

Assignment Title: Vehicle Dynamics Analysis

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Abstract - This report covers standard dynamic manoeuvres in the VI-Grade software to compare different spring stiffnesses and damping rates as well as evaluate the performance around a circuit. Furthermore, there is the derivation of the equation of motion for both the sprung and unsprung mass. Using this equation to develop a Simulink and input appropriate acceleration values in Simulink from VI-Grade.

Equation 2

$$C_{crit} = 2 * \sqrt{k * m} \text{ to}$$

Equation 3

$$\xi = \frac{C}{2 * \sqrt{k * m}}.$$

These equations then allow me to identify different damping rates while changing the springs stiffness by the same rate between setups. As “the average damping ratio for body motion is typically in the range of 0.2 to 0.6” (Dixon, 1996). From these values I have chosen to use damping ratio of 0.6, as an under damped coefficient, this is due to the vehicle model I am using in the VI-Grade being a sports car, which is typically of a higher performance than a road car. Moreover, I will be using coefficients of 1 to identify a critically damped set up and using 1.4 to identify the damping rates for an over damped setup. Using the previous equations, as well as the damping coefficients mentioned I have calculated the damping rates needed to keep the same damping coefficients for changing spring rates between setups.

1. INTRODUCTION (Size 11, Cambria font)

In this report, I’ll be conducting a range of standard dynamic manoeuvres to evaluate different suspension spring and damping setups. This then allows me to assess the performance and tire response of an example vehicle in the VI-Grade. Additionally, I’ll be conducting a performance lap simulation of Pembrey Circuit to assess the impacts of spring rates and damping on the vehicles performance and handling.

For Task 2 I will be deriving and explaining equations of motion used to later develop a half car model in Simulink that represents the car used in the VI-Grade. Using the half car model developed, I will be simulating the vehicle’s ride performance under different bump inputs for various spring and damper setups. Finally, I will be further developing my Simulink model to include acceleration inputs to compare results to data collected from the VI-Grade.

2. VI-GRADE

2.1 Identifying Setup Changes

To begin, I had to identify variables that had been changing during the great VI grade setup. To do this I had rearranged the equation

Equation 1

$$\xi = C/C_{crit} \text{ and}$$

Setup Type	Spring Rate (N/m)	Spring Rate (N/m)	Damping Rate (Ns/m)	Damping Rate (Ns/m)	Damping Coefficient
Under Damped					
Softer	47500	52500	6819.970674	7169.937238	0.6
Baseline	95000	105000	9644.895023	10139.82248	
Harder	190000	210000	13639.94135	14339.87448	
Critically Damped					
Softer	47500	52500	11366.61779	11949.8954	1
Baseline	95000	105000	16074.82504	16899.70414	
Harder	190000	210000	22733.23558	23899.79079	
Over Damped					
Softer	47500	52500	15913.26491	16729.85356	1.4
Baseline	95000	105000	22504.75505	23659.5858	
Harder	190000	210000	31826.52981	33459.70711	

Table 1- Damping Rate Calculations

2.2 Standard Dynamic manoeuvres

2.2.1 Acceleration

The first standard manoeuvres I had conducted is straight line acceleration from rest. The standard vehicle model I had used is the sports car which has a full car sprung mass of 1360kg but has been run at 680kg to model as a half car, however keeping the unsprung mass the same. Furthermore, as shown in table 1, between setups I am changing spring rates between a soft medium and hard setups for each damping coefficients. simulated sports car in the VI-Grade

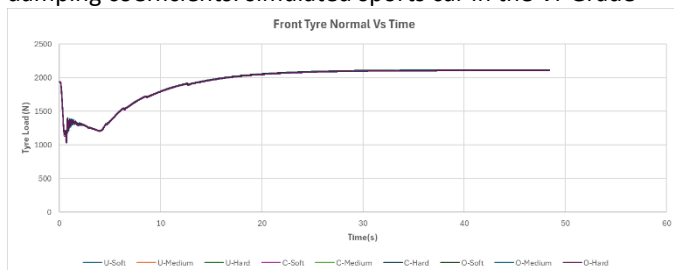


Figure 1- Graph Showing Front Tyre Load Vs Time

The Graph above is showing the front tyre loads of the simulated sports car in the VI-Grade. The graph shows that for every setup the general rate at which the car is accelerating is the same. However, differences are shown in the initial phase of the acceleration as the set up with largest change in tyre load is the hard over damped setup. This is due to more force having to be applied to the spring to get the same compression upon initial squat but being over damped means that it takes longer for the car to settle back into its initial position while load is being applied and unapplied.

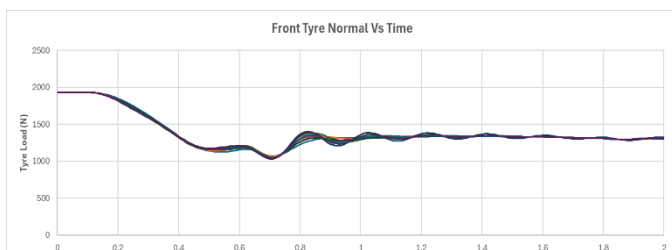


Figure 3- Graph Showing the front Tyre Load Vs Time for the First 2s of Acceleration

The graph above shows that during the first 2s of acceleration where the car is under pitch the harder spring rate setups ups have larger peaks than the softer spring rate setups.

