

Stress Analysis

9/5/22

$$A=2$$

$$B=0$$

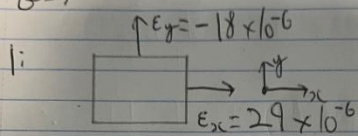
$$C=0$$

$$D=0$$

$$E=8$$

$$F=8$$

$$G=9$$



$$\epsilon_x = 29 \times 10^{-6}$$

$$\epsilon_y = -18 \times 10^{-6}$$

$$\nu = 0.28$$

$$E = 170 \text{ GPa} \rightarrow 170 \times 10^9 \text{ Pa}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_x = \frac{170 \times 10^9}{1-0.28^2} (29 \times 10^{-6} + 0.28 \times -18 \times 10^{-6})$$

$$\sigma_x = 4.42 \times 10^6 \text{ Pa}, \therefore \text{Tensile}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} (-18 \times 10^{-6} + 0.28 \times 29 \times 10^{-6})$$

$$\sigma_y = -1.82 \times 10^6 \text{ Pa}, \therefore \text{Compressive}$$

$\sigma_x = 88 \text{ MPa} \rightarrow 88 \times 10^6 \text{ Pa, tensile} \therefore +88 \times 10^6 \text{ Pa}$
 $\sigma_y = 49 \text{ MPa} \rightarrow 49 \times 10^6 \text{ Pa, compressive} \therefore -49 \times 10^6 \text{ Pa}$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

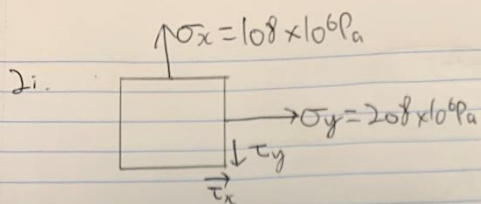
$$\epsilon_x = \frac{1}{170 \times 10^9} (88 \times 10^6 - (0.28 \times -49 \times 10^6))$$

$$\epsilon_x = 598.35 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x)$$

$$\epsilon_y = \frac{1}{170 \times 10^9} (-49 \times 10^6 - (0.28 \times 88 \times 10^6))$$

$$\epsilon_y = -433.18 \times 10^{-6}$$



$$\sigma_x = 108 \times 10^6 \text{ Pa}$$

$$\sigma_y = 208 \times 10^6 \text{ Pa}$$

$$\tau_{xy} = 125 \text{ MPa} \rightarrow 125 \times 10^6 \text{ Pa}$$

$$\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_1 = \frac{(108 \times 10^6 + 208 \times 10^6)}{2} + \frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 292.63 \times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_2 = \frac{(108 \times 10^6 + 208 \times 10^6)}{2} - \frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_2 = 23.37 \times 10^6 \text{ Pa}$$

$$2ii. \tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

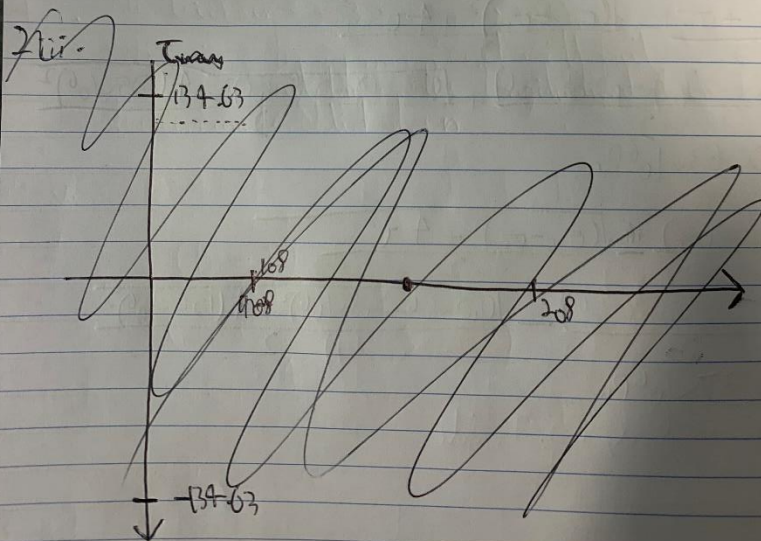
$$\tau_{max} = \frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

