, so we can say that

$$\text{Copper losses} = k_c T^2 \quad (7.10)$$

where k_c is a constant depending on the resistance of the brushes and the coil, and also the magnetic flux Φ . These copper losses are probably the most straightforward to understand and, especially in smaller motors, they are the largest cause of inefficiency.

The second major source of losses is the **iron losses**, because they are caused by magnetic effects in the iron of the motor, particularly in the rotor. There are two main causes of these iron losses, but to understand both it must be understood that the magnetic field in the rotor is continually changing. Imagine a small ant clinging to the edge of the rotor of Figure 7.2. If the rotor turns round one turn then this ant will pass a north pole, then a south pole, and then a north pole, and so on. As the rotor rotates, the magnetic field supplied by the magnets may be unchanged, but that seen by the turning rotor (or the ant clinging to it) is always changing. Any one piece of iron on the rotor is thus effectively in an ever-changing magnetic field. This causes two types of loss. The first is called 'hysteresis' loss, and is the energy required to magnetise and demagnetise the iron continually, aligning and realigning the magnetic dipoles of the iron. In a good magnetically soft iron this should be very small, but will not be zero. The second iron loss results from the fact that the changing magnetic field will generate a current in the iron, by the normal methods of electromagnetic induction. This current will result in heating of the iron. Because these currents just flow around and within the iron rotor they are called

'eddy currents'. These eddy currents are minimised by making the iron rotor not out of one piece, but using thin sheets all bolted or glued together. Each sheet is separated from its neighbour by a layer of paint. This greatly reduces the eddy currents by effectively increasing the electrical resistance of the iron.

It should be clear that these iron losses are proportional to the *frequency* with which that magnetic field changes – a higher frequency results in more magnetising and demagnetising, and hence more hysteresis losses. Higher frequency also results in a greater rate of change of flux, and hence greater induced eddy currents. However, the rate of change of magnetic flux is directly proportional to the speed of the rotor – to how quickly it is turning. We can thus say that

$$\text{Iron losses} = k_i \omega \quad (7.11)$$

where k_i is a constant. In fact, it will not really be constant, as its value will be affected by the magnetic field strength, among other non-constant factors. However, a single value can usually be found which gives a good indication of iron losses. The degree to which we can say k_i is constant depends on the way the magnetic field is provided – it is more constant in the case of the permanent magnet motor than the separately excited one.

The third category of loss is that due to **friction and windage**. There will of course be a friction torque in the bearings and brushes of the motor. The rotor will also have a wind resistance, which might be quite large if a fan is fitted to the rotor for cooling. The friction force will normally be more or less constant. However, the wind resistance force will increase with the square of the speed. To get at the power associated with these forces, we must multiply by the speed, as

$$\text{Power} = \text{torque} \times \text{angular speed}$$

The power involved in these forces will then be

$$\text{Friction power} = T_f \omega \quad \text{and} \quad \text{windage power} = k_w \omega^3 \quad (7.12)$$

where T_f is the friction torque and k_w is a constant depending mainly on the size and shape of the rotor, and whether or not a cooling fan is fitted.

Finally, we address those losses that occur even if the motor is totally stationary, and that vary neither with speed nor torque. In the case of the separately excited motor these are definitely not negligible, as current (and hence power) must be supplied to the coil providing the magnetic field. In the other types of motor to be described in the sections that follow, power is needed for the electronic control circuits that operate at all times. The only type of motor for which this type of loss could be zero is the permanent magnet motor with brushes. The letter C is used to designate these losses.

It is useful to bring together all these different losses into a single equation that allows us to model and predict the losses in a motor. When we do this it helps to combine the terms for the iron losses and the friction losses, as both are proportional to motor speed. Although we have done this for the brushed DC motor, it is important to note that *this equation is true, to a good approximation, for all types of motor*, including the more sophisticated types to be described in later sections.

