

assignment 2

by leuan Walker

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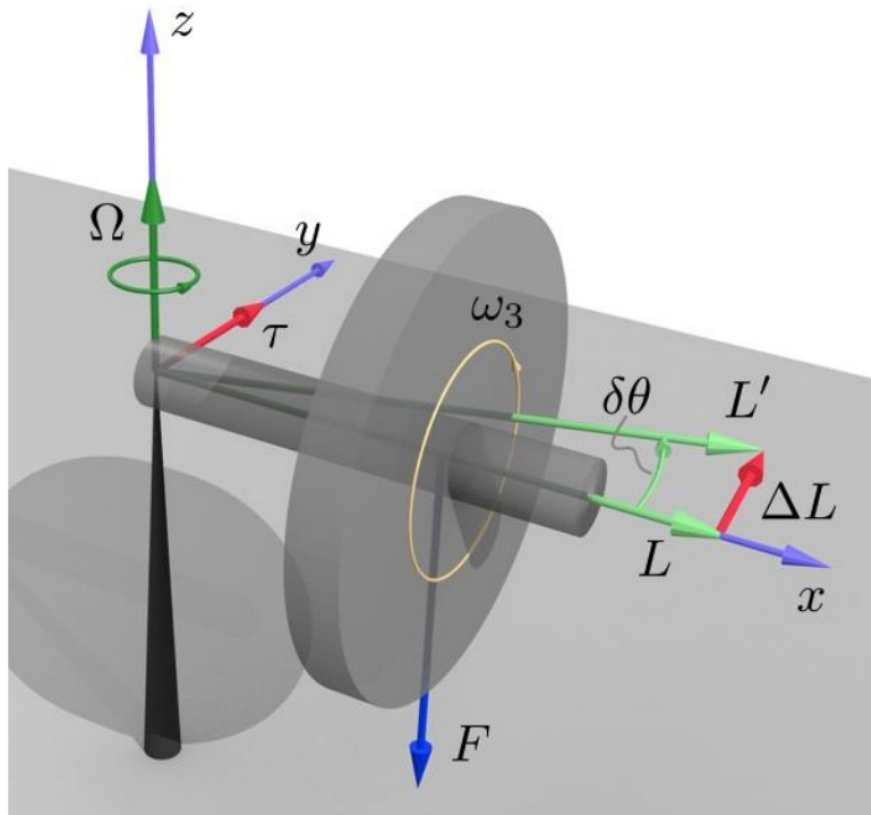
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Mechanical Science 2
SGDM5011
Static and dynamic principles



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Candidate: Ieuan Walker
Assignment 2

Introduction

This report covers various mechanical based situations and how they are solved mathematically or graphically. The first task is calculating the bending moment of a beam which is simply supported from both ends and under a localized load and a distributed load. The second task of the report calculates the force required to buckle a strut under a load whilst explaining the use and differences for Rankine Gordon method and Euler's method.

Task 3 is a report on calculating gyroscopic torque produced from a single engine plane when it is turning through a radius. The report also describes how gyroscopic motion is produced and how it is used in everyday things and the simplest forms of gyroscopes.

Task 4 depicts a scenario involving balancing masses on a co-planar system and how these masses centrifugal forces can be countered to balance a system. The report briefly explains the difference between static and dynamic balancing and how this use of balancing can be used in real world applications.

Task 5 shows a shaft which is under a torque and bending moment, the candidate shows calculations on stress due to bending and the max shearing force for the shaft depicted in the brief.

Task 6 depicts an undamped system which when moved has an undamped response to the force. The candidate shows calculations for the dampening coefficient and the spring stiffness to show the systems properties.

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Task 1- simply supported beams

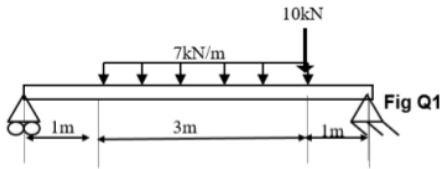


Figure 1 shows the beam used in the brief to depict the forces on the simply supported beam.

From the assignment brief the following parameters were obtained.

$E = 200 \text{ GN/m}^2 (\times 10^9)$ – modulus of elasticity

$I = 3 \times 10^{-5} \text{ m}^4$ – second moment of area

$L = 5$ – Length

The Brief showed there was two moments in the clockwise around the left hand support.

10KN @ 4m

21KN (7KN/m x 3m) @ 2.5 (centre of UD load)

As shown in **Q1** in the appendix the forces around R1 were used to calculate R2 (18.5KN) after calculating R2 the forces were added and R2 subtracted to obtain R1 (12.5KN)

$$\text{Using EI} = 12.5x - 10(x-4) - \frac{7(x-1)^2}{2} + \frac{7(x-4)^2}{2}$$

This was differentiated twice to obtain the slope and the deflection equations as shown in **Q1** in the appendix. Using the value $X = 5$ this allowed the candidate to use all of the equation to calculate the A value as all of the terms were used as the beams entire length was being used. The candidate then used $X=2.5$ in the deflection equation to obtain the deflection of the beam at 2.5m which is the centre of the beam. This meant any terms that were not needed as the $(x (2.5) - 4)$ term would result as 0.

