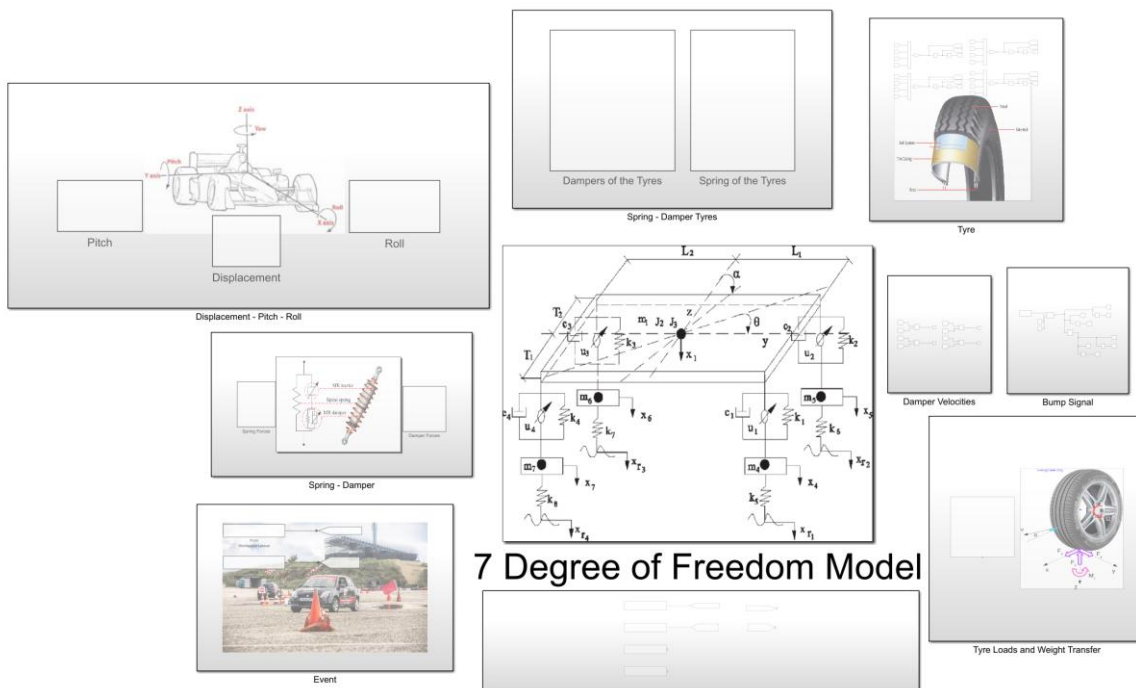


University of Wales Trinity Saint David
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Transient Vehicle Response Modelling

Advanced Vehicle Dynamic Behaviour / Advanced Motorsport Dynamics ACEA7002 /
ACEA7003



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1 Introduction

1.1 Overview of Vehicle Dynamics and previous models

Vehicle Dynamics are the motions of the vehicle generate by the steering action, through which the vehicle is capable of independent motion (Abe, 2015). Vehicle dynamics is a broad and complex field that can be examined at various levels of detail, depending on the specific objectives of the study. It encompasses the study of motion, forces, and interactions that govern the behaviour of vehicles in different environments. The dynamics of a car, for instance, differ significantly from those of a bicycle or an aircraft, despite sharing fundamental physical principles such as roll, pitch, and yaw. Each system experiences unique forces and constraints that influence its motion, requiring tailored approaches for analysis and optimization.

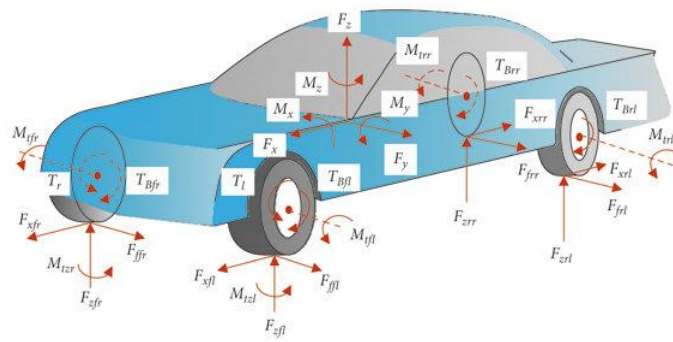


Figure 1 External Forces and Momentum in a Standard Road Car (He et al., 2020)

In recent years, the study of vehicle dynamics has become increasingly sophisticated due to the vast amount of data available. Figure 1 illustrates the external forces and moments acting on a standard road car, excluding the influence of the suspension and damper system. Even without considering internal forces, the system becomes highly complex due to the numerous interacting variables.

The inclusion of internal forces, such as those generated by the suspension system, would further increase the complexity, necessitating advanced computational models for accurate analysis. Thus, why traditional hand calculations, once used to analyse motion and forces, are no longer sufficient to handle the complexity of modern dynamic systems. The interdependence of equations governing motion and system interactions makes it impractical

to solve these problems manually for every data point. Instead, advanced computational tools and simulations, such as multi-degree-of-freedom models (DoF), have become essential for accurately predicting vehicle behaviour and optimizing performance.

Among these models, two degrees of freedom (DoF) quarter vehicle models are widely used to analyse the vertical vibrations while a 4-DoF half vehicle model is generally used to analyse both the vertical, translational behaviour and the pitching of the vehicle. Finally, a 7-DoF full vehicle model is preferred when analysing the yawing dynamics of the vehicle in addition to the vertical and pitching motions (Cem Onat et al., 2007).

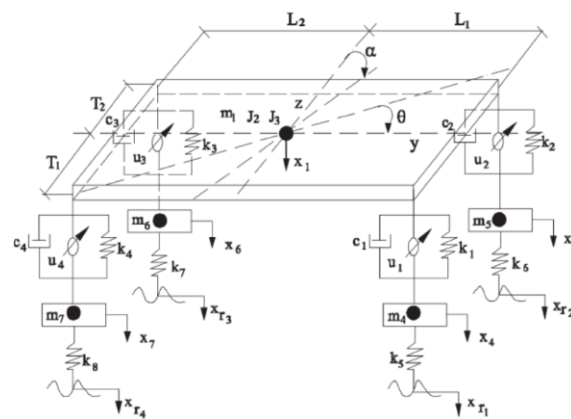


Figure 2 7 Degree of Freedom (Cem Onat et al., 2007)

Considering the dynamics behaviour of a vehicle requires accurate modelling (using computational power) of the vehicle dynamics. Vehicle dynamics models, depending on the type of analysis such as the study of riding behaviour or handling, consists of simple models to models with a higher number of degrees of freedom (Azadi, Kazemi and Hamidreza Rezaei Nedamani, 2021).