Looking between the different damping coefficients of under, critically and under damped:

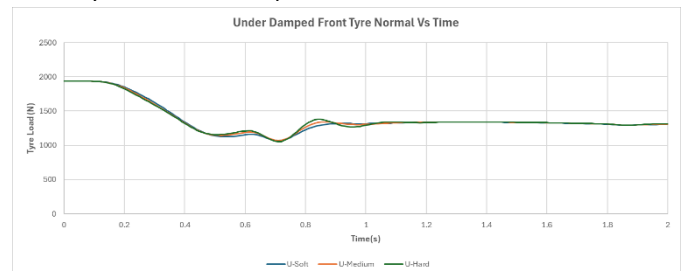
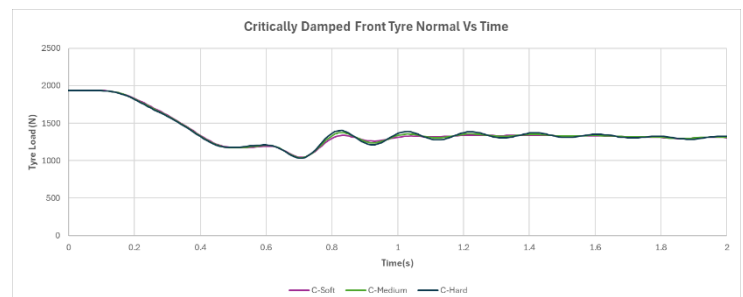


Figure 4- Graph showing the Under Damped Front Tyre Load

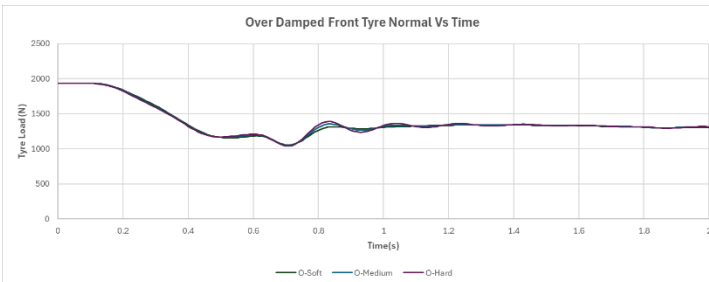
The underdamped setups show that they have very little variation between peak and minimum tyre loads during the initial acceleration while quickly resetting to the natural ride position due the damper encouraging the spring to reset by taking energy from it.

Figure 5- Graph showing the Critically damped setups



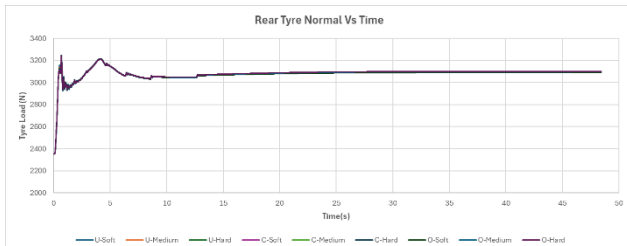
The graph above is showing the critically damped setups during the initial acceleration. The graph is showing the loads varying constantly throughout the initial acceleration. Moreover, by the end of the initial phase of acceleration the tyre load is still oscillating showing that the damper is having very little to no effect on the spring, so it is using the frictional forces created to dissipate energy.

Figure 2-Graph showing the Over Damped tyre Loads



The graph above is showing the tyre load response for the over damped setups. The graph is showing that for these setups take longer for the car to return to its natural ride height than the under damped setups. This is due to the damper not being able to dissipate enough kinetic energy from the spring in the time required.

Figure 6-Graph showing the rear Normal Tyre Loads vs Time



The graph above is showing the rear normal tyre loads of the simulated VI-Grade model. The graph shows that like the front tyre loads the over damped hard spring rate set up has the highest peaks of tyre load. This is a result of during the initial phase of acceleration the car is pitching rearwards meaning that there is high compression on the rear. Consequently, the harder setup means that to get the same quantity of compression from the springs more force is required.

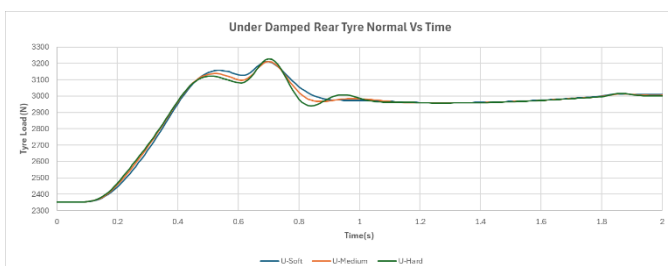
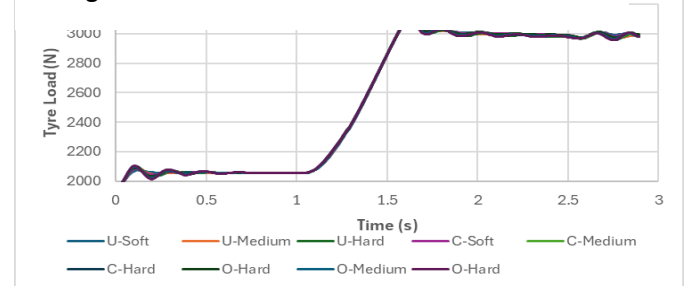


Figure 8- Under Damped Rear Tyre Loads

The graph above is showing the under damped rear tyre loads during the initial acceleration phase. The graph shows that for the other setups the harder spring rates once more has the largest peaks when compared to the medium and softer spring rates. This results in slower initial acceleration due to the car taking longer in returning to the natural ride height.

