

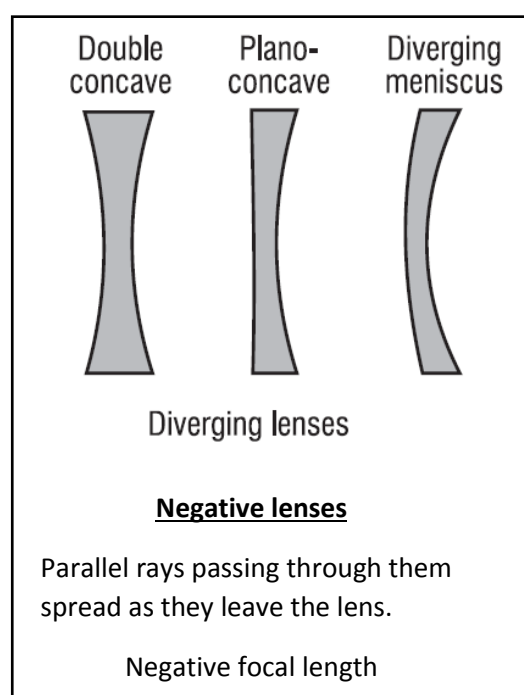
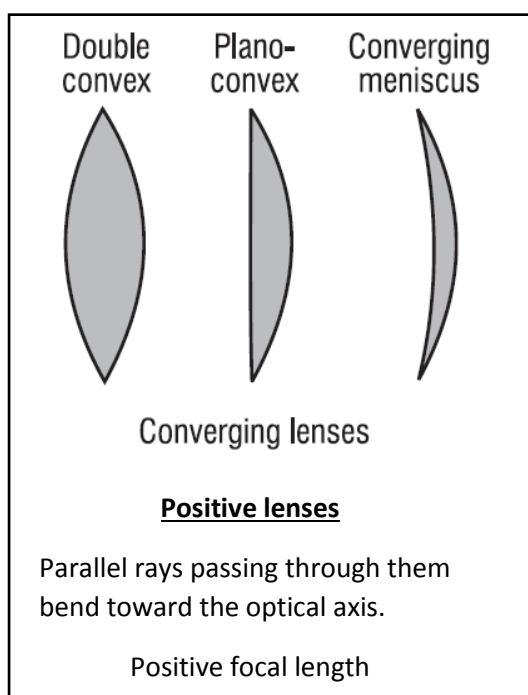
THIN LENSES

Lenses are at the heart of many optical devices, camera, microscopes, binoculars, telescopes etc. Just as the law of reflection determines the imaging properties of mirrors, Snell's law of refraction determines the imaging properties of lenses. In this section we shall look first at the types and properties of lenses, then the use of graphical ray-tracing techniques to locate images, and finally use of mathematical formulas to locate the size, orientation and position of images in simple lens systems.

I. TYPES OF LENSES

If the axial thickness of a lens is small compared with the radii of curvature of its surfaces, it can be treated as a **thin** lens. Ray tracing techniques and lens formulae are relatively simple for thin lenses. If the thickness of a lens is not negligible compared with the radii of curvature of its faces, it must be treated as a thick lens. Ray tracing techniques and lens imaging formulae are more complicated for thick lenses.

