

LOCATION SWANSEA

LLEOLIAD ABERTAWE

LEVEL 4

LEFEL 4

**EXAMINATION PERIOD
APRIL 2018**

**MODULE CODE: M4X02193
ANALYTICAL METHODS WITH STUDY SKILLS TEST 2**

TIME: 1 ½ HOURS

ANSWER ALL QUESTIONS

Equipment or materials required –

Calculators allowed

Formula booklet provided

QUESTION ONE

a) Given the following matrices:

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 2 \\ 6 & 3 \\ 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 5 & -1 \\ 0 & 2 \end{pmatrix}$$

Find, if possible, the following:

(i) AB

(ii) BA

(iii) A+B

(iv) A+C

(v) 4A

(13 marks)

b) Find the determinant of the following matrix:

$$\begin{pmatrix} 2 & 1 & 5 \\ 4 & -1 & 0 \\ 2 & 1 & 2 \end{pmatrix}$$

(3 marks)

c) Solve the following set of simultaneous equations using Cramer's rule:

$$3x + 2y + z = 17$$

$$x + y + 2z = 9$$

$$2x - y + z = 0$$

(8 marks)

QUESTION TWO

a) Given the following complex numbers:

$$z_1 = 5 + 2j \quad z_2 = 3 - 7j$$

Calculate the following:

(i) $z_1 + z_2$ (ii) $z_1 - z_2$ (iii) $z_1 z_2$ (iv) $\frac{z_1}{z_2}$

(7 marks)

b) Express the following complex number in polar form:

$$6 - 5j$$

(3 marks)

c) Calculate the following using De Moivre's theorem, expressing your answer in rectangular form:

$$(3 + 2j)^5$$

(5 marks)

QUESTION THREE

a) Integrate the following:

(i) $\int 5x^5 - 3x^2 + 2 \, dx$

(ii) $\int (x + 7)^2 \, dx$

(iii) $\int 7 \sec^2 3x \, dx$

(iv) $\int_0^3 3x^3 - x + e^x \, dx$

(17 marks)

b) Use integration by parts to integrate the following:

$$\int 2x \cos x \, dx$$

(4 marks)

c) Use integration by substitution to integrate the following, using the substitution $u = \sin x$:

$$\int \sin x \cos x \, dx$$

(4 marks)

Q1)

a) (i) $AB = \begin{pmatrix} 35 & 18 \\ -3 & 3 \\ 28 & 14 \end{pmatrix}$

6
(-1 incorrect)

(ii) BA Cannot do

1

(iii) $A+B$ Cannot do

1

(iv) $A+C$ Cannot do

1

b) $4A = \begin{pmatrix} 8 & 16 & 4 \\ 0 & -4 & 12 \\ 12 & 8 & 4 \end{pmatrix}$

4
(-1 incorrect)

b) $\begin{vmatrix} 2 & 1 & 5 \\ 4 & -1 & 0 \\ 2 & 1 & 2 \end{vmatrix} = 2(-2-0) - 1(8-0) + 5(4+2)$
 $= -4 - 8 + 30$
 $= 18$

1
1
1

$$c) \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 17 \\ 9 \\ 0 \end{pmatrix}$$

↑
A

$$|A| = 12$$

$$A_1 = \begin{pmatrix} 17 & 2 & 1 \\ 9 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix} \quad |A_1| = 24$$

$$A_2 = \begin{pmatrix} 3 & 17 & 1 \\ 1 & 9 & 2 \\ 2 & 0 & 1 \end{pmatrix} \quad |A_2| = 60$$

$$A_3 = \begin{pmatrix} 3 & 2 & 17 \\ 1 & 1 & 9 \\ 2 & -1 & 0 \end{pmatrix} \quad |A_3| = 12$$