As shown in **Q1** in the appendix the maximum deflection at 2.5m was 10.1977mm.

Task 2 – buckling under load

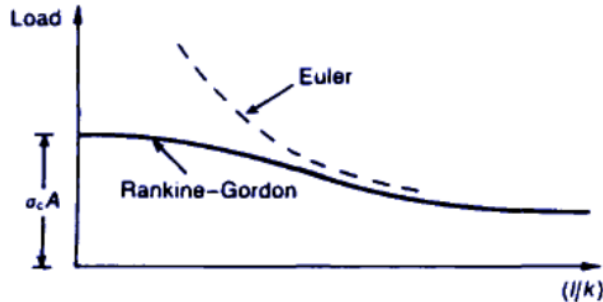


Figure 2 shows a graph showing the accuracy of Rankine Gordon's and Euler's methods of calculating the buckling point of a strut.

Figure 2

Accessed (29/04/20)

Figure 1 depicts the theoretical methods of calculating the buckling force, as shown Euler's method is more accurate when the beam is a higher slenderness ratio, as the beam in Q2 was considered a slender beam Euler's method was used. For struts with a lower slenderness ratio Euler's method becomes less accurate, Euler's method of calculation is optimistic when calculating the force required to buckle a low slenderness ratio strut, whereas Rankine Gordon's shows more realistic values which are more accurate when calculating struts which fall under this slenderness ratio.

$$\text{Euler's rule} = FE = \frac{n^2 \pi^2 EI}{L^2}$$

$$\text{Second moment of area: } \frac{\pi(D^4 - d^4)}{64}$$

As shown in Q2 in appendix

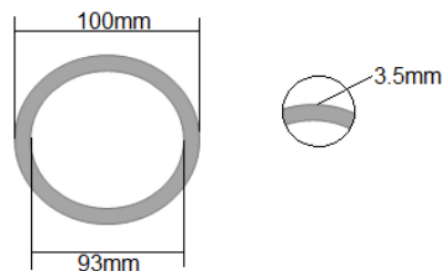
$$I = 1.236746804 \times 10^{-6}$$

$$E = 200 \text{ GN/m}^2$$

$$L = 3.5 \text{ M}$$

$$N = 1 \text{ (pin jointed)}$$

$$FE(\text{critical load}) = \frac{1^2 \pi^2 (200 \times 10^9) (1.236746804 \times 10^{-6})}{3.5^2} = 199.2849257 \text{ KN}$$



Task 3 – Gyroscopic motion

Gyroscopic motion is the tendency for a rotating component to maintain its orientation of rotation, a rotating object will have an angular momentum and this momentum must be conserved. The rotating component will resist a change in its axis of rotation as the change in the orientation will change the angular velocity. Gyroscopic motion is used in many inertial navigation systems that are found in ships and aeroplanes. Gyroscopic systems are often used to help a ship or aeroplane to keep the correct orientation and to stop capsizing the gyroscopic effect can also have negative effects on components such as wind turbines, as they have the tendency to keep spinning in the same direction.

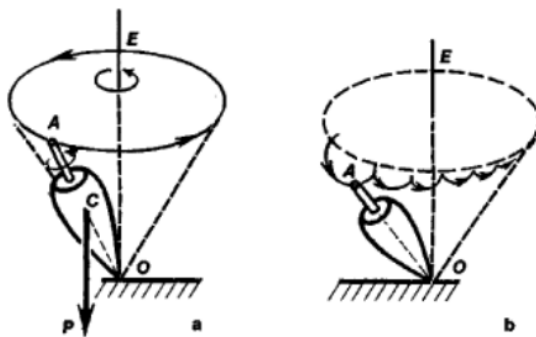


Figure 3

<https://encyclopedia2.thefreedictionary.com/Gyroscopic+Motion>

Figure 3 shows a basic representation of the path of a toy table top spinner which is the simplest form of gyroscopic motion. The axis of the gyroscope can also be influenced by nutation, which consists of small oscillations of the axis about its mean position as shown in figure 3. The faster the gyroscope spins the smaller the oscillations, as for table top spinners which usually spin at a lower velocity this property can be observed. In addition to this resistance slowly decreased the spinning gyroscope which causes a rapid decay in oscillations. This principle can be used in many industry such as in projectile launching. as used in rifles, rifling allows the projectile to spin at a high speed to create a gyroscopic motion this keeps the projectile in the orientation of exit from the barrel or force source. The spinning projectile will keep its orientation which allows the projectile to stay aerodynamic which will reduce resistance to the projectile allowing it to travel further. This can be shown in figure 4.