If we combine Equations (7.10)–(7.12), we have

$$\text{Total losses} = k_c T^2 + k_i \omega + k_w \omega^3 + C \quad (7.13)$$

However, it is usually the motor efficiency η_m that we want. This is found as follows:

$$\eta_m = \frac{\text{output power}}{\text{input power}}$$

$$\eta_m = \frac{\text{output power}}{\text{output power} + \text{losses}} = \frac{T\omega}{T\omega + k_c T^2 + k_i \omega + k_w \omega^3 + C} \quad (7.14)$$

This equation will be very useful when we come to model the performance of electric vehicles in Chapter 8. Suitable values for the constants in this equation can usually be found by experimentation, or by regression using measured values of efficiency. For example, typical values for a permanent magnet motor of the 'Lynch' type that we were considering in Section 7.1.2, that might be fitted to an electric scooter, are as follows:

$$k_c = 0.8 \quad k_i = 0.1 \quad k_w = 10^{-5} \quad C = 20$$

It is useful to plot the values of efficiency on a torque/speed graph, giving what is sometimes known as an 'efficiency map' for the motor, which gives an idea of the efficiency at any possible operating condition. Such a chart is shown in Figure 7.7. MATLAB is an excellent program for producing plots of this type, and in Appendix 1 we have included the script file used to produce this graph.

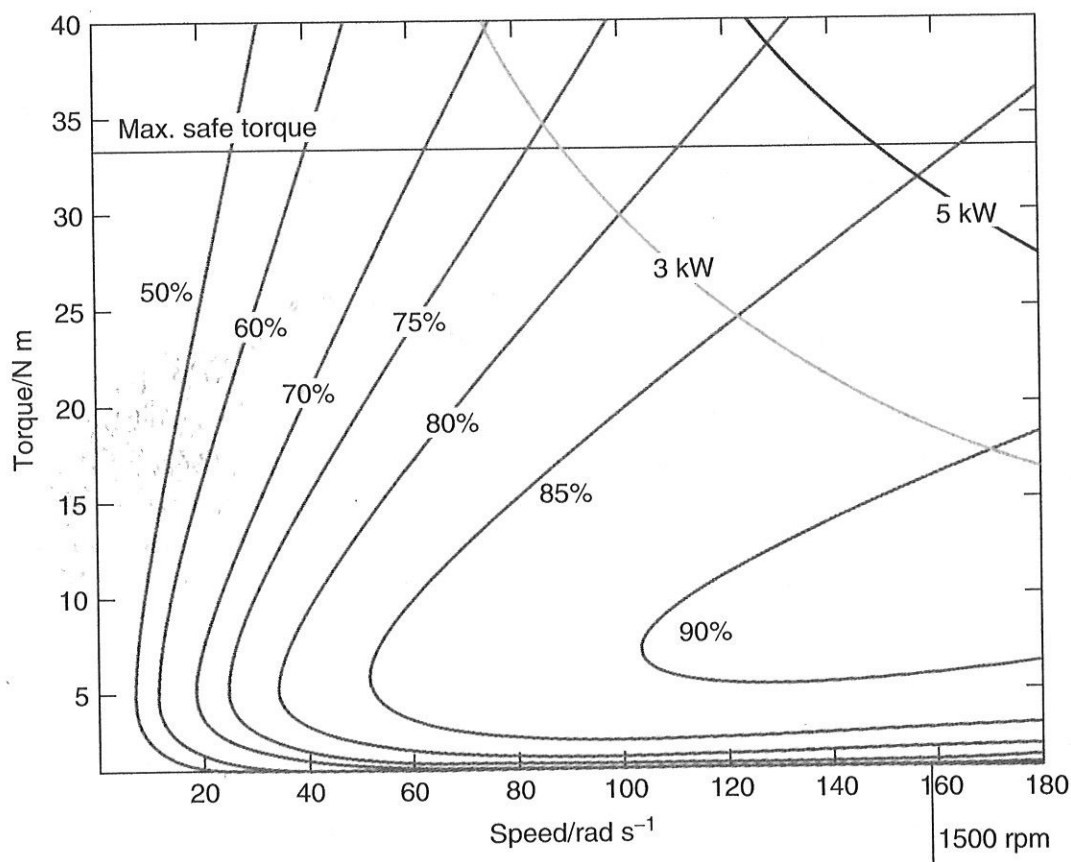


Figure 7.7 Efficiency map for a typical permanent magnet DC motor, with brushes

7.1.6 Motor Losses and Motor Size

While it is obvious that the losses in a motor affect its efficiency, it is not so obvious that the losses also have a crucial impact on the maximum power that can be obtained from a motor of any given size.