Going back in time and as mentioned previously there are models that require less computational power and serves as a starting point to get to the standard 7 DoF. Which are the 1 DoF (First degree of Freedom), 2 DoF (quarter car model) and 4 DoF (half car model).

In 2019 a team of engineering developed a First Degree of Freedom takes only into consideration the vertical movement of the sprung mass (Deac et al., 2019).

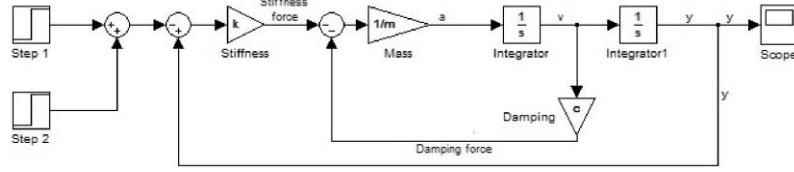


Figure 3 Simulink model for 1 DoF Suspension System (Deac et al., 2019)

$$\ddot{y} = \frac{c(\dot{z} - \dot{y}) + k(z - y)}{m}$$

Equation 1 First Degree Suspension Model - Z displacement

As observed in the provided image, determining the applied forces in the Simulink model is relatively straightforward. In contrast, the two-degree-of-freedom model, as shown in Figure 6, presents a more intricate system due to its additional dynamic interactions.

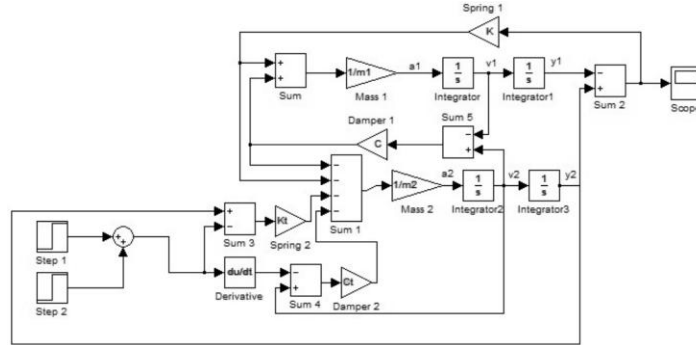


Figure 4 Simulink model for 2 DoF Suspension System (Deac et al., 2019)

$$\ddot{y}_1 = \frac{c(\dot{y}_2 - \dot{y}_1) + k(y_2 - y_1)}{m_1}$$

Equation 2 2 DoF Z - Displacement of the sprung mass

$$\ddot{y}_2 = \frac{c(\dot{y}_1 - \dot{y}_2) + k(y_1 - y_2) + c_t(\dot{z} - \dot{y}_2) + k_t(z - y_2)}{m_2}$$

Equation 3 2 DoF Z - Displacement of the unsprung mass

Moving forwards to the half car model which is not only considering the suspension model of 1 section of the car it takes into consideration 2 suspension from 1 side of the car, the front and rear of the car. This following figure is the Simulink model for the half car system.

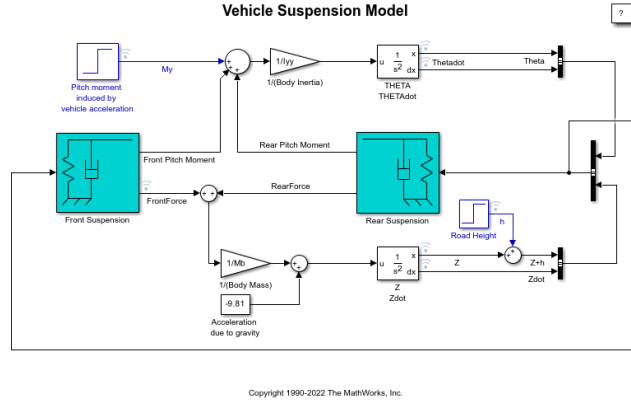


Figure 5 Half Car Suspension Model (Mathworks.com, 2020)

$$-m_s \ddot{z}_s = k_f(z_s - z_f - a\theta) + k_r(z_s - z_r + b\theta) + C_f(\dot{z}_s - \dot{z}_f - a\dot{\theta}) + C_r(\dot{z}_s - \dot{z}_r + b\dot{\theta})$$

Equation 4 z_s corresponds to displacement in Z of the sprung mass

$$-I\ddot{\theta} = bk_r(z_s - z_r + b\theta) - ak_f(z_s - z_f - a\theta) + bC_r(\dot{z}_s - \dot{z}_r + b\dot{\theta}) - aC_f(\dot{z}_s - \dot{z}_f - a\dot{\theta})$$

Equation 5 θ corresponds to the pitch in degrees or radians of the system

$$-m_f \ddot{z}_f = k_f(z_f - y_f) - k_f(z_s - z_f - a\theta) + C_f(\dot{z}_f - \dot{y}_f) - C_f(\dot{z}_s - \dot{z}_f - a\dot{\theta})$$

Equation 6 z_f is the displacement in Z axis of the unsprung mass

$$-m_r \ddot{z}_r = k_r(z_r - y_r) - k_r(z_s - z_r + b\theta) + C_r(\dot{z}_r - \dot{y}_r) - C_r(\dot{z}_s - \dot{z}_r + b\dot{\theta})$$

Equation 7 z_r is the displacement of the Z axis of the unsprung mass

As suspension system models evolved from single-degree-of-freedom to two-degree-of-freedom and later four-degree-of-freedom, they provided better insight into vehicle behaviour. These advancements have significantly improved handling and stability by allowing more accurate simulations of weight transfer, pitch dynamics, and tire interactions.

1.2 Significance of 7DOF Modelling

The seven-degree-of-freedom (7DOF) model is essential for accurately capturing heave, pitch, roll, and independent wheel movements, providing a more precise representation of weight transfer, load distribution, and suspension dynamics. Unlike lower-degree models, it enables better handling, stability, and ride comfort optimization, particularly in high-performance and safety-critical applications. A full-vehicle model with non-single suspension shown in figure 8 is a seven-degree-of-freedom model, which consists of heave, pitch, roll modes of the sprung mass, vertical motions of the front suspensions, vertical motion and roll of the rear non-single suspension (Fang et al., 2011).