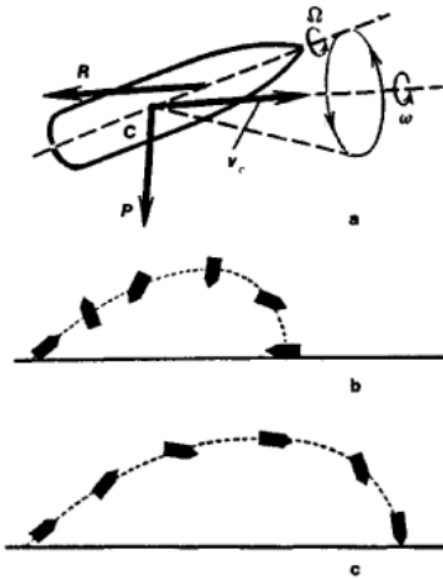


Figure 4

As shown in **figure 4** a spinning projectile can reduce air resistance and improve distance travelled. This principle can also be used in sports as ovulated playing balls e.g. American football & rugby are often spun by the thrower as this gyroscopic motion allows the ball to minimize air resistance staying in the correct orientation and allowing to travel further. This principle in gyroscopic motion is applied to lots of real world applications.

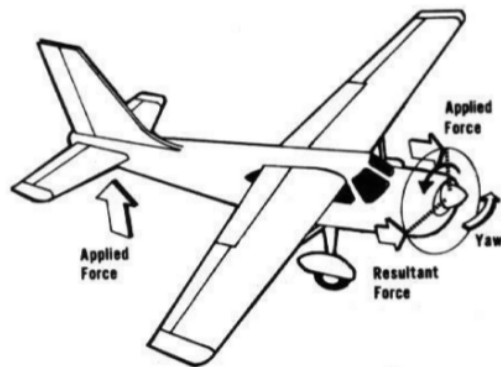


Figure 5

As shown by **figure 5**, the gyroscopic effect can be applied to a flying aircraft with a single engine.

Figure 17-45 Raising Tail Produces Gyroscopic Precession

The moment of inertia for the aircraft was given in the brief as 1.4 Kg m^2 and the propeller was spinning at 1600 RPM. The operator converted the 1600 RPM into Rad/s so that the equation was correct. As seen in the appendix the candidate multiplied the RPM value by $\frac{2\pi}{60}$ which gave 167.5 Rad/s.

To obtain Ω the candidate used $\Omega = \frac{V}{R}$

The candidate used the speed given in the brief (200km/hr) and converted the value to M/s using

$$\frac{200 \text{ Km/hr} \times 1000}{3600} = 55.5 \text{ m/s}$$

$$\Omega = \frac{55.5}{200} = 0.2775$$

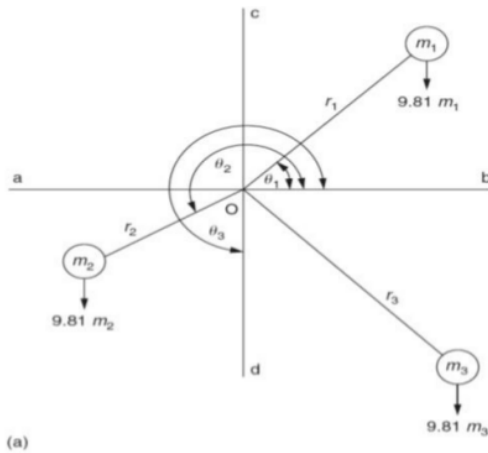
$$T = 1.5 \times 167.5 \times 0.2775 = 65.07 \text{ Nm (as shown in the appendix)}$$

Task 4 - static and dynamic balancing in a non-coplanar system

Static

Co-planar refers to masses that are in the same plane. For an object to be in the state of static equilibrium there must be no resultant force or moment acting on the object. The algebraic sum of the moments acting on the plane must be equal to zero.

Figure 6 depicts three co-planar masses positioned about an axis (O). For practical applications used on earth each mass would be subject to a 9.81N of gravitational



force. For this system to be balanced these forces interact, they apply forces to one another that are equal in magnitude and opposite in direction. This is also stated in Newton's third law. When these forces are equal to zero the masses will be in a state of static equilibrium.

Figure 6

Dynamic

When masses which are co-planar (within the same plane) are rotating, each of the masses exerts a centrifugal force which is equal to $mr\omega^2$ acting on the centre axis of the co-planar masses. The rotational speed (ω) of these masses are expressed in rad/s. As stated in static balancing of co-planar masses, the algebraic sum of the moments acting around the centre axis must be equal to zero in both directions acting on the plane at right angles also shown in **Figure 6**.

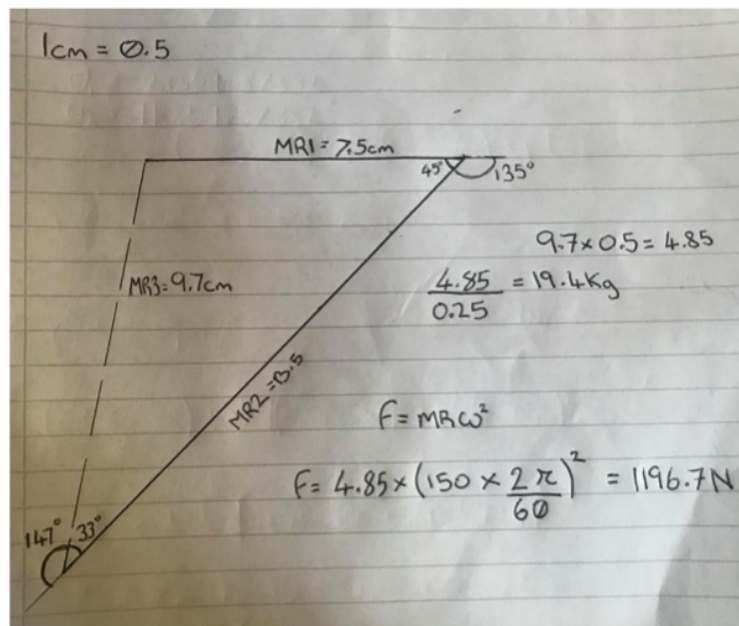
$$\therefore m_1 r_1 \omega^2 \cos \theta_1 + m_2 r_2 \omega^2 \cos \theta_2 + m_3 r_3 \omega^2 \cos \theta_3 = 0$$