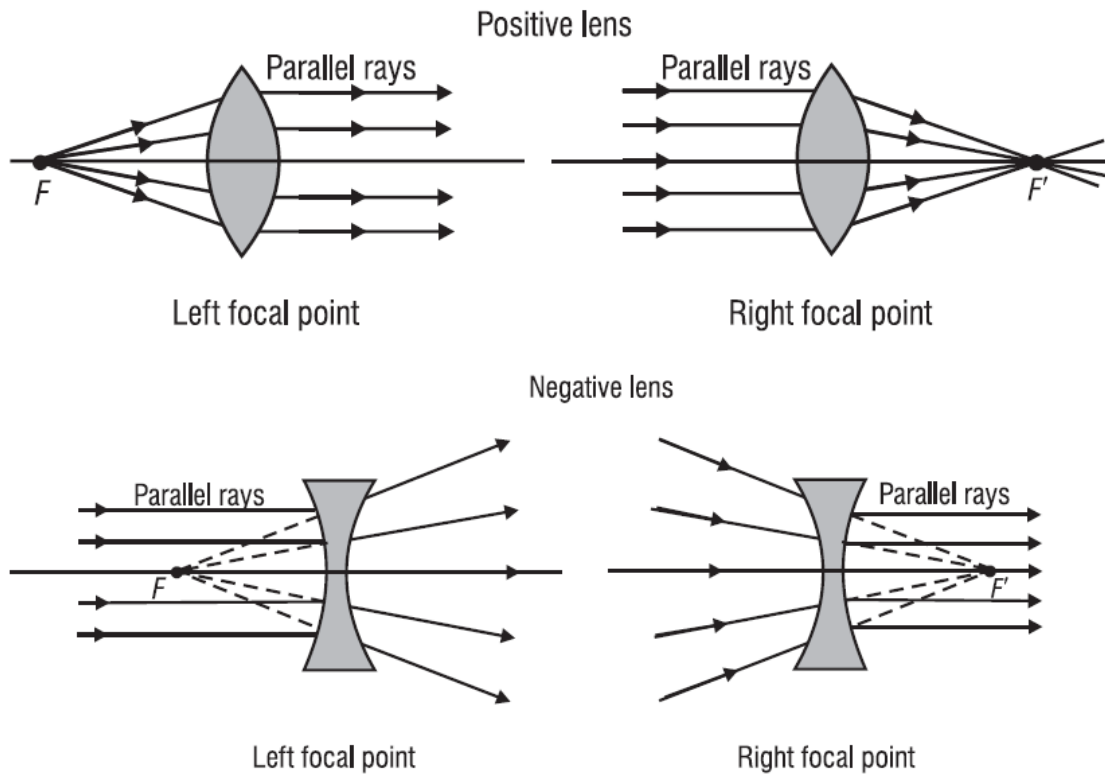
1. Converging and Diverging thin lenses



2. Focal points of thin lenses

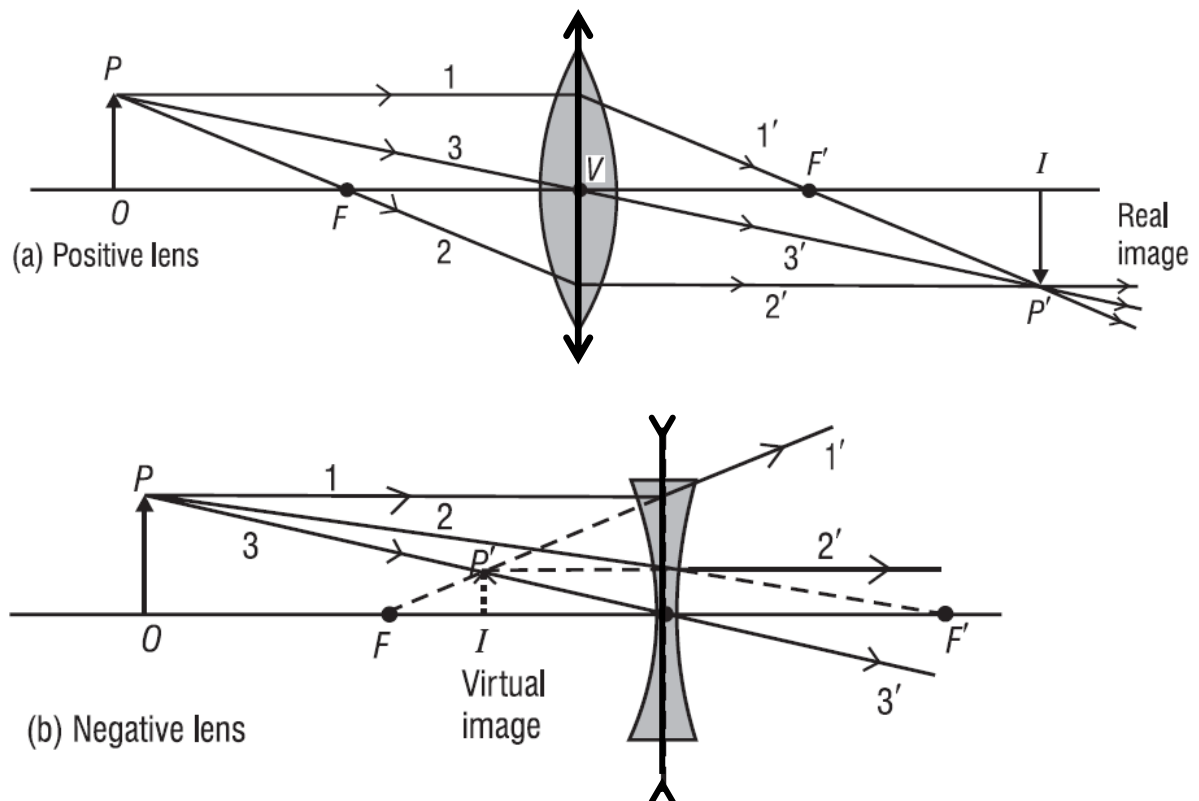
For thin lenses, there are two focal points, symmetrically located on each side of the lens, since light can approach from either side of the lens.