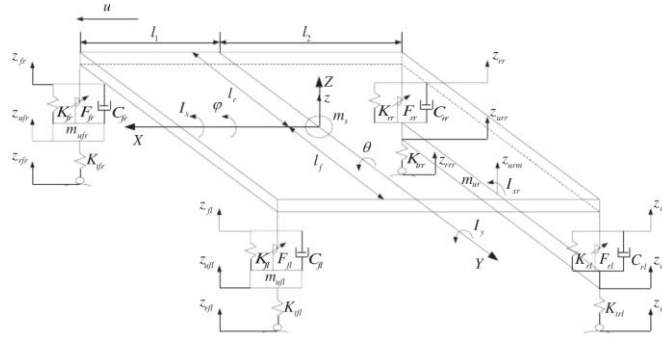


Figure 6 7 DoF for a Suspension System (Fang et al., 2011)

$$\begin{aligned}
 -m\ddot{z} &= k_{f1}(z - z_{f1} - a\theta - \beta\phi) + k_{f2}(z - z_{f2} - a\theta + \alpha\phi) + k_{r1}(z - z_{r1} + b\theta - \beta\phi) \\
 &\quad + k_{r2}(z - z_{r2} + b\theta + \alpha\phi) \\
 +C_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) &+ C_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} + \alpha\dot{\phi}) + C_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) \\
 &\quad + C_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} + \alpha\dot{\phi})
 \end{aligned}$$

Equation 8 $\ddot{Z}$ - Is the vertical Acceleration of the sprung mass, while $\dot{Z}$ and Z are the corresponding velocity and displacement while the $\dot{Z}_f$ and Z_f are vertical displacements of the unsprung masses of each wheel

$$\begin{aligned}
 -I_x\ddot{\theta} &= bk_{r1}(z - z_{r1} + b\theta - \beta\phi) + bk_{r2}(z - z_{r2} + b\theta + \alpha\phi) \\
 &\quad - ak_{f1}(z - z_{f1} - a\theta - \beta\phi) - ak_{f2}(z - z_{f2} - a\theta + \alpha\phi) \\
 +bC_{r1}(\dot{z} - \dot{z}_{r1} + b\dot{\theta} - \beta\dot{\phi}) &+ bC_{r2}(\dot{z} - \dot{z}_{r2} + b\dot{\theta} + \alpha\dot{\phi}) - aC_{f1}(\dot{z} - \dot{z}_{f1} - a\dot{\theta} - \beta\dot{\phi}) \\
 &\quad - aC_{f2}(\dot{z} - \dot{z}_{f2} - a\dot{\theta} + \alpha\dot{\phi})
 \end{aligned}$$

Equation 9 m - Mass of the sprung body - θ is the pitch, relating to the acceleration and angle

$$\begin{aligned}
 -I_y\ddot{\phi} &= \alpha k_{f1}(z - z_{f1} - a\theta - \beta\phi) + \alpha k_{r2}(z - z_{r2} + b\theta + \alpha\phi) \\
 &\quad - \beta k_{f1}(z - z_{f1} - a\theta - \beta\phi) - \beta k_{r1}(z - z_{r1} + b\theta - \beta\phi)
 \end{aligned}$$

$$+ \alpha C_{f_1}(\dot{z} - \dot{z}_{f_1} - a\dot{\theta} - \beta\dot{\phi}) + \alpha C_{r_2}(\dot{z} - \dot{z}_{r_2} + b\dot{\theta} + \alpha\dot{\phi}) - \beta C_{f_1}(\dot{z} - \dot{z}_{f_1} - a\dot{\theta} - \beta\dot{\phi}) \\ - \beta C_{r_1}(\dot{z} - \dot{z}_{r_1} + b\dot{\theta} - \beta\dot{\phi})$$

Equation 10 Φ - Roll Angle - Roll Acceleration - Also I_x, I_y are the moments of inertia for pitch and roll motion

$$-m_{f_1}\ddot{z}_{f_1} = k_{f_1}(z_{f_1} - y_{f_1}) - k_{f_1}(z - z_{f_1} - a\theta - \beta\phi) + C_{f_1}(\dot{z}_{f_1} - \dot{y}_{f_1}) \\ - C_{f_1}(\dot{z} - \dot{z}_{f_1} - a\dot{\theta} - \beta\dot{\phi})$$

Equation 11 k_f - k_r / c_f - c_r Spring stiffness constants and damping coefficients for each wheel

$$m_{f_2}\ddot{z}_{f_2} = k_{f_2}(z_{f_2} - y_{f_2}) - k_{f_2}(z - z_{f_2} - a\theta + \alpha\phi) + C_{f_2}(\dot{z}_{f_2} - \dot{y}_{f_2}) \\ - C_{f_2}(\dot{z} - \dot{z}_{f_2} - a\dot{\theta} + \alpha\dot{\phi})$$

Equation 12 y_f - y_r / $\dot{z}$ Ground height at each wheel and Ground velocity at each wheel position

$$m_{r_1}\ddot{z}_{r_1} = k_{r_1}(z_{r_1} - y_{r_1}) - k_{r_1}(z - z_{r_1} + b\theta - \beta\phi) + C_{r_1}(\dot{z}_{r_1} - \dot{y}_{r_1}) \\ - C_{r_1}(\dot{z} - \dot{z}_{r_1} + b\dot{\theta} - \beta\dot{\phi})$$

Equation 13 a-b Distances from the centre of mass to the front and rear axle

$$m_{r_2}\ddot{z}_{r_2} = k_{r_2}(z_{r_2} - y_{r_2}) - k_{r_2}(z - z_{r_2} + b\theta + \alpha\phi) + C_{r_2}(\dot{z}_{r_2} - \dot{y}_{r_2}) \\ - C_{r_2}(\dot{z} - \dot{z}_{r_2} + b\dot{\theta} + \alpha\dot{\phi})$$

Equation 14 α, β Lateral offsets of the suspension elements from the roll centre (Fang et al., 2011)

In motorsport, particularly in Formula One, the **seven-degree-of-freedom (7DOF) model** is instrumental in analysing and optimizing vehicle dynamics. This model accounts for the vehicle's heave, pitch, roll, and the independent vertical movements of each wheel, providing a comprehensive understanding of how a car responds under various conditions (Santos et al., 2006).

As the pursuit of performance gains intensifies, teams are advancing beyond the 7DOF model to more complex simulations, such as **10DOF** or even **14DOF** models. These higher-order models incorporate additional factors like detailed suspension kinematics, compliance, and tire dynamics, offering a more precise representation of real-world conditions. Research comparing different model complexities found that while a 7DOF model provides valuable insights, higher-order models like the 14DOF offer increased accuracy in lap time simulations, capturing nuances that simpler models might miss (Lot and Dal Bianco, n.d.).