$$m_1 r_1 \omega^2 \sin \theta_1 + m_2 r_2 \omega^2 \sin \theta_2 + m_3 r_3 \omega^2 \sin \theta_3 = 0$$

Dynamic balancing is a common practise to remove unbalanced components from a system to reduce vibration and other problems un-balanced systems can cause. Balancing car wheels is a common use of dynamic co-planar masses as unbalanced wheels create vibrations to the car which can deteriorate suspension and other dampening components on the system. Using dynamic balancing a resultant can be obtained at an angle to create a balanced system. With wheel balancing weights are used to counterbalance the force which is unbalancing the system. For Task 4 in the brief the candidate used a graphical method, using the known masses and there radius the candidate created a scaled representation of the directions of the centrifugal forces using the angles to determine the resultant force. As shown in **Q4** the resultants mass was 19.4Kg acting at an angle of 147° from MR2. The centrifugal force generated from this mas at 150 RPM was 1196.7N.

As shown in the **appendix Q4**

kg	(M) Radius	KgM	Angle θ
15	0.25	3.75	0
27	0.25	6.75	135°
19.4	0.25	4.85	147°



Q4

The out of balance force of the system at 150 RPM was worked out to be 1196.7N

Task 5 - complex stresses

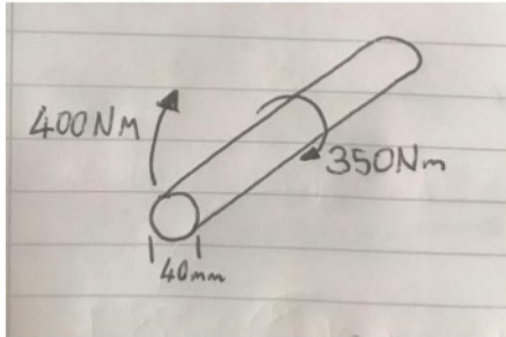


Figure 7 depicts a solid shaft which has a diameter of 40mm under a torque and bending moment as shown described in the brief. Also shown in **appendix Q5**.

$$\text{Torsion equation} = \frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{Tr}{J}$$

$$\tau = \frac{350 \times 0.02}{2.513274123 \times 10^{-7}} = 27.855153 \text{ MNm}^2$$

Torsion constant

$$J = \frac{\pi D^4}{32}$$

$$J = \frac{\pi \times 0.04^4}{32}$$

$$J = 2.513274123 \times 10^{-7}$$

Stress due to bending

$$\frac{M}{I} = \frac{\sigma}{y} = \sigma = \frac{MY}{I}$$

$$\sigma_x = \frac{400 \times 0.02}{1.256637061 \times 10^{-7}} = 63.66 \text{ MN/m}^2$$

$$\sigma_y = 0$$

Second moment of area

$$I = \frac{\pi D^4}{64}$$

$$I = \frac{\pi \times 0.04^4}{64}$$

$$I = 1.256637061 \times 10^{-7}$$

Principle stresses

$$\sigma_1, \sigma_2 = \left(\frac{\sigma_x - \sigma_y}{2} \right) \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau)^2}$$

$$\left(\frac{63.66 - 0}{2} \right) \pm \frac{1}{2} \sqrt{(63.66 - 0)^2 + 4(27.855)^2}$$

$$31.83 \pm \frac{1}{2} \sqrt{(4052.5956 + 3103.6041)}$$

$$\sigma_1 = 74.12716214$$

$$\sigma_2 = -10.46716214$$

Shearing τ max

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{74.12716214 - (-10.46716214)}{2} = 42.297$$

As seen in **appendix Q5**

Using the brief the candidate create a diagram of the described shaft and annotated the moment and torsion being applied to the shaft. Using the torsion and bending equation the candidate calculated the second moment of area and the torsion constant which are similar apart from the torsion constant divided by 32 whilst second moment of area is over 64. Using the torsion equation, the candidate rearranged the equation for $\tau = \frac{Tr}{J}$ using the torsion constant the tau (τ) was obtained.

Using the bending equation the candidate re arranged it to obtain $\sigma = \frac{MY}{I}$ which gave the stress due to bending in the scenario. Using these values the candidate calculated the principle stresses of the shaft.

$$\sigma_1, \sigma_2 = \left(\frac{\sigma_x - \sigma_y}{2} \right) \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau)^2}$$

Using the principle stresses the candidate the used $\tau = \frac{\sigma_1 - \sigma_2}{2}$ to obtain Shearing τ max of the shaft.