$$x = \frac{24}{12}$$

$$y = \frac{60}{12}$$

$$z = \frac{12}{12}$$

$$x = 2$$

$$y = 5$$

$$z = 1$$

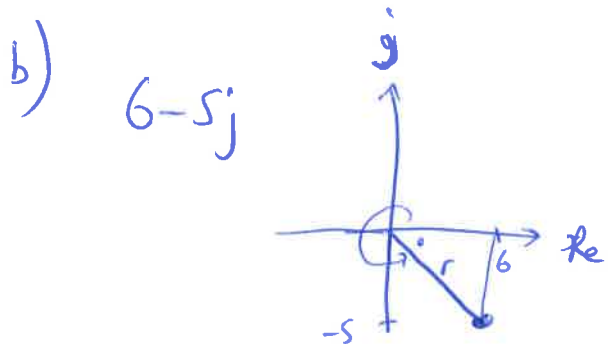
Q2) $z_1 = 5 + 2j$ $z_2 = 3 - 7j$

(i) $z_1 + z_2 = 8 - 5j$

(ii) $z_1 - z_2 = 2 + 9j$

(iii) $z_1 z_2 = (5 + 2j)(3 - 7j)$
 $= 15 - 35j + 6j - 14j^2$
 $= 29 - 29j$

(iv) $\frac{z_1}{z_2} = \frac{(5 + 2j)(3 + 7j)}{(3 - 7j)(3 + 7j)}$
 $= \frac{15 + 35j + 6j + 14j^2}{9 - 49j^2}$
 $= \frac{1 + 41j}{58} = \frac{1}{58} + \frac{41}{58}j$



$$r = \sqrt{6^2 + 5^2}$$

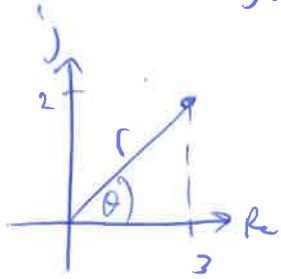
$$r = \sqrt{61} = \underline{7.81}$$

$$\tan \theta = 5/6 \quad \theta = 39.81^\circ$$

$$\theta = \underline{320.19^\circ}$$

$$\therefore 7.81 (\cos 320.19^\circ + j \sin 320.19^\circ)$$

c) $(3+2j)^5$



$$r = \sqrt{3^2 + 2^2}$$

$$r = \sqrt{13} = 3.61$$

$$\tan \theta = 2/3$$

$$\theta = 33.69^\circ$$

$$\begin{aligned} \text{DMT} &= (3.61)^5 (\cos 33.69 \times 5 + j \sin 33.69 \times 5) \\ &= 613.11 (\cos 168.45 + j \sin 168.45) \end{aligned}$$

$$= -600.69 + 122.76j$$

Q3) a)

$$(i) = \frac{5x^6}{6} - \frac{3x^3}{3} + 2x + C$$

$$(ii) = \int (x^2 + 14x + 49) dx$$

$$= \frac{x^3}{3} + \frac{14x^2}{2} + 49x + C$$

$$(iii) = \frac{7}{3} \tan 3x + C$$

$$(iv) \left[\frac{3x^4}{4} - \frac{x^2}{2} + e^x \right]_0^3$$

3

$$= \left[\frac{3(3)^4}{4} - \frac{(3)^2}{2} + e^3 \right] - \left[0 - 0 + e^0 \right]$$

$$= [60.75 - 4.5 + 20.09] - [1]$$

$$= 76.34 - 1$$

$$= 75.34$$

$$b) \int \underset{\substack{\uparrow \\ u}}{2x} \underset{\substack{\uparrow \\ \frac{dv}{dx}}}{\cos x} dx$$

$$\frac{du}{dx} = 2 \quad v = \sin x$$

$$= 2x \sin x - \int 2 \sin x dx$$

$$= 2x \sin x + 2 \cos x + C$$

$$c) \int \sin x \cos x dx \quad u = \sin x$$

$$\int u \cos x \frac{du}{\cancel{\cos x}}$$

$$\frac{du}{dx} = \cos x$$

$$\int u du$$

$$\frac{du}{\cos x} = dx$$

$$= \frac{u^2}{2} + C \rightarrow \left[\frac{\sin^2 x}{2} + C \right]$$

(4)

(4)