Consider a brushed motor of the type we have described in this section. The power produced could be increased by increasing the supply voltage, and thus the torque, as per Figure 7.6. Clearly, there must be a limit to this, as the power cannot be increased to infinity. One might suppose the limiting factor is the voltage at which the insulation around the copper wire breaks down, or some such point. However, that is not the case. The limit is in fact temperature related. Above a certain power the heat generated as a result of the losses, as given by Equation (7.13), becomes too large to be conducted, convected and radiated away, and the motor overheats.

An important result of this is that the key electric motor parameters of *power density* and *specific power*, being the power per unit volume and the power per kilogram mass, are not controlled by electrical factors so much as how effectively the waste heat can be removed from the motor.³

This leads to a very important disadvantage of the classical brushed DC motor. In this type of motor virtually all the losses occur in the *rotor* at the centre of the motor. This means that the heat generated is much more difficult to remove. In the motors to be considered in later sections the great majority of the losses occur on the *stator*, the stationary outer part of the motor. Here they can much more easily be removed. Even if we stick with air cooling it can be done more effectively, but in larger motors liquid cooling can be used to achieve even higher power density.

This issue of motor power being limited by the problem of heat removal also explains another important feature of electric motors. This is that they can safely be driven well in excess of their rated power for short periods. For example, if we take a motor that has a rated power of 5 kW, this means that if it is run at this power for about 30 min, it will settle down to a temperature of about 80 °C, which is safe and will do it no harm. However, being fairly large and heavy, a motor will take some time to heat up. If it is at, say, 50 °C, we can run it in excess of 5 kW, and its temperature will begin to increase quite rapidly. However, if we do not do this for more than about 1 min, then the temperature will not have time to rise to a dangerous value. Clearly this must not be overdone, otherwise local heating could cause damage, nor can it be done for too long, as a dangerous temperature will be reached. Nevertheless, in electric vehicles this is particularly useful, as the higher powers are often only required for short time intervals, such as when accelerating.

7.1.7 Electric Motors as Brakes

The fact that an electric motor can be used to convert kinetic energy back into electrical energy is an important feature of electric vehicles. How this works is easiest to understand in the case of the classical DC motor with brushes, but the broad principles apply to all motor types.

³ Though obviously the losses are affected by electrical factors, such as coil resistance.

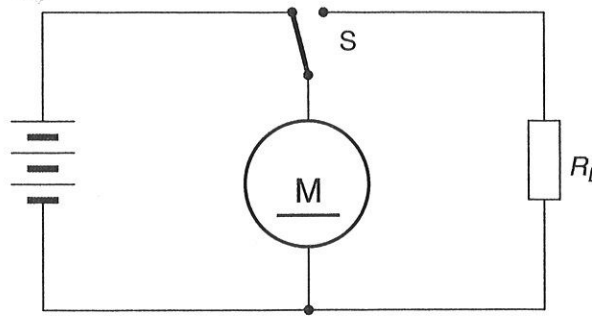


Figure 7.8 Motor circuit with resistor to be used for dynamic braking

Consider Figure 7.8. A DC motor is connected to a battery of negligible internal resistance, and voltage E_s . It reaches a steady state, providing a torque T at a speed ω . These variables will be connected by Equation (7.7). Suppose the switch S is now moved over to the right. The motor will continue to move at the same angular speed. This will cause a voltage to be generated, as given by Equation (7.6). This voltage will be applied to the resistor R_L , as in Figure 7.8, with the current further limited by the resistance of the rotor coil (armature). The result is that the current will be given by the formula

$$I = \frac{K_m \Phi \omega}{R_a + R_L} \quad (7.15)$$

This current will be flowing *out* of the motor, and will result in a negative torque. The value of this torque will still be given by the torque equation produced earlier as Equation (7.4). So, the negative torque, which will slow the motor down, will be given by the equation

$$T = -\frac{(K_m \Phi)^2 \omega}{R_a + R_L} \quad (7.16)$$

We thus have a negative torque, whose value can be controlled by changing the resistance R_L . The value of this torque declines as the speed ω decreases. So, if R_L is constant we might expect the speed to decline in an exponential way to zero.