The following sketches indicate the role that the two focal points play, for positive lenses and negative lenses. Study these figures carefully.



II. IMAGE LOCATION BY RAY TRACING

The following figures show **three key rays** that are used to locate an image point corresponding to a given object.



Any two rays, drawn from object tip P, will locate the corresponding image point P'. This is normally sufficient to draw the entire image.

For a positive lens:

- Incoming ray 1 parallel to optical axis transmits toward focal point F'.
- Incoming ray 2 passing through focal point F emerges parallel to optical axis.
- Incoming ray 3 passing through lens vertex (centre) V is not deviated.

For a convex mirror:

- Incoming ray 1 parallel to optical axis transmits away from optical axis as if to come from focal point F.
- Incoming ray 2 headed towards focal point F' emerges parallel to optical axis.
- Incoming ray 3 passing through lens vertex (centre) V is not deviated.

Example:

Determine size and location of the image your eye would see after the following two lens system,

- ✓ Lens 1: converging lens of focal length 2 cm.
- ✓ Lens 2: same lens as lens 1.
- ✓ Object location: 6 cm before lens 1.
- ✓ Distance between lenses: 7 cm.
- ✓ Object height: 2 cm.

III. LENS FORMULAE FOR THIN LENSES

As with mirrors, convenient formulas can be used to locate the image mathematically. The derivation of such formulae can be easily carried out through geometry of triangles considerations.

See Khan Academy videos "Object image and focal distance relationship (proof of formula)" + "Object image height and distance relationship".

1. Equations for thin lenses

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Where,

- d_o is the object distance (from object to lens vertex V)
- d_i is the image distance (from image to lens vertex V)
- f is the focal length (from either focal point F or F' to lens vertex V)

The magnification m produced by a thin lens is given by,

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

Where,

- m is the magnification (ratio of image size to object size)
- h_i is the transverse size of the image
- h_o is the transverse size of the object

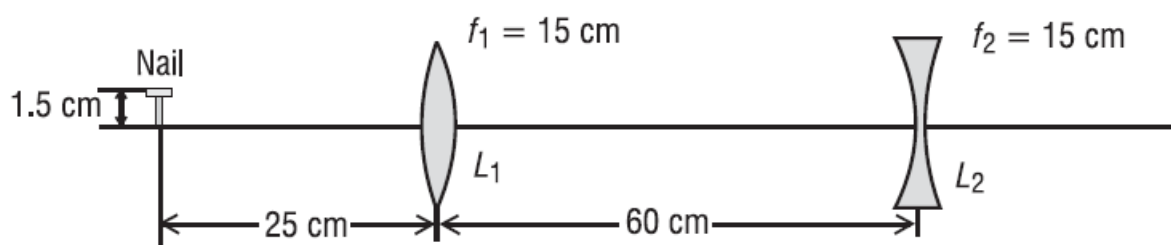
2. Sign convention

The sign convention to be used in conjunction with the thin lenses equations is as follows,

- Light travels initially from left to right toward the lens.
- Object distance d_o is **positive** for **real** objects located to the **left** of the lens.
- Object distance d_o is **negative** for **virtual** objects located to the **right** of the lens.
- Image distance d_i is **positive** for **real** images formed to the **right** of the lens.
- Image distance d_i is **negative** for **virtual** images formed to the **left** of the lens.
- The focal length f is **positive** for **converging** lens.
- The focal length f is **negative** for **diverging** lens.
- Transverse distances h_o and h_i are positive above the optical axis, negative below.

IV. EXAMPLES

1. Example 1



A two lens system is made up of a converging lens followed by a diverging lens, each of focal length 15 cm. The system is used to form an image of a short nail, 1.5 cm high, standing erect, 25 cm from the first lens. The two lenses are separated by a distance of 60 cm as shown in above figure. Locate the final image, determine its size, and state whether it is real or virtual, erect or inverted.

2. Example 2

Let's consider a magnifying glass that is equivalent to a thin converging lens. It has a focal length of 0.1 m. One observes a journal with letters having a height $h_o = 2 \times 10^{-3} \text{ m}$.

What is the nature and height of the image when the magnifying glass is at a distance $d_o = 7 \text{ cm}$ from the journal?