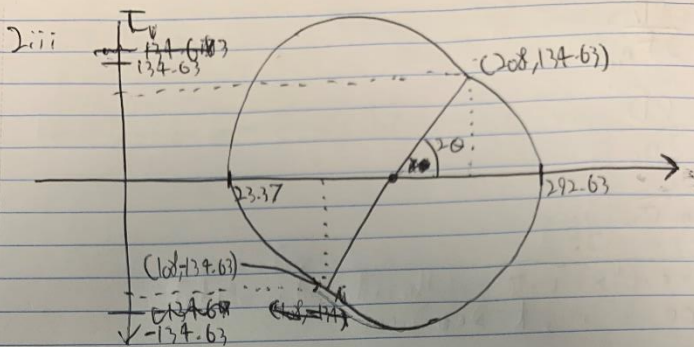
$$\tau_{max} = 134.63 \times 10^6 \text{ Pa}$$

$$\tau_{min} = -\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{min} = -\frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\tau_{min} = -134.63 \times 10^6 \text{ Pa}$$





4. $D = 280 \text{ mm} \rightarrow 0.28 \text{ m}$

$L = 600 \text{ mm} \rightarrow 0.6 \text{ m}$

$\sigma_{\text{max}} = 150 \text{ MPa} \rightarrow 150 \times 10^6 \text{ Pa}$

$\eta = 58\% \rightarrow 0.58$

$\eta_L = 69\% \rightarrow 0.69$

$P = 14.8 \text{ Bar} \rightarrow 14.8 \times 10^5 \text{ Pa}$

$\sigma_c = \frac{PD}{2t\eta_L}$

$t\sigma_c = \frac{PD}{2\eta_L}$

$t = \frac{PD}{2\eta_L\sigma_c}$

$t = \frac{14.8 \times 10^5 \times 0.28}{2 \times 0.58 \times 150 \times 10^6}$

$t = 2.38 \times 10^{-3} \text{ m}$

$t = 2.38 \times 10^{-3} \text{ m}$

$t = 2.38 \text{ mm}$

$$4.ii. \sigma_L = \frac{PD}{2t\eta_c}$$

$$t \sigma_L = \frac{PD}{2\eta_c}$$

$$t \theta = \frac{PD}{2\eta_c \sigma_L}$$

$$t = \frac{14.8 \times 0.28}{2 \times 0.69 \times 150 \times 10^6}$$

$$t = 2.00 \times 10^{-3} \text{ m}, t = 2.00 \text{ mm}$$

$$4.iii. \text{Min thickness} = 2.38 \times 10^{-3} \text{ m} \rightarrow \text{from } 2.38 \text{ mm}$$

Otherwise the pressure vessel will fail

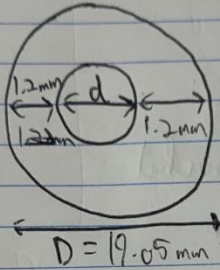
2.00 mm is suitable for the longitudinal stress, but not the circumferential stress, $\therefore$ min thickness = 2.38 mm

$$5. l = 1.38 \text{ m}$$

$$D = 19.05 \text{ mm} \rightarrow 19.05 \times 10^{-3} \text{ m}$$

$$t = 1.2 \text{ mm} \rightarrow 1.2 \times 10^{-3} \text{ m}$$

$$E = 200 \text{ GPa} \rightarrow 200 \times 10^9 \text{ Pa}$$



$$d = D - 2t$$

$$d = 19.05 - 2(1.2)$$

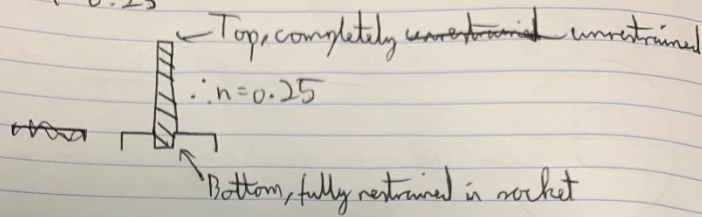
$$d = 16.55 \text{ mm} \rightarrow 16.55 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi (D^4 - d^4)}{64}$$

$$I = \frac{\pi (19.05 \times 10^{-3})^4 - (16.55 \times 10^{-3})^4}{64}$$

$$I = 2.78 \times 10^{-9} \text{ m}^4$$

5ii. Bottom: Fully restrained in a rocket
 Top: Completely unrestrained
 $\therefore n = 0.25$



$$5iii. F_{CR} = \frac{n\pi^2 EI}{L^2}$$

$$F_{CR} = \frac{0.25 \times \pi^2 \times 200 \times 10^8 \times 2.78 \times 10^{-9}}{1.38^2}$$