This way of slowing down an electric motor, using a resistor, is known as 'dynamic braking'. Note that all the kinetic energy of the motor (and the vehicle connected to it) is ultimately converted into heat, just like normal friction brakes. However, we do have control of where the heat is produced, which can be useful. We also have the potential of an elegant method of controlling the braking torque. Nevertheless, the advance over normal friction brakes is not very great, and it would be much better if the electrical energy produced by the motor could be stored in a battery or capacitor.

If the resistor of Figure 7.8 were replaced by a battery, then we would have a system known as 'regenerative braking'. However, the simple connection of a battery to the motor is not practical. Suppose the voltage of the battery is V_b , and the motor is turning at speed ω ; then the current that will flow out of the motor will be given by the equation

$$I = \frac{V}{R} = \frac{K_m \Phi \omega - V_b}{R_a} \quad (7.17)$$

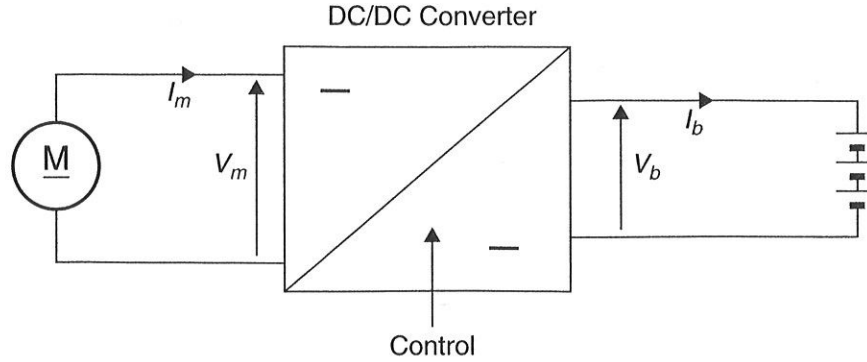


Figure 7.9 Regenerative braking of a DC motor

The slowing down torque will be proportional to this current. Once the value of ω reaches the value where the voltage generated by the motor ($=K_m \Phi \omega$) reaches the battery voltage, then there will be no more braking effect. Unless the battery voltage is very low, then this will happen quite soon. If the battery voltage is low, then it will be difficult to use the energy stored in it, and the braking effect might well be far too strong at high speeds, with the current given by Equation (7.16) being impracticably large.

A solution lies in a voltage converter circuit as in Figure 7.9. The converter unit, known as a 'DC/DC converter', draws a current from the motor I_m , which will occur at a voltage V_m . This voltage V_m will change with motor (and hence vehicle) speed. The current I_m will change with the desired braking torque. The DC/DC converter will take this electrical power ($= V_m \times I_m$) and put it out at an increased voltage (and reduced current) so that it matches the rechargeable battery or capacitor that is storing the energy. The battery might well be the same battery that provided the electricity to make the motor go in the first place. The key point is that the motor voltage might be considerably *lower* than the battery voltage, but it can still be providing charge to the battery.

Such a converter circuit sounds as if it is a little 'too good to be true' and the possibility of it remote. It seems to be like getting water to flow up hill. However, such circuits are quite possible with modern power electronics. We are not producing power, we are exchanging a low voltage and high current for a higher voltage and lower current. It is like a transformer in AC circuits, with the added facility of being able to vary continuously the ratio of the input and output voltages.

Although voltage converter circuits doing what is described above can be made, they are not 100% efficient. Some of the electrical power from the braking motor will be lost. We can say that

$$V_b \times I_b = \eta_c \times V_m \times I_m \quad (7.18)$$

where η_c is the efficiency of the converter circuit.

We have thus seen that a motor can be used to provide a controllable torque over a range of speeds. The motor can also be used as a brake, with the energy stored in a battery or capacitor. To have this range of control we need power electronics circuits that can control the voltages produced. The operation of these circuits is considered in the section that follows.