1.3 Objectives of the Study

The objective of this study is to develop and analyse a 7 Degrees of Freedom (7DOF) Spring-Mass-Damper model of a Formula Ford Race Car using Simulink, focusing on its transient vehicle response behaviour under various driving conditions. The model will incorporate longitudinal, lateral, and vertical motion, as well as roll, pitch, to provide an accurate representation of the car's performance.

Table 1 Formula Ford Race Car data – Data used to create the 7 DoF

Vehicle Mass (kg)	M_T	514
Front Corner Unsprung Mass (kg)	M_{usf}	22
Rear Corner Unsprung Mass (kg)	M_{usr}	22
Vehicle Sprung Mass (kg)	M_s	426
Centre of Gravity Height (m)	CoG	0.277
Weight Split F:R (%)	-	45:55
Wheelbase (m)	W	2.443
Front Track (m)	T_f	1.62
Rear Track (m)	T_r	1.57
Front Tyre Rolling Radius (m)	r_f	0.2667
Rear Tyre Rolling Radius (m)	r_r	0.2794
Front Motion Ratio	MR_f	1
Rear Motion Ratio	MR_r	1
Front Default Spring Rate (N/mm)	K_{sf}	18
Rear Default Spring Rate (N/mm)	K_{sr}	16
Front Roll Centre Height (mm)	h_{rf}	0.9
Rear Roll Centre Height (mm)	h_{rr}	59
Front Slow Bump	N/m/s	1660
Front Fast Bump	N/m/s	511
Front Slow Rebound	N/m/s	1915
Front Fast Rebound	N/m/s	511
Rear Slow Bump	N/m/s	1477
Rear Fast Bump	N/m/s	537
Rear Slow Rebound	N/m/s	1612
Rear Fast Rebound	N/m/s	537



Race Car parameters, including mass distribution, suspension properties, and tire characteristics, will be used to refine the model. Analysis will be conducted to assess the impact of variations in suspension damping and tire stiffness on the vehicle's response. The model's accuracy will be validated through comparisons with theoretical predictions or available experimental data, obtained from the Base Performance Simulator.

2 Methodology

2.1 Equations of Motion

To develop a seven-degree-of-freedom (7DOF) model in Simulink (using MATLAB), it was first essential to thoroughly understand the governing equations of motion and how to implement them within a block diagram framework. This involved analysing the sprung and unsprung mass dynamics, the interactions between forces and moments, and the vehicle's response to different driving conditions.

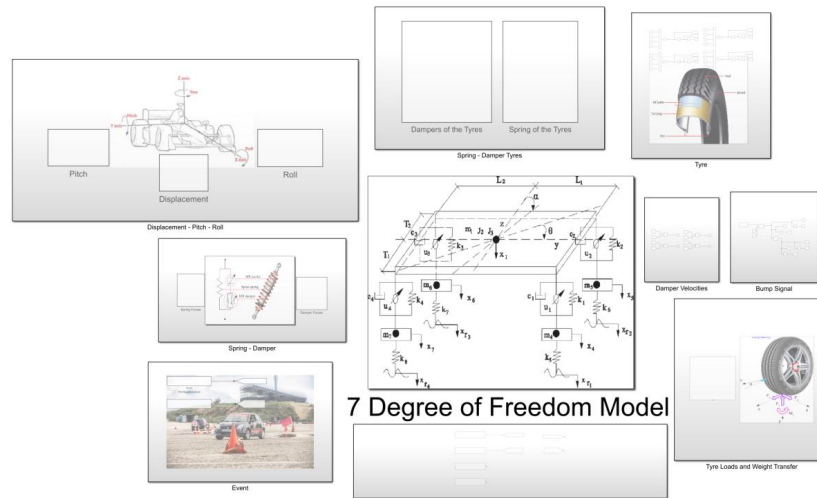


Figure 7 Overview of all the Simulink Model with all the different subsystems

These governing equations are displayed in Equation 4 for the complete seven-degree model. Establishing these equations for vertical motion, pitch, roll, and independent wheel displacements based on the spring-damper system it was possible to start creating the model inside Simulink using a similar approach to how the previous models were developed.

2.2 Development of the Simulink Model

1. Input Parameters

Before constructing the Simulink model, the first essential step is to define the input parameters that will govern the behaviour of the 7DOF vehicle dynamics simulation. These values, which include vehicle mass, suspension characteristics, tire properties, and geometric parameters, are inputted into MATLAB as shown in Figure 10. By assigning these parameters directly in MATLAB and ensuring that the variable names remain consistent throughout the model, the process of building the Simulink model becomes significantly more streamlined.

```
% Geometric parameters
alphaf = 1.62/2; % 1/2 front track
alphafr = 1.57/2; % 1/2 rear track
betaf = 1.62/2; % 1/2 front track
betar = 1.57/2; % 1/2 rear track
a = 2.443 * 0.45; % Distance to COG from front axle
b = 2.443 * 0.55; % Distance to COG from rear axle

% Suspension and tyre parameters (front corners)
kf1 = 18000; % Spring stiffness front corner
ktf1 = 190000; % Tyre stiffness front corner
cf1 = 1156.375; % Damping front corner
ctf1 = 330; % Tyre damping front corner
kf2 = 18000; % Spring stiffness front corner (duplicate if needed)
ktf2 = 190000; % Tyre stiffness front corner
cf2 = 1156.375; % Damping front corner
ctf2 = 330; % Tyre damping front corner

% Suspension and tyre parameters (rear corners)
kr1 = 16000; % Spring stiffness rear corner
ktr1 = 190000; % Tyre stiffness rear corner
cr1 = 1055.563; % Damping rear corner
ctr1 = 330; % Tyre damping rear corner
kr2 = 16000; % Spring stiffness rear corner (duplicate if needed)
ktr2 = 190000; % Tyre stiffness rear corner
cr2 = 1055.563; % Damping rear corner
ctr2 = 330; % Tyre damping rear corner

% Mass properties
m = 426; % Sprung mass
mf1 = 22; % Unsprung mass front corner
mf2 = 22; % Unsprung mass front corner
mr1 = 22; % Unsprung mass rear corner
mr2 = 22; % Unsprung mass rear corner

% Inertia properties
ixx = 0.28*(m+mf1+mf2+mr1+mr2) - 71.6; % Roll moment of inertia
iyy = 2.56*(m+mf1+mf2+mr1+mr2) - 1103; % Pitch moment of inertia

% Additional geometry
h = 0.02609; % Distance between RC height and GOG height
```