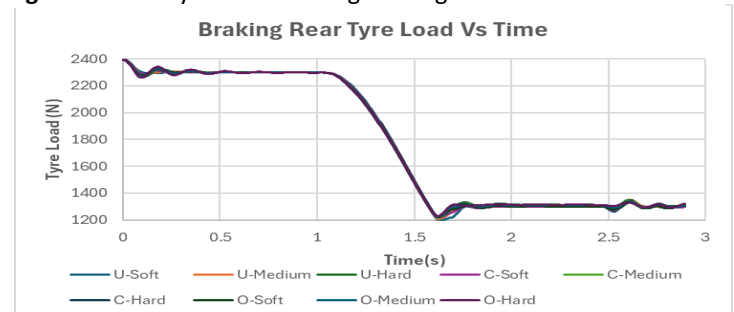
2.2.2 Braking

Figure 9- Graph Showing Front Tyre Loads During Braking



The Graph above is showing the front tyre loads during braking. The graph shows that once the braking has started the harder spring setups have greater changes in the tyre loads due to needing more force to compress the springs. However, after 1.5s the medium and the softer spring rate setups have a larger reduction in the tyre loads than the harder springs. This is a result of the normal force being moved towards the front of the car and the harder springs requiring more force to achieve the same level of damping.

Figure 7- Rear Tyre Loads During Braking

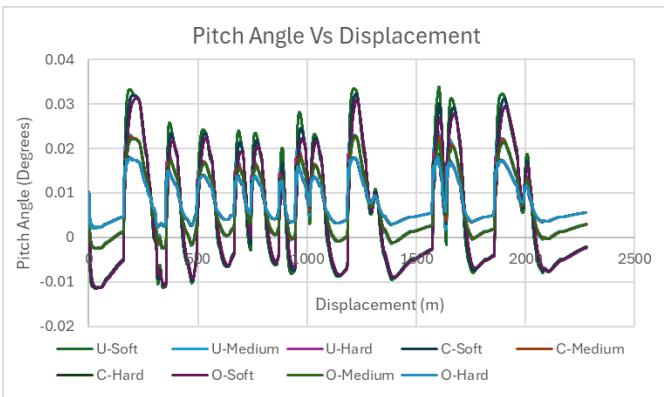


The graph above is showing the rear tyre loads during braking. Similar to the front the harder spring setups initially have the greatest changes in tyre loads. However, as the load transfer shifts more rearwards, the under damped soft set ups has the reduction in tyre loads kept

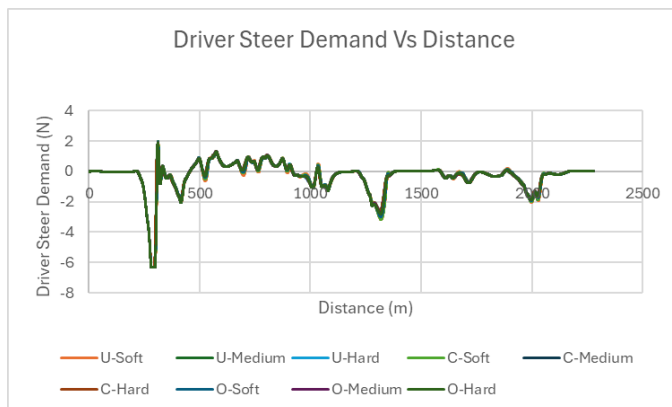
lower for the longest period. This is a results of the softer springs requiring less force to be applied to reach the same compression as the other setups as well as being underdamped which is helps to reduce energy dissipated from the compression requiring less changes in tyre load.

2.3 Prembrey

Figure 10- graph showing pitch angle Vs distance



The graph above is showing the pitch angle against displacement. The graph is showing that during braking events the softer spring rate setups have the largest pitch angle whereas, the harder spring rate setups have the least variation in pitch angle. This is caused by the same brake force through each setup in the corners therefore meaning the softer springs will have the largest displacement causing greater changes in pitch angle. This is in contrast to the harder setups which mean that the same force would cause a small change in spring length and therefore pitch change relative to the softer springs.



This is then shown in the driver steer demands as the car is pitching more with the softer spring rate setups the

simulation is having to provide more steering demand to compensate for the higher changes in pitch angle .

3 Simulink

3.1.1 4DOF Model

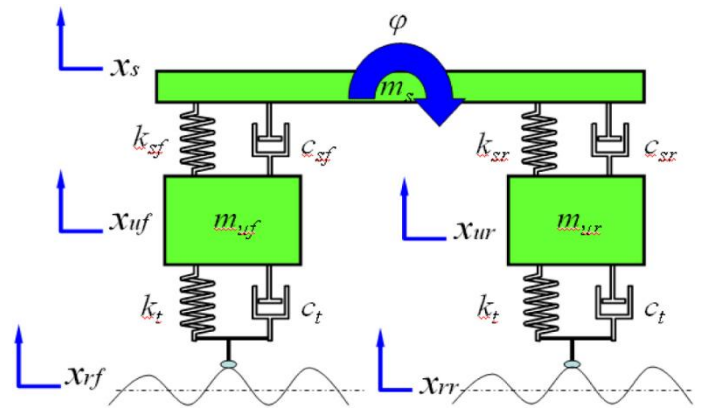


Figure 11 - 4 Degree of Freedom Model (Talukdar, et al., 2012)

The four degrees of freedom model is used to show the relationship between the sprung and sprung mass with the inclusion of springs and dampers. Moreover, this model simulates both the pitch and heave a car would experience during both bump and rebound.