Task 6- logarithmic decrement

Logarithmic decrement is used to find the dampening ratio of an underdamped system in the time domain, the logarithmic decrement can be defined as the natural log of the amplitude of two peaks in the systems underdamped response. The logarithmic decrement measures the systems response and how the amplitude of the response deteriorates over a period of time. The logarithmic decrement of an under dampened can be expressed as.

$$\delta = \frac{1}{n} \log \frac{x(t)}{x(t+nT)}$$

Where n defines the number of cycles, T which represents the period of oscillation and t which expresses the first amplitude. The dampening coefficient of the system can also be expressed as,

$$\ln \frac{x^1}{x^2} = \frac{2\pi \epsilon M}{\sqrt{1-\epsilon^2}}$$

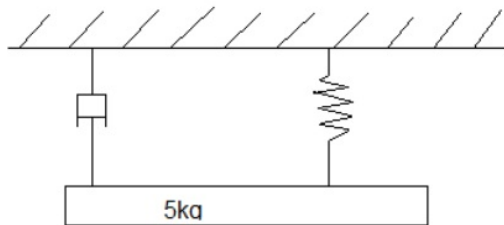


Figure 8 depicts the under dampened system in described in Task 6 in the brief.

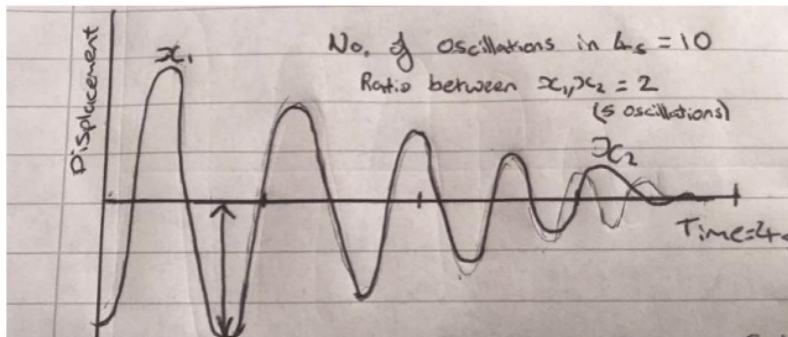


Figure 9 shows the candidates rough under dampened response from the system showing the oscillating system decaying.

$\omega_n = 15.7 \text{ rad/s}$
$C_c = 50\pi$
$K = 1233.7 \text{ N/m}$
$R = \text{amplitude}$
$T = \text{time constant}$
$F = \text{frequency } \frac{1}{T}$

$$\text{Frequency} = \frac{10}{4} = 2.5\text{Hz}$$

Natural frequency

$$\omega_n = 2.5 \times 2\pi$$

$$\omega_n = 5\pi \text{ Rad/s}$$

$$\omega_n = 15.70796327 \text{ rad/s}$$

Critical dampening

$$C_c = 2 \times M \times \omega_n$$

$$2 \times 5 \times 5\pi$$

$$C_c = 50\pi \text{ N/M/s}$$

Spring stiffness

$$\omega_n = \sqrt{\frac{K}{M}} = 5\pi = \sqrt{\frac{K}{5}}$$

$$5\pi^2 = \frac{K}{5}$$

$$K = 5\pi^2 \times 5$$

$$K = 1233.70055 \text{ N/m}$$

$$\varepsilon = \frac{c}{C_c}$$

$$\ln \frac{x^1}{x^2} = \frac{2\pi\varepsilon M}{\sqrt{1-\varepsilon^2}}$$

Dampening coefficient

$$\ln \frac{x^1}{x^2} = \frac{2\pi\varepsilon M}{\sqrt{1-\varepsilon^2}}$$

$$\ln(2) = \frac{2\pi\varepsilon 5}{\sqrt{1-\varepsilon^2}}$$

$$0.6931471806 = \frac{10\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\sqrt{1-\varepsilon^2} = \frac{10\pi\varepsilon}{0.6931471806}$$

$$\sqrt{1-\varepsilon^2} = 45.32360142\varepsilon$$

$$1 - \varepsilon^2 = 2054.228846\varepsilon^2$$

$$1 = 2055.228846\varepsilon^2$$

$$\varepsilon^2 = \frac{1}{2055.228846} = 4.865638208 \times 10^{-4}$$

$$\varepsilon = 0.02205819169$$

$$\varepsilon = \frac{c}{C_c} = C = C_c \times \varepsilon$$

$$50\pi \times 0.02205819169$$

$$C = 3.464892649 \text{ N/m/s (dampening coefficient)}$$

also shown in **appendix Q6**

Reference page

Front cover- Gyroscopic Motion I . 2020. Gyroscopic Motion I . [ONLINE] Available at:
<http://www.entropy.energy/scholar/node/gyroscope-via-torque>. [Accessed 4 April 2020].