Figure 8 MATLAB inputted data of the Formula Ford Race Car

Now properly make the model work it not only needs the data from the car but also the inputs for each session which in this case are the Lateral Forces and Longitudinal Forces applied in the car, in the code below is it a representation of how from a .mat file the right variable from each session was retrieved.

```

numSessions = 18;

% Preallocate a structure array to hold data for each session. ***
sessionData = struct();

for i = 1:numSessions
    % Construct the file name for the current session
    fileName = sprintf('Session %d.mat', i);

    % Load the variables 'G_Force_Lat' and 'G_Force_Long' from the file
    loadedData = load(fileName, 'G_Force_Lat', 'G_Force_Long');

    % Extract 'G_Force_Lat'
    if isstruct(loadedData.G_Force_Lat)
        latData = loadedData.G_Force_Lat.Value;
    else
        latData = loadedData.G_Force_Lat;
    end
    % Ensure the data is a column vector
    sessionData(i).Lat = latData(:);

    % Extract 'G_Force_Long'
    if isstruct(loadedData.G_Force_Long)
        longData = loadedData.G_Force_Long.Value;
    else
        longData = loadedData.G_Force_Long;
    end
    sessionData(i).Long = longData(:);
end

```

Figure 9 Code to retrieve the Lat & Long Forces from each session having a .mat file

Setup number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
front spring rate (N/mm)	18	22	26	14	9	26	9	18	18	18	18	18	18	18	18	18	18	18
rear spring rate (N/mm)	16	20	24	12	8	8	24	16	16	16	16	16	16	16	16	16	16	16
front slow bump (N/m/s)	1645	1645	1645	1645	1645	1645	1645	1645	1645	1245	2045	1645	1645	1645	1645	1645	1645	1645
front fast bump (N/m/s)	533.25	533.25	533.25	533.25	533.25	533.25	533.25	383.25	683.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25	533.25
front slow rebound (N/m/s)	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1936.25	1436.25	2436.25	1936.25	1936.25	1936.25
front fast rebound (N/m/s)	511	511	511	511	511	511	511	511	511	511	511	383.25	638.75	511	511	511	511	511
rear slow bump (N/m/s)	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75	1107.75	1907.75	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75	1507.75
rear fast bump (N/m/s)	552.75	552.75	552.75	552.75	552.75	552.75	552.75	402.75	702.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75
rear slow rebound (N/m/s)	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1609	1209	2009	1609	1609	1609
rear fast rebound (N/m/s)	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	552.75	402.75	702.75	552.75	552.75	552.75	552.75	552.75
ARB front (N/mm)	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	8	8	Detached
ARB rear (N/mm)	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	Detached	8	Detached	8

Figure 10 Setups for each run changing front and rear spring rate, front and rear slow and fast bump reaction and front and rear ARB

To ensure a valid comparison between the kinematics model (simulation using the simulator) and the 7 Degree of Freedom (7DOF) model in Simulink, a total of 18 runs were conducted, systematically altering the available vehicle parameters. Each run incorporated variations in front and rear spring rates, slow and fast bump reactions, and front and rear anti-roll bars, as shown in Figure 11. To streamline the data input process, color-coding was applied to the table, making it easier to identify parameter changes and efficiently transfer values into MATLAB.

that all governing equations receive the correct data, starting from predefined vehicle parameters inputted in MATLAB.

Since each equation depends on specific inputs, the model must first establish the source of the data before performing any calculations. This step-by-step execution prevents errors and undefined values while maintaining clarity and ease of debugging while ensuring that all subsystems are modular, organized, and interact seamlessly within the 7DOF model, contributing to a coherent and accurate vehicle dynamics simulation.

It is important to mention that this left-to-right structure is applied both at the system level and within each subsystem. Each governing equation—whether for Z-Pitch-Roll motion, suspension dynamics, tire behavior, or weight transfer calculations—follows the same logical flow.

3. Spring – Damper Equations

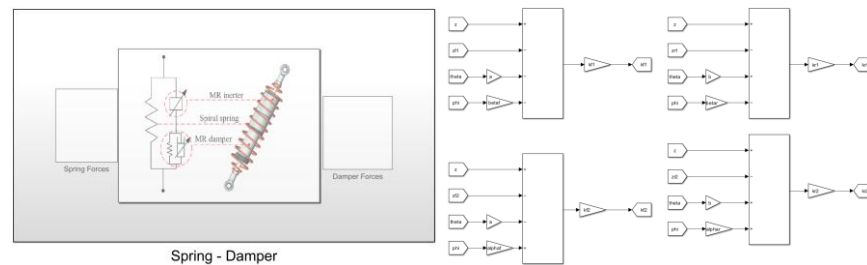


Figure 14 Subsystem 2, Spring - Damper Equations - Spring Equations as for this study there were 18 different set ups in which the value of $kf1$, $kf2$, $kr1$ and $kr2$ changed for each set up

The change in variable was made using a for loop which determined by the number of the set up / session it used the correct spring stiffness for all 4 springs of the car it can be seen in figure 13.

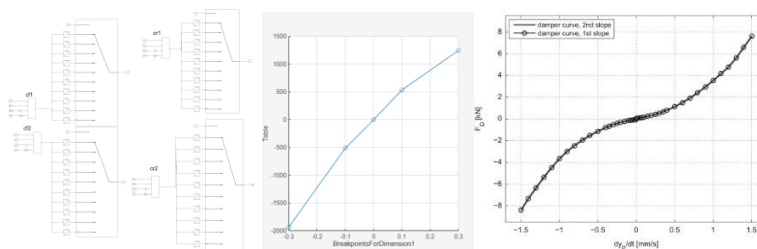


Figure 15 Damper Equations (Including the damper curves) – Example of Damper static characteristics determines from the tests with triangular displacement excitations (Titurus et al., 2010)

Since the spring rate (stiffness) changes for each setup were successfully implemented using a MATLAB script, the same approach was applied to automate the assignment of damper curve

parameters. This ensured that for each run, the correct damper values were assigned alongside the spring stiffness values, maintaining consistency across all 18 setups.

4. Tyre Equations

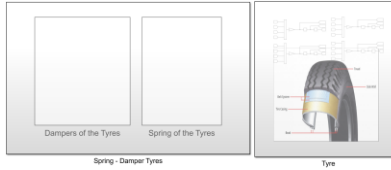


Figure 16 Tyre Subsystem 3-4 Equations taking into account the spring and damper elements of the tyres

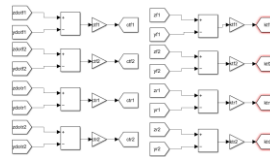


Figure 17 Damper and Spring Element of the tyres of each tyre

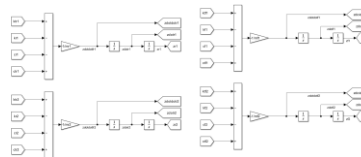


Figure 18 Tyre Equation for each tyre

The characteristics of the tyres did not change throughout the sessions so it was the quickest and easiest section of the car to simulate.