$$F_{CR} = 720.37 \text{ N}$$

$$5iv. K = \frac{I}{A}$$

$$A = \frac{\pi(D^2 - d^2)}{4}$$

$$A = \frac{\pi(14.05 \times 10^{-3})^2 - (6.55 \times 10^{-3})^2}{4}$$

$$A = 6.99 \times 10^{-5} \text{ m}^2$$

$$S.R. = \frac{L}{K}$$

$$S.R. = \frac{1.38}{6.99 \times 10^{-5}}$$

$$S.R. = 19742.49$$

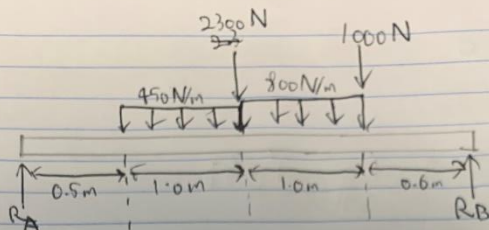
$$S.R. = \frac{1.38}{6.99 \times 10^{-5}}$$

$$S.R. = 19742.49$$

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$19742.49 > 180, \therefore \text{strut}$

3i.



$$R_A = 1969.4 \text{ N}$$

$$R_B = 2580.6 \text{ N}$$

$$EI \frac{d^2 y}{dx^2} = 1969.4[x] - \frac{450[x-0.5]^2}{2} + \frac{450[x-1.5]^2}{2} - 2300[x-1.5] - \frac{800[x-1.5]^2}{2} + \frac{800[x-2.5]^2}{2} - 1000[x-2.5]$$

$$3.ii. EI \frac{dy}{dx} = \frac{1969.4[x]^2}{2} - \frac{450[x-0.5]^3}{6} + \frac{450[x-1.5]^3}{6} - \frac{2300[x-1.5]^2}{2}$$

$$- \frac{800[x-1.5]^3}{6} + \frac{800[x-2.5]^3}{6} - \frac{1000[x-2.5]^2}{2} + A$$

$$3.iii. EI y = \frac{1969.4[x]^3}{6} - \frac{450[x-0.5]^4}{24} + \frac{450[x-1.5]^4}{24} - \frac{2300[x-1.5]^3}{6}$$

$$- \frac{800[x-1.5]^4}{24} + \frac{800[x-2.5]^4}{24} - \frac{1000[x-2.5]^3}{6} + Ax + B$$

3iv. @ $x=0, y=0$

Substitute into 3iii.

$$0 = \frac{1969.4[0]^3}{6} - \frac{450[0-0.5]^4}{24} + 4$$

$$0 = 0 - 0 + 0 - 0 - 0 + 0 - 0 + A(0) + B$$

$$B = 0$$

@ $x=2, y=0$

$$0 = 1969.4[2]$$

@ $x=3$

@ $x=3, y=0$, substitute into 3iii.

$$0 = \frac{1969.4[3]^3}{6} - \frac{450[2.5]^4}{24} + \frac{450[1.5]^4}{24} - \frac{2200[1.5]^3}{6}$$

$$0 = \frac{-800[1.5]^4}{24} + \frac{800[0.5]^4}{24} - \frac{1000[0.5]^4}{24} + 3A + 0 + 3.1A + 0$$

$$0 = 3A + 6759.98$$

$$3.1A = -6759.98$$

$$A = -2180.64$$

$$3A = -6759.98$$

$$A = -2252.99$$

$$A = -2252.99$$

$$3v. EI = 1.88 \text{ MPa} \rightarrow 1.88 \times 10^6 \text{ Pa}$$

$$x = 1.5$$

$$\cancel{x = 3.1}$$

$$x = \frac{3.1}{2} = 1.55 \text{ m}$$

$$1.88 \times 10^6 y = \frac{1969.4 [1.55]^3}{6} - \frac{450 [1.55]^4}{24} + \frac{450 [0.5]^4}{24} - \frac{2300 [0.5]^3}{6}$$

$$\rightarrow - \frac{800 [0.5]^4}{24} - 2180.64 [1.55]$$

$$1.88 \times 10^6 y = -2180.78$$

$$y = \frac{-2180.78}{1.88 \times 10^6}$$

$$y = 1.16 \times 10^{-3} \text{ m}$$

$$\cancel{y = 1.16}$$

$$y = 1.16 \times 10^{-3} \text{ m}$$