Figure 1 – report task 1

Figure 2 – Rankine Gordon Equation I Engineers Edge I www.engineersedge.com. 2020.
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Figure 4 -TheFreeDictionary.com. 2020. Gyroscopic Motion I Article about
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Figure 5- TheFreeDictionary.com. 2020. Gyroscopic Motion I Article about
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Figure 6 – Engineersfield. 2020. Balancing rotating masses - Engineersfield. [ONLINE]
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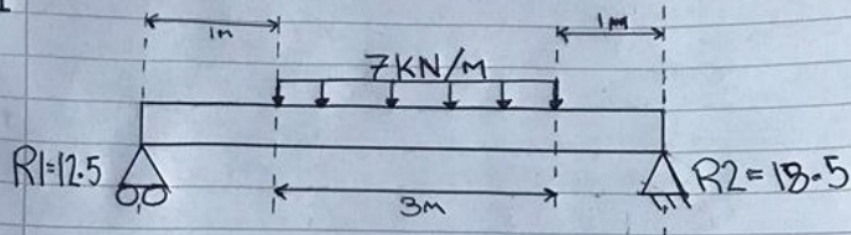
Figure 7 - complex stresses representation (appendix)

Figure 8 – task 6 system representation (appendix)

Figure 9 – task 6 system response diagram (appendix)

Appendix

Q1



$$E = 200 \text{ GN/m}^2 (\times 10^9)$$

$$I = 3 \times 10^{-5} \text{ m}^4$$

$$L = 5 \text{ m}$$

$$R_1 = 12.5$$

$$R_2 = 18.5$$

$$7 \times 3 = 21 \text{ kN}$$

$$(21 \text{ kN} \times 2.5 \text{ m}) / (10 \text{ kN} \times 4) = 5 R_2$$

$$\frac{92.5}{5} = 18.5 = R_2$$

$$21 + 10 = 31 - 18.5 = 12.5 = R_1$$

$$EI = 12.5x - \frac{10(x-4)^2}{2} - \frac{7(x-1)^2}{2} + \frac{7(x-4)^2}{2}$$

$$\frac{dy}{dx} = \frac{12.5x^2}{2} - \frac{10(x-4)^2}{2} - \frac{7(x-1)^3}{6} + \frac{7(x-4)^3}{6} + A$$

$$EIY = \frac{12.5x^3}{6} - \frac{10(x-4)^3}{6} - \frac{7(x-1)^4}{24} + \frac{7(x-4)^4}{24} + Ax + B$$

$$0 = 0 - 0 - 0 + 0 + B = 0$$

Q1

$$x = 5 \quad y = 0$$

$$\frac{12.5 \times 5^3}{6} - \frac{10(5-4)^3}{6} - \frac{7(5-1)^4}{24} + \frac{7(5-4)^4}{24} + (A \times 5) + B$$

$$260.416 - 1.6 - 74.6 + 0.296 + 5A + B$$

$$0 = 184.512 + 5A + B$$

$$\frac{184.512}{5} = -36.9024 = A$$

$$EIY = \frac{12.5x^3}{6} - \frac{10(x-4)^3}{6} - \frac{7(x-1)^4}{24} + \frac{7(x-4)^4}{24} - 36.9024x$$

$$\frac{12.5(2.5)^3}{6} - \cancel{\frac{10(2.5-4)^3}{6}} - \frac{7(2.5-1)^4}{24} + \cancel{\frac{7(2.5-4)^4}{24}} - (36.9024 \times 2.5)$$

$$32.552 - 1.4765625 - 92.256 = -61.1865625$$

$$\frac{-61.1865625}{(200 \times 10^9) \times (3 \times 10^{-5})} = -10.1977 \text{ mm}$$

Q2

$$\text{Euler's Rule} = F_e = \frac{n^2 \pi^2 EI}{L^2}$$

$$I = 1.237$$

$$E = 200 \times 10^9$$

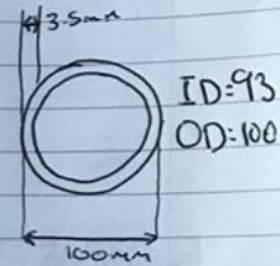
$$L = 3.5 \text{ m}$$

$$n = 1 (\text{pin})$$

I = Second moment of area

$$I = \frac{\pi (D^4 - d^4)}{64}$$

$$\frac{\pi (100 \times 10^3)^4 - (\overset{93}{93.5} \times 10^3)^4}{64}$$



$$I = \frac{\pi (100)^4}{64} - \frac{\pi (93)^4}{64}$$

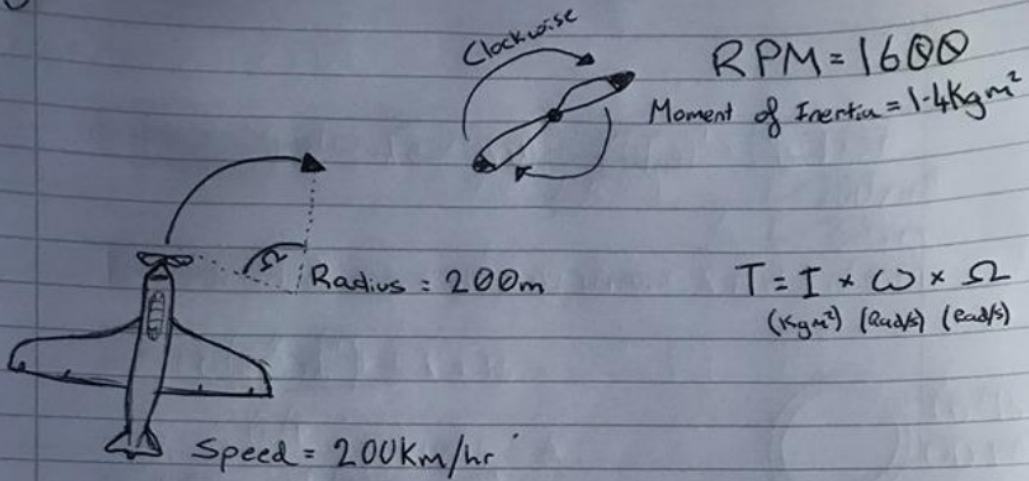
$$(4.90873852 \times 10^6) - (3.671991717 \times 10^6) = 1.236746804 \times 10^6$$