5. Bump Input

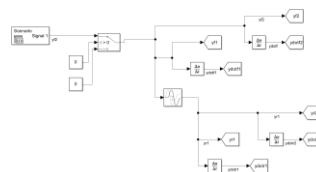


Figure 19 Bump Input using Signal Editor

Before introducing lateral and longitudinal forces into the simulation, the 7DOF model first needed to be tested against a bump input to verify the correct implementation of the suspension system. This validation step ensured that the system responded as expected, with no errors in sign conventions, force calculations, or parameter assignments.

6. Damper Velocities

2.3 Assumptions and Limitations

Several simplifications and assumptions were made to ensure computational feasibility while maintaining accuracy.

1. Idealized Tyre Models

- a. Assumed that the contact patch remains constant regardless of load variations.
- b. Ignored tyre temperature, pressure and grip variations

These factors significantly influence tyre behaviour, but their inclusion requires a nonlinear tyre model, such as the Pacejka "Magic Formula," which drastically increases complexity. Even in a simplified Ford model, implementing this approach would require accounting for parameters like sideways coefficient of friction, wheel friction coefficient, surface friction, wheel lateral stiffness, lateral peak factor, lateral shape factor, and lateral curvature, all of which govern tyre grip and response under different conditions (Blundell and Harty, 2015).

These parameters are often derived from extensive experimental data, making their integration computationally demanding. A higher-order model, such as a 10DOF or 14DOF system, would be necessary to fully incorporate these effects for a more accurate tyre-road interaction.

2. Neglecting Aero Effects

- a. Although some runs considered aero loads, the aerodynamic forces were simplified using a basic lift and drag equation with assumed coefficients
- b. The only reliable input was the car's velocity (m/s), while frontal area and coefficients were estimated
- c. A higher order model would require detailed CFD simulations or wind tunnel data which were not available

3. Simplified Suspension Representation

- a. Assumed the suspension system as a for single point model (one per wheel)
- b. Neglected complex weight distribution effects in suspension geometry

- c. A 10 or 14 DoF would be required to incorporate suspension linkage kinematics, compliance, and flexibilities

4. No Errors in Calculation Procedures

- a. The 7 DoF model follows a force momentum approach, while the kinematics model uses a geometric displacement-based method which means that both models use different mathematical frameworks, a percentage of deviation is expected and does not indicate a calculation error

5. Rigid Body for the Chassis

- a. Assumed the chassis is a perfectly rigid body, ignoring chassis flex and torsional stiffness effects
- b. This flex influences suspension loads, but accounting for it requires a multi-body dynamics (MBD) approach with a finite element model (FEM), increasing computational power requirements

6. No Asymmetry in Front and Rear Suspension Geometry

- a. The model treats front and rear suspensions symmetrically, assuming identical behaviour in left and right suspension elements
- b. In real world applications, caster angles, camber gain, and roll centre heights differ between front and rear suspensions, affecting weight transfer and handling

7. No Lateral Load Transfer Due to Roll Centre Migration

- a. Assumed that lateral load transfer occurs only due to roll stiffness and CG height, without considering roll centre migration effects
- b. A more advanced suspension kinematics model would account for instantaneous centre variations, affecting roll moment distribution and chassis behaviour.

3. Results and Discussion

3.1 Direct Data Correlation – Visual Representation

This section provides a visual correlation between the 7DOF Simulink model and MoTeC data, comparing key parameters: Roll Angle, Pitch Angle, Damper Velocities and Tyre Loads. While multiple datasets were analysed, representative graphs are presented. However, the overall trends and specific cases observed throughout the study will be discussed to highlight key insights and discrepancies between the two models.

3.1.1 Roll Angle

The 7DOF Simulink model closely follows the kinematics model (MoTeC data) in overall trends, confirming its validity in predicting vehicle dynamics. However, deviations occur at peak values, where the model underestimates some higher oscillations in this specific case, due to simplified suspension compliance and tire damping assumptions.

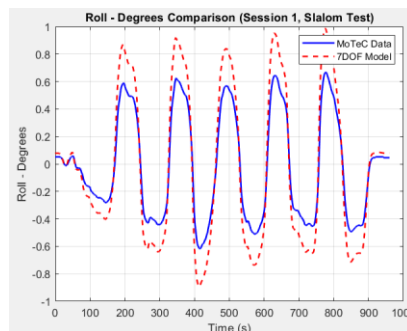


Figure 23 Roll Degrees - First Session no changes

In Session 5 (Figure 26), the 7DOF model nearly matches the MoTeC data, but a slight time shift is observed. This is due to an intentional adjustment in the suspension system, where initial inputs cause peak reactions at the first-time step. Despite these minor differences, the model effectively captures roll, pitch, and damper behavior, with refinements in higher DOF models potentially improving peak accuracy.

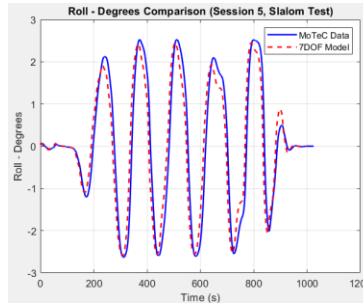


Figure 24 Roll Degrees - Session #5 - Lower Front and Rear Spring Stiffness

Adjusting the front and rear spring stiffness significantly improved the visual accuracy of the 7DOF Simulink model, as shown in the roll angle comparison for Session 5 (Slalom Test). The revised stiffness values enhanced the model's ability to match the oscillatory behavior observed in the MoTeC data, reducing deviations in roll amplitude. This adjustment suggests that suspension stiffness plays a key role in accurately capturing roll dynamics, particularly in transient maneuvers like slalom tests, where weight transfer effects are critical.