3.1.2 derivation of equation

Spring rate is defined as the normal force required to move a spring a set distance whereas dampers are in relation to the spring's velocity therefore

Equation 4- Spring Rate

$$F = kx$$

Equation 5- Damping Rate

$$F = C\dot{x}$$

Table 2- Spring and Dampers Extensions

Springs	Extension	Dampers	Extension
kf	$kf(Zs - Zf - a\theta)$	Cf	$Cf(\dot{Z}s - \dot{Z}f - a\dot{\theta})$
kr	$kr(Zs - Zr + b\theta)$	Cr	$Cr(\dot{Z}s - \dot{Z}r + b\dot{\theta})$
ktf	$ktf(Zf - yf)$	Ctf	$Ctf(\dot{Z}r - \dot{y}f)$
ktr	$ktr(Zr - yr)$	Ctr	$Ctr(\dot{Z}r - \dot{y}r)$

The above table is showing the extensions the springs and dampers undergo during bump and rebound. For the tyre spring and damping rate, the extension has been calculated by the distance that the unsprung mass has moved upwards taking away the distance at which the tyre is pulling the car back down to the floor by. Whereas, the sprung spring and damping rates extension is calculated by how much the sprung mass is lifted from its natural ride taking away the distance that the unsprung mass moves down as well as the pitch angle created.

The sprung mass's extension is then denoted by $kf+kr+Cf+Cr$ to give the equation ($\ddot{Z} = Z\text{DotDot}$):

Equation 6- $-M\ddot{Z}$

$$\begin{aligned}
 &kf(Zs - Zf - a\theta) + kr(Zs - Zr + b\theta) \\
 &\quad + Cf(\dot{Z}s - \dot{Z}f - a\dot{\theta}) \\
 &\quad + Cr(\dot{Z}s - \dot{Z}r + b\dot{\theta})
 \end{aligned}$$

Next, the inertia of the Sprung's Mass's pitch is denoted by the rotational moment of $kf+kr+Cf+Cr$ to give the equation

Equation 7- $-I*\ddot{\theta}$

$bkr(Zs - Zr + b\theta) - akf(Zs - Zf - a\theta) + bCr(\dot{Z}s - \dot{Z}r + b\dot{\theta}) - Cf(\dot{Z}s - \dot{Z}f - a\dot{\theta})$ the introduction of the coefficients and are because the inertia is treated as a moment from the centre of the car.

Finally, the unsprung masses extension is denoted by $ktr+kr+Ctr+Cr$ or $ktf+kf+Ctf+Cf$ to give the equations

Equation 8- $-mf\ddot{Z}$

$$\begin{aligned}
 &ktf(Zf - yf) - kf(Zs - Zf - a\theta) + Cf(\dot{Z}f - \dot{y}f) \\
 &\quad - Cf(\dot{Z}s - \dot{Z}f - a\dot{\theta})
 \end{aligned}$$

And

Equation 9- $-mr\ddot{Z}$

$$\begin{aligned}
 &ktr(Zr - yr) - kr(Zs - Zf + b\theta) + Cr(\dot{Z}f - \dot{y}f) \\
 &\quad - Cr(\dot{Z}s - \dot{Z}f + b\dot{\theta})
 \end{aligned}$$

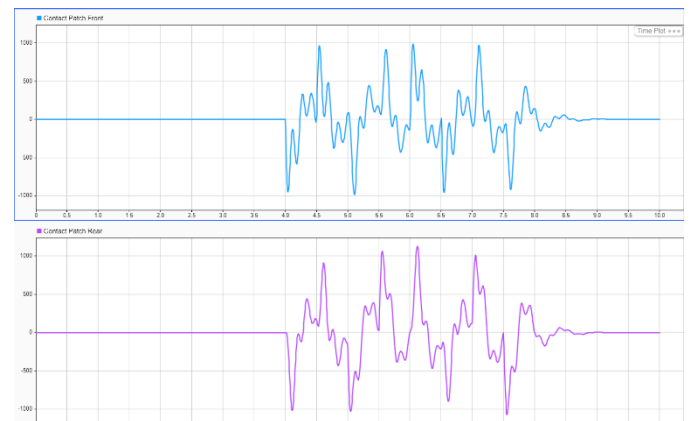
3.2 Simulink Model

The simulink model will be in the appendix with the workspaces:

Acceleration, differentials, Bump, Zs, Theta, Zf and Zr

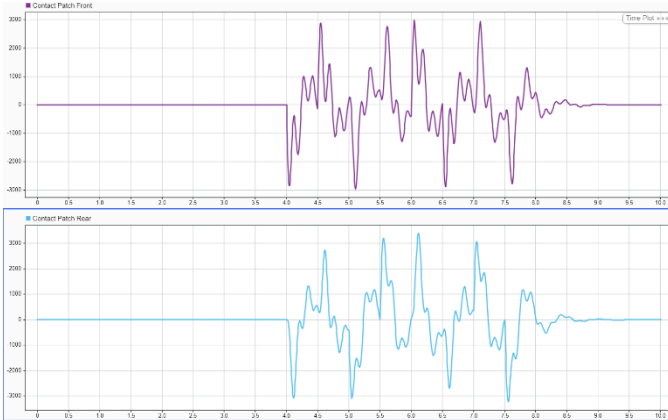
3.3 Bump Inputs

Figure 12-Bump @.1



At bump is equal to 0.1 the peak are around 1000 to -1000N

Figure 13-Bump @.3



At bump is equal to 0.3 the peak tyre loads is simulated around 3000 and -3000N with loads oscillating between 1000 and -1000N compared to 0.1 where it is between 500 and -500N

Figure 14-Bump@.5



The peaks are around 5000N and -5000N and the range when both are in bump between 2000N and -2000N.