$$F_e (\text{critical load}) = \frac{n^2 \pi^2 (200 \times 10^9) (1.236746804 \times 10^6)}{3.5^2}$$

$$= 199,284,925.7 \text{ N}$$

$$199,284,925.7 \text{ KN}$$

Q3



$$\omega = 1600 \times \frac{2\pi}{60} = 167.5 \text{ Rad/s}$$

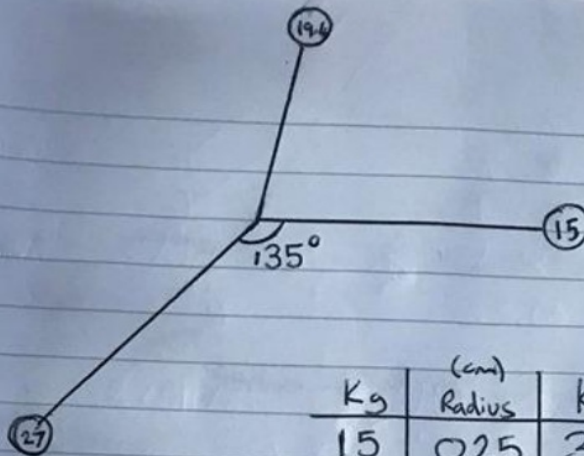
$$\alpha = \frac{v}{R} =$$

$$\frac{200 \text{ km/hr} \times 1000}{3600} = 55.5 \text{ m/s}$$

$$\alpha = \frac{55.5}{200} = 0.2775$$

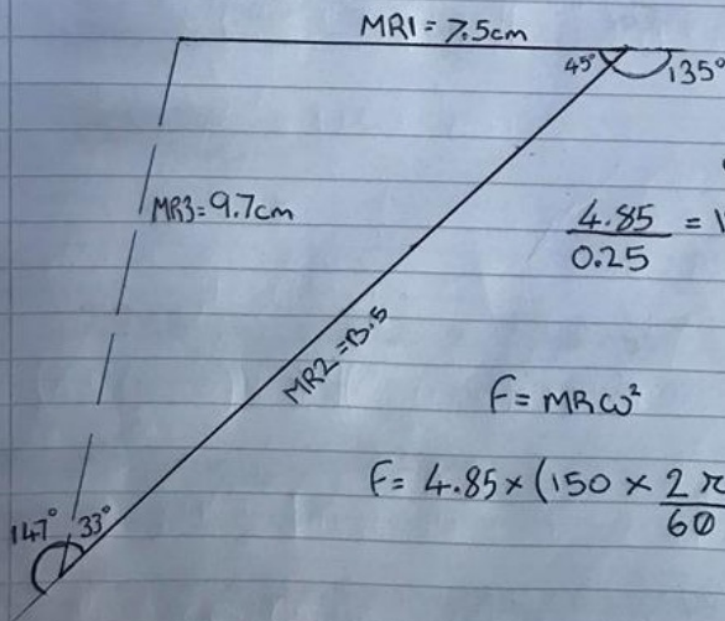
$$T = 1.4 \times 167.5 \times 0.2775 = 65.07 \text{ Nm}$$

Q4



Kg	(cm) Radius	Kgm	θ
15	0.25	3.75	0°
27	0.25	6.75	135°
19.4	0.25	4.85	147°

$1\text{cm} = 0.5$



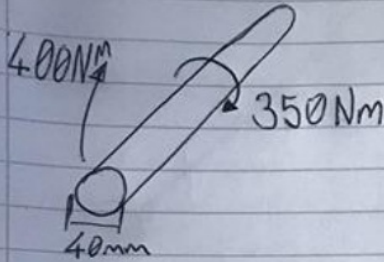
$9.7 \times 0.5 = 4.85$

$\frac{4.85}{0.25} = 19.4\text{Kg}$

$F = MR\omega^2$

$F = 4.85 \times \left(150 \times \frac{2\pi}{60}\right)^2 = 1196.7\text{N}$

Q5



$$OD = 40 \text{ mm}$$

$$B.M = 400 \text{ Nm}$$

$$T = 350 \text{ Nm}$$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow \tau = \frac{T r}{J}$$

$$\tau = \frac{350 \times 0.02}{2.513274123} = 2.7855153 \text{ MN/m}^2$$

$$I = \frac{\pi D^4}{32}$$

$$J = \frac{\pi \times 0.04^4}{32}$$

$$= 2.7855153 \text{ MN/m}^2$$

$$J = 2.513274123 \times 10^{-7} \text{ m}^4$$

Stress due to bending

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M y}{I}$$

$$I = \frac{\pi d^4}{64} \Rightarrow \frac{\pi \times 0.04^4}{64} = 1.25663706 \times 10^{-7}$$