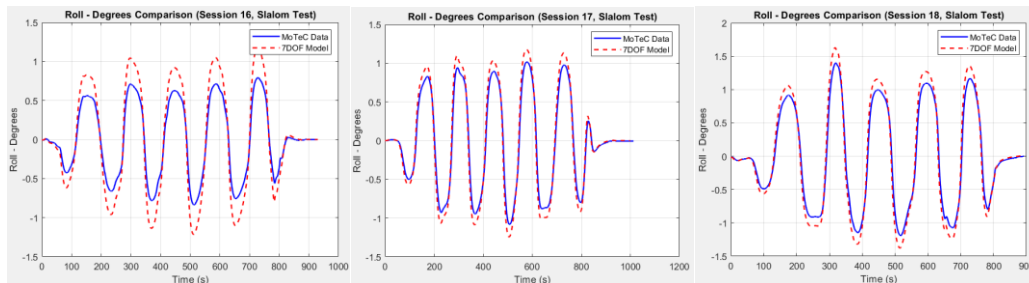


Figure 25 Roll Degrees - Session #16-17-18 - Anti Roll Bar Attached

As seen in Figure 27, the 7DOF Simulink model consistently overestimates roll angle in Sessions 16, 17, and 18, where the anti-roll bar was attached to the car. This discrepancy arises because the 7DOF model does not include anti-roll bar equations, leading to higher predicted roll values than those recorded in MoTeC data.

Despite this limitation, the model accurately captures the overall trend, demonstrating strong correlation in oscillatory behavior and dynamic response. This suggests that, while anti-roll bar effects are missing, the core suspension and weight transfer modeling remain valid, highlighting the robustness of the 7DOF framework in replicating vehicle dynamics.

3.1.2 Pitch Angle

The 7DOF Simulink model does not account for certain real-world suspension setup factors, as outlined in the assumptions section. One key omission is the rake angle, which refers to the static pitch inclination of the vehicle due to suspension geometry. This results in a constant pitch offset, even when the car is at rest, which can be observed across all sessions.

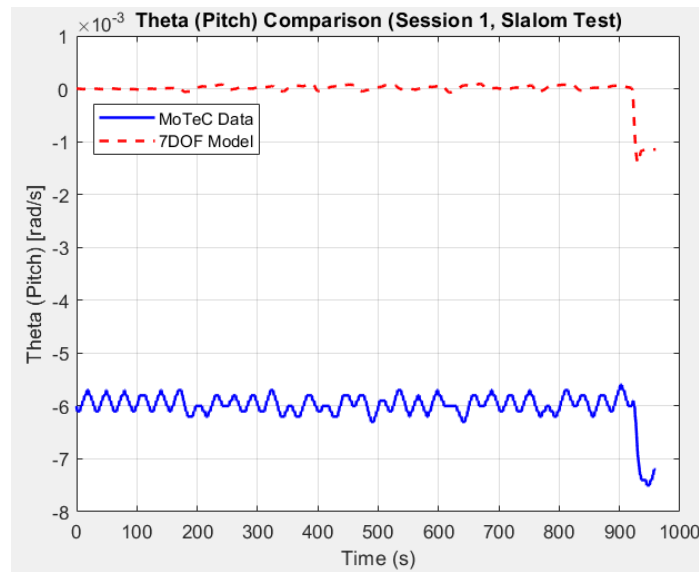


Figure 26 Pitch Angle - Session #3

In Figure 28 (Session 3), the offset between MoTeC data and the 7DOF model is 0.006, showing that while the model accurately follows pitch dynamics, it lacks the static rake angle present in the real system. Despite this, the trend and transient responses remain well-correlated. The pitch angle stays nearly constant at 48.5 mph, with no major variations, but decreases under braking as weight transfers forward, compressing the front suspension and unloading the rear, aligning with expected vehicle dynamics. Including rake effects and refined suspension kinematics could improve accuracy.

3.1.3 Damper Velocities

As observed in the damper velocity comparison graphs, the 7DOF Simulink model accurately replicates the MoTeC simulation data, demonstrating a strong correlation between the two. The model successfully captures both small and large velocity changes [mm/s], maintaining alignment with the kinematics-based simulation model throughout the test.

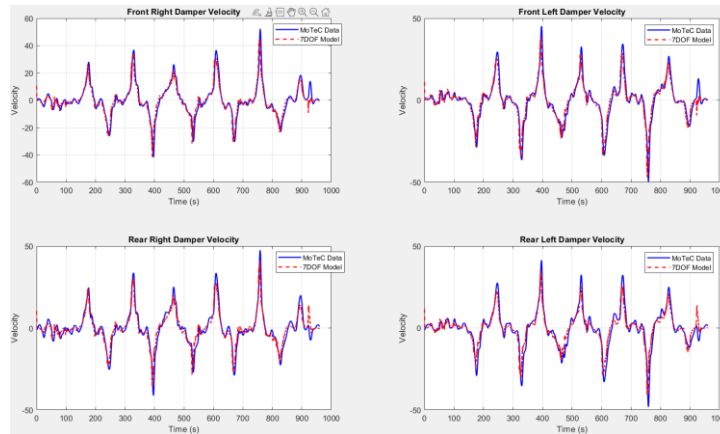


Figure 27 Damper Velocity - First Session - No Changes of the dampers

The consistency across all four dampers—front right, front left, rear right, and rear left—indicates that the suspension dynamics and damping characteristics were well-represented in the 7DOF model. The precise replication of damper velocity fluctuations confirms the model's ability to simulate transient suspension behavior, making it a reliable tool for further vehicle dynamics analysis.

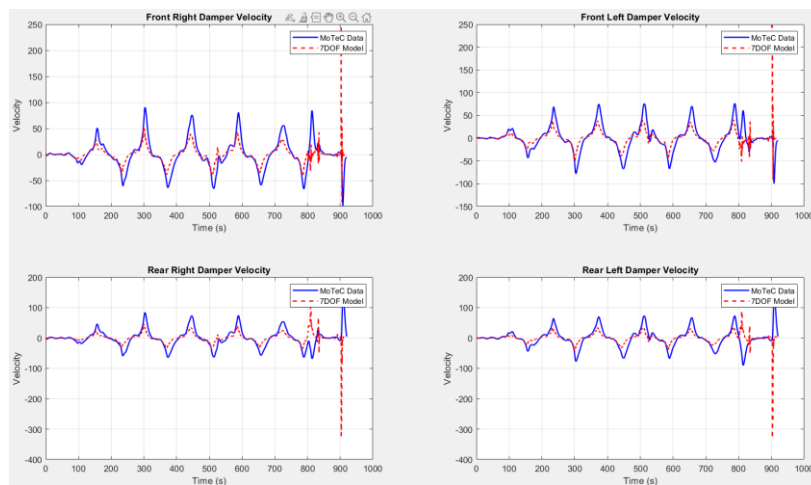


Figure 28 Damper Velocity - Session #10 - Changing the damper curves as from the initial

3.1.4 Tyre Loads

Visually analyzing the tyre load comparison, the 7DOF Simulink model accurately predicts the front tyre loads, closely matching the MoTeC data in trend and magnitude. However, the rear tyre loads show significant deviations, likely due to neglecting full load-weight transfer effects in the model.