3.4 Acceleration Inputs

Figure 15- Acceleration input in simulink

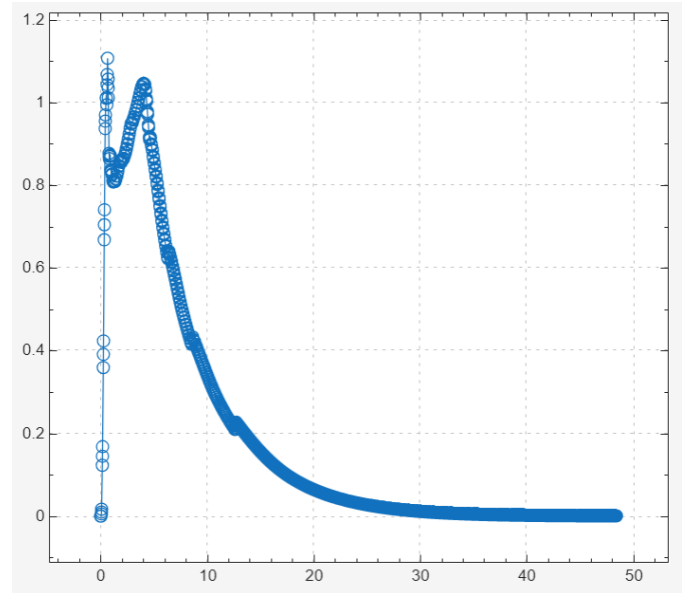


Figure 16-Critically Damped Medium Acceleration Simulink (Front)

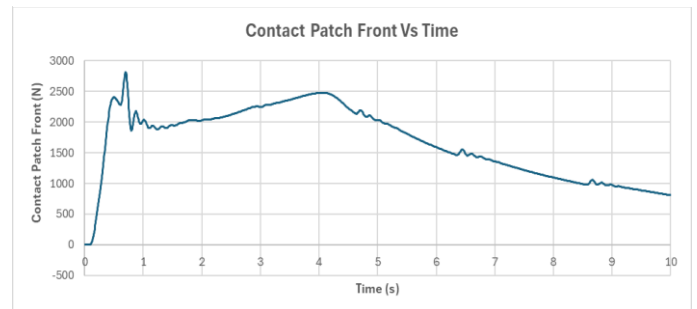
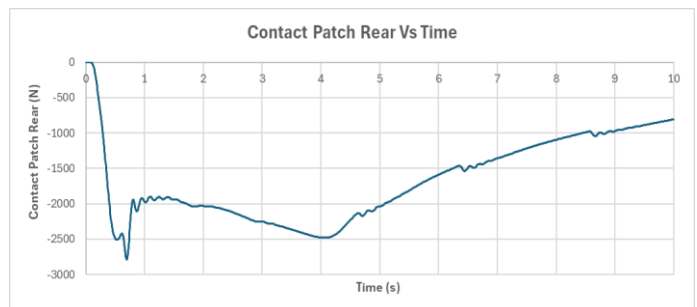
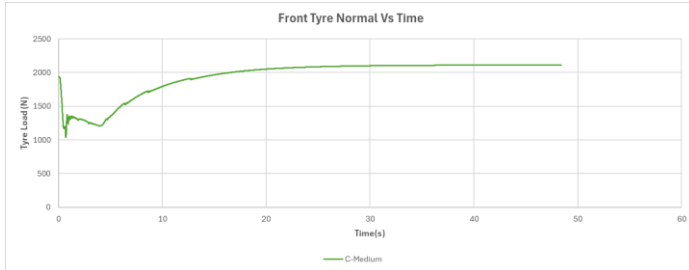


Figure 17-Critically Damped Medium Acceleration Simulink (Rear)



Above are the Simulated values of tyre load data from Simulink for the critically damped coefficient and medium spring rate setup.



more visible. The rear tyre loads both have a peak of around 3000N but with the VI-Grade having around 3100N and Simulink having around 2750N

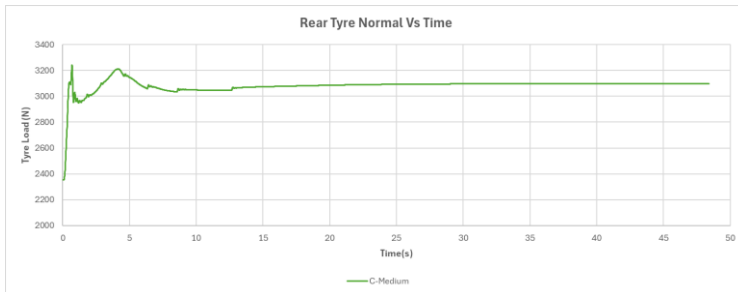


Figure 19-Critically Damped Medium Acceleration VI-Grade (Rear)