$$\sigma_x = \frac{400 \times 0.02}{1.25663706 \times 10^{-7}} = 63.66 \text{ MN/m}^2$$

$$\sigma_y = 0$$

Q5

Principle Stresses

$$\sigma_1, \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2} \right) \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau)^2}$$

$$\left(\frac{63.66 - 0}{2} \right) \pm \frac{1}{2} \sqrt{(63.66 - 0)^2 + 4(27.855)^2}$$

$$31.83 \pm \frac{1}{2} \sqrt{4052.8956 + 3103.6041}$$

$$\sigma_1 = 74.12716214$$

$$\sigma_2 = -10.46716214$$

Shearing τ_{max}

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{74.12716214 - (-10.46716214)}{2}$$

$$= 42.297$$

Principle Plane

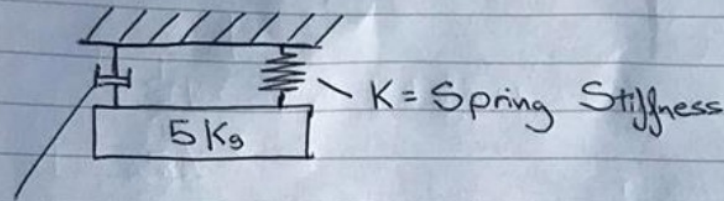
$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2 \times 27.855}{63.66 - 0} = 0.8751178$$

$$\tan 2\theta = 0.8751178 \rightarrow 2\theta = \tan^{-1} 0.8751178$$

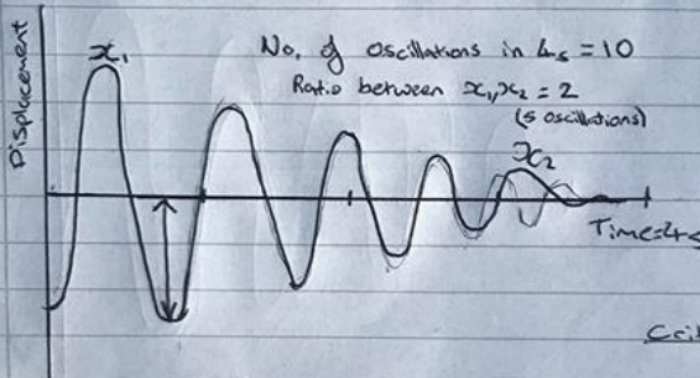
$$\theta = \frac{41.18974766}{2} = \theta_1 = 20.59487382^\circ$$

$$\theta_2 = 110.59487382^\circ$$

Q6



$C_c = \text{Critical Damping}$



$$\omega_n = 15.7 \text{ Rad/s}$$

$$C_c = 50\pi \text{ N/m/s}$$

$$K = 1233.7 \text{ N/m}$$

$R = \text{amplitude}$

$T = \text{time constant}$

$$F = \text{Frequency} = \frac{1}{T}$$

Critical Damping

$$\text{Frequency} = \frac{10}{4} = 2.5 \text{ Hz}$$

Natural Frequency

$$\omega_n = 2.5 \times 2\pi$$

$$\omega_n = 5\pi \text{ Rad/s}$$

$$\omega_n = 15.70796327 \text{ Rad/s}$$

Spring Stiffness

$$\omega_n = \sqrt{\frac{K}{M}} = 5\pi = \sqrt{\frac{K}{5}}$$

$$5\pi^2 = \frac{K}{5}$$

$$K = 5\pi^2 \times 5$$

$$K = 1233.70055 \text{ N/m}$$

$$C_c = 2 \times M \times \omega_n$$

$$2 \times 5 \times 5\pi$$

$$C_c = 50\pi \text{ N/m/s}$$

$$\epsilon = \frac{C}{C_c}$$

$$\ln \frac{x_1}{x_2} = \frac{2\pi \epsilon M}{\sqrt{1 - \epsilon^2}}$$

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Q6

Damping Coefficient

$$\ln \frac{x_1}{x_2} = \frac{2\pi \xi m}{\sqrt{1-\xi^2}}$$

$$\ln(2) = \frac{2\pi \times \xi \times 5}{\sqrt{1-\xi^2}}$$

$$0.6931471806 = \frac{10\pi \xi}{\sqrt{1-\xi^2}}$$

$$\sqrt{1-\xi^2} = \frac{10\pi \xi}{0.6931471806}$$

$$\sqrt{1-\xi^2} = 45.32360142 \xi$$

$$1-\xi^2 = 2054.228846 \xi^2$$

$$1 = 2055.228846 \xi^2$$

$$\xi^2 = \frac{1}{2055.228846} = 4.865638208 \times 10^{-4}$$

$$\xi = 0.02205819169$$

$$\xi = \frac{c}{c_c} = C = c_c \times \xi$$

$$50\pi \times 0.02205819169$$

$$C = 3.464892649 \text{ N/m/s (damping Coefficient)}$$