To begin the two sets of data are in the opposite direction due to which direction the Z axis is in. the Simulink model has clearer representation of damping due to the peaks being

Appendix

Figure 20-1DOF

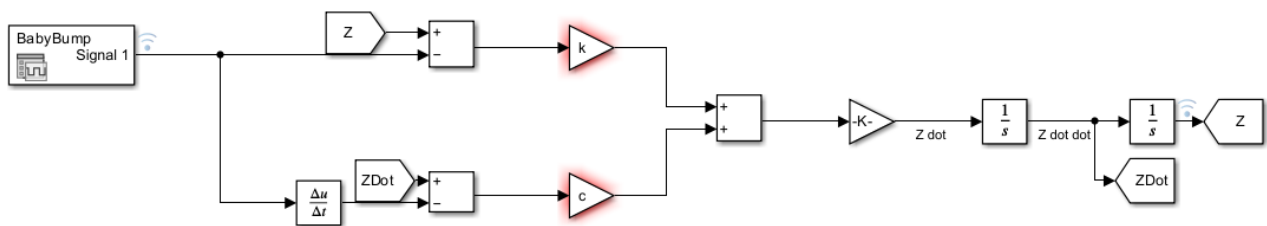


Figure 21-2DOF Sprung

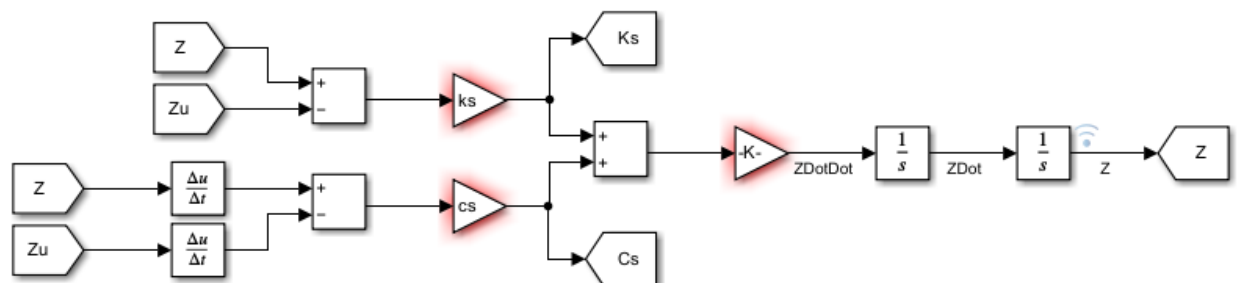


Figure 22-2DOF Unsprung

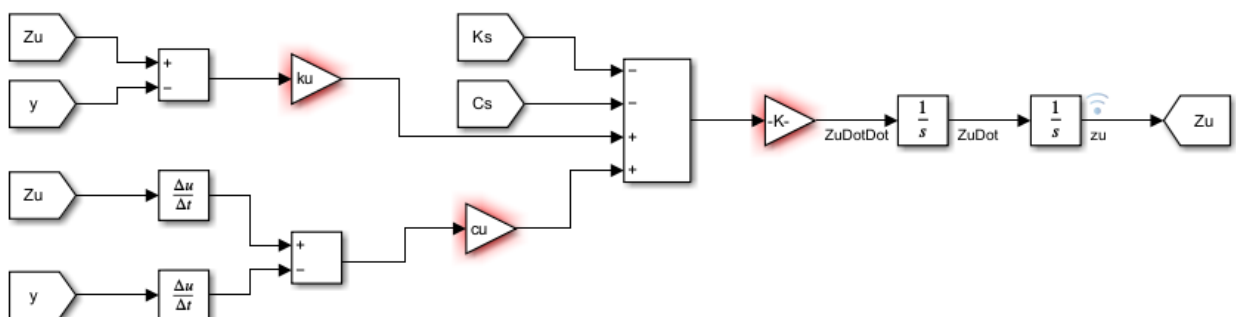


Figure 23-4DOF Acceleration

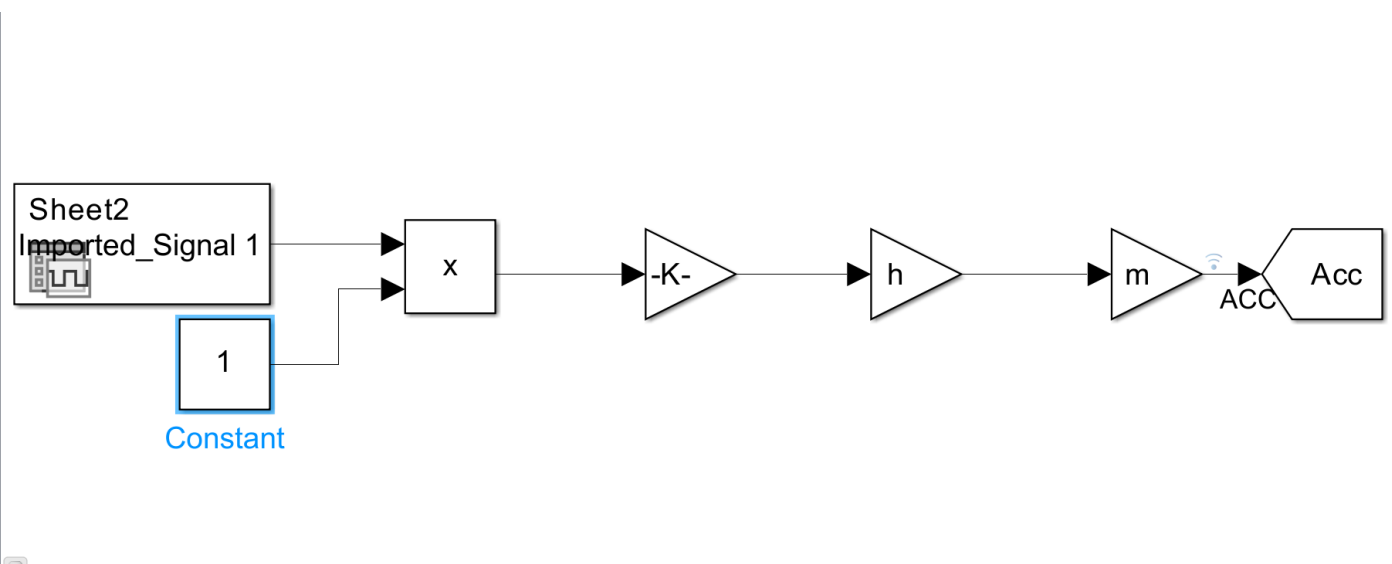


Figure 24-4DOF Bump

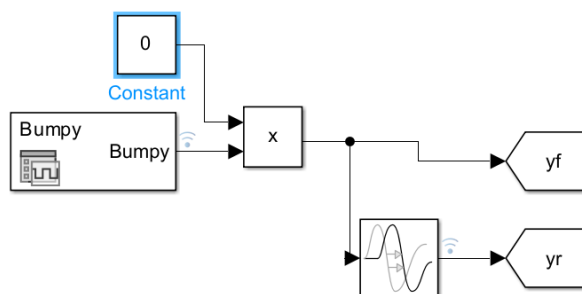


Figure 25- Derivatives

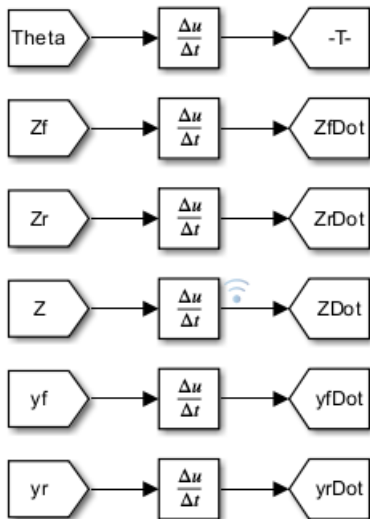


Figure 26-4DOF Theta

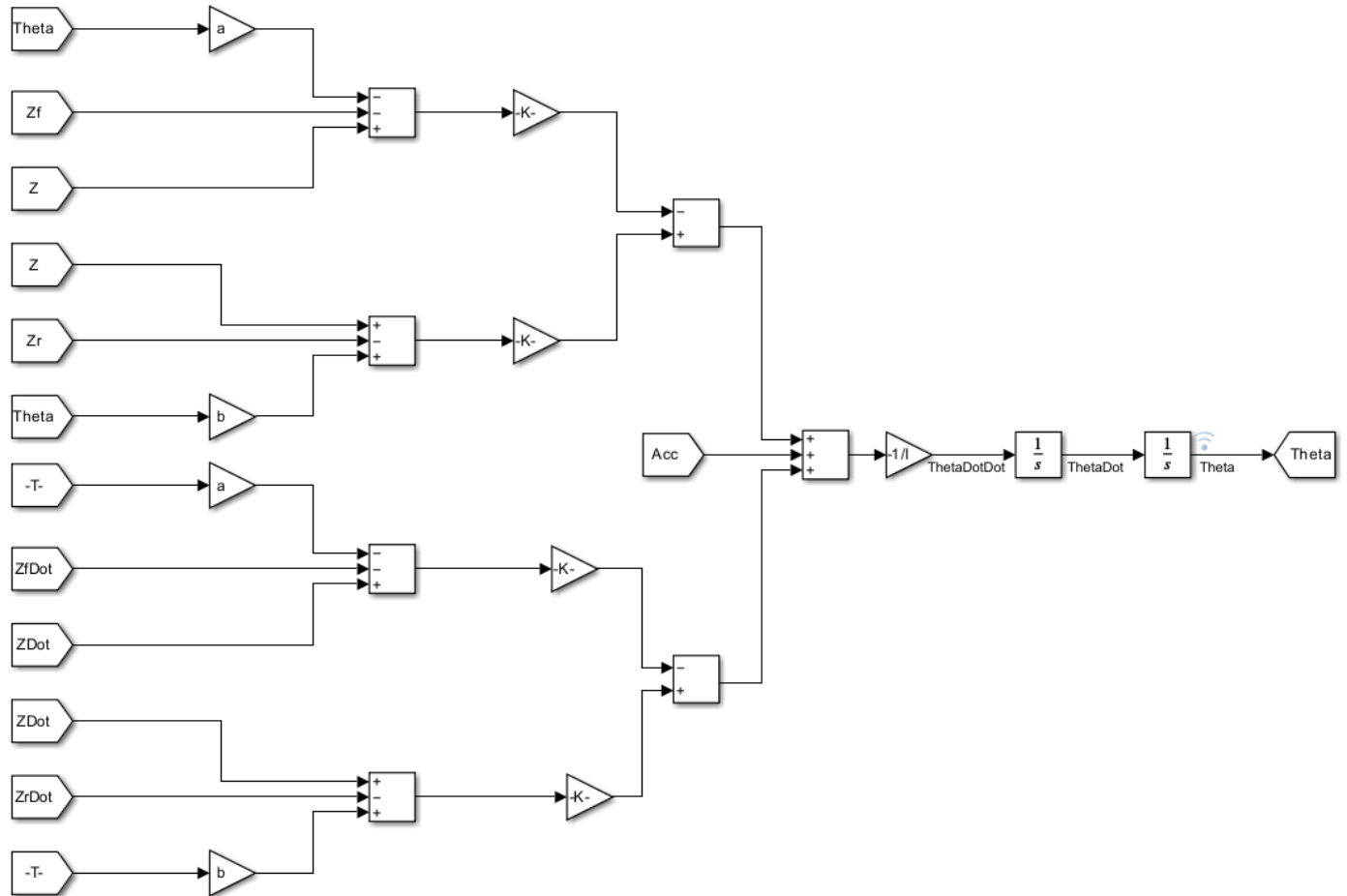


Figure 27- Unsprung Front

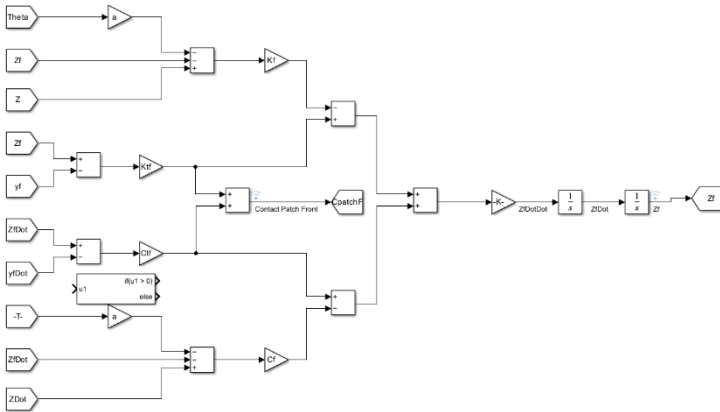


Figure 28-Unsprung Rear

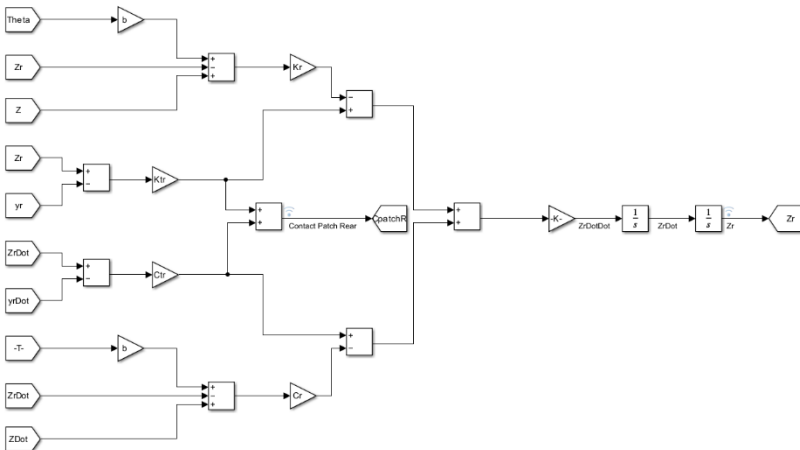
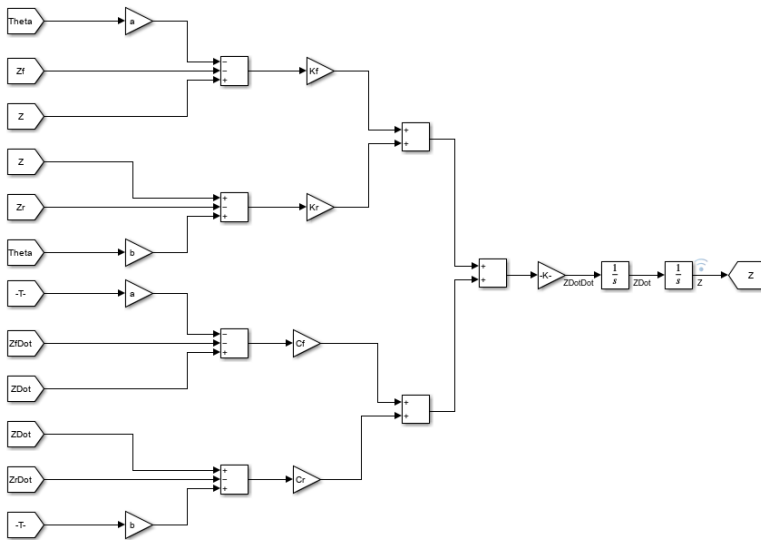


Figure 29- Sprung Mass



Bibliography

Dixon, J., 1996. *Tires, suspension and handling*. 2nd ed. London: Society of Automotive Engineering.

Talukdar, S. et al., 2012. *Mathematical Modeling in Vehicle Ride Dynamics*. [Online]

Available at: <https://saemobilus-sae-org.ezproxy.uwtsd.ac.uk/papers/mathematical-modeling-vehicle-ride-dynamics-2012-01-0056#view>

[Accessed 7 December 2025].