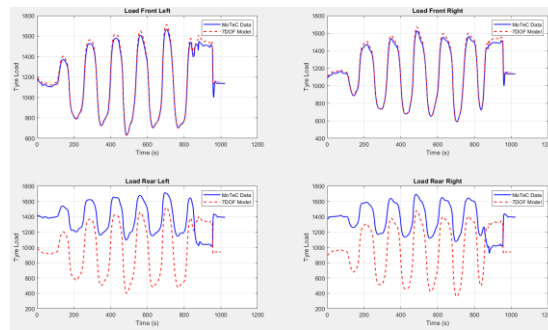


Figure 29 Tyre Loads - Session #6

Despite these discrepancies, the general trend remains consistent, suggesting that the core load transfer dynamics are partially captured. The rear loads are consistently overestimated, with a constant offset of approximately 500 N, indicating a systematic shift in the model's weight distribution assumptions. While the front tyre loads were well-modeled, refining weight transfer equations and incorporating more complex suspension interactions could improve accuracy in the rear load predictions.

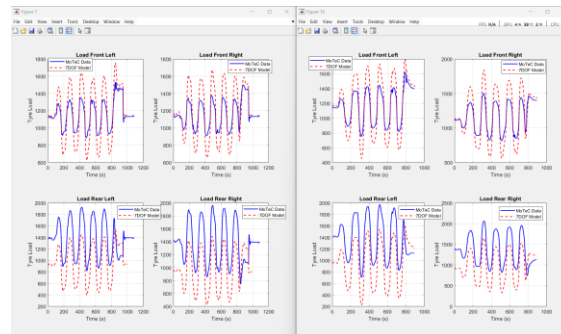


Figure 30 Tyre Loads - Session #7-18

A consistent pattern appears across all sessions, where front loads are underestimated, and rear loads are overestimated. Swapping these values would likely improve correlation. Adjusting load transfer equations and weight distribution assumptions could enhance accuracy.

3.2 Root Mean Square Error Correlation (RMSE)

The root means square error is a widely used metric for evaluating models. RMSE is optimal for normal (Gaussian) which are the errors (Hodson, 2022). The Gaussian is a theoretically derived distribution of random errors of measurement when the same object is measured with the same instrument under constant conditions (Andrich, 2018).

3.2.1 Roll Angle

The average RMSE of 0.82 suggests a moderate correlation between the 7DOF Simulink model and MoTeC data, with Session 5 showing the highest deviation (~ 1.55), likely due to suspension settings and weight transfer effects. Early sessions (1-3) have lower RMSE (~ 0.4 - 0.8), indicating a better match, while Sessions 6-15 stabilize around 0.8 - 1.0 , suggesting consistent but moderate deviations, possibly due to simplifications in tire modeling and roll center assumptions. The higher RMSE values in certain setups suggest that simplifications in weight transfer and suspension dynamics contribute to deviations, particularly in high-roll scenarios.

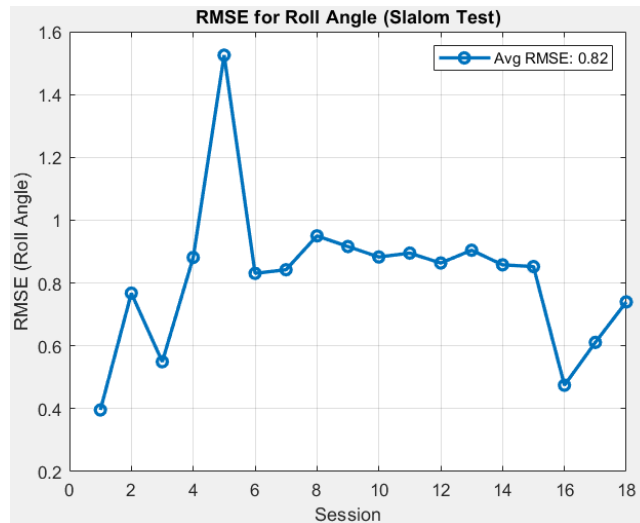


Figure 31 RMSE for Roll Angle for all sessions

A sensitivity analysis was conducted to assess the impact of suspension damping variations on the vehicle's dynamic response. Increasing damping values reduced oscillation amplitudes, leading to lower roll and pitch angles, while lower damping values resulted in more pronounced oscillations and longer settling times. This confirms that damping directly influences transient response and stability, particularly during rapid directional changes. The analysis highlights the importance of selecting appropriate damping coefficients to balance ride comfort and handling

performance, as excessive damping can limit compliance, while insufficient damping can lead to instability.

3.2.2 Pitch Angle

The average RMSE of 0.51 indicates a strong correlation between the 7DOF model and MoTeC data for pitch dynamics. Session 6 exhibits the highest RMSE (~ 0.65), likely due to simplified weight transfer effects and suspension compliance assumptions, while Sessions 1-3 show the lowest RMSE (~ 0.35 - 0.45), meaning the model closely follows real pitch behavior in these cases.

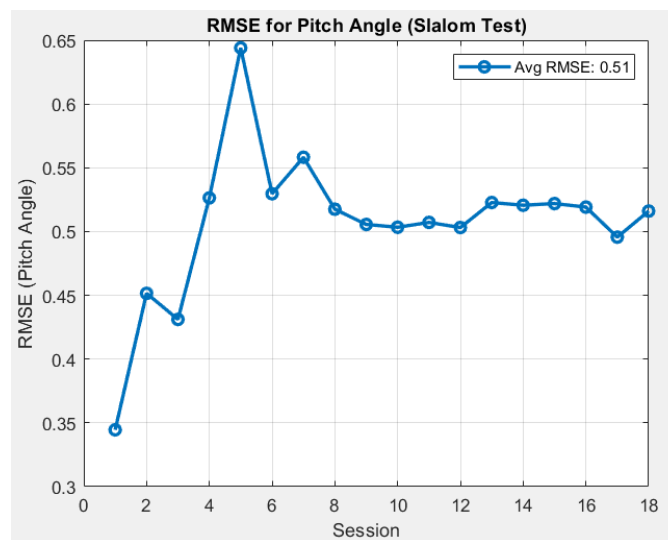


Figure 32 RMSE for Pitch Angle

From Sessions 7-18, RMSE stabilizes around 0.5, suggesting consistent but minor deviations, possibly due to neglected aerodynamic effects and the absence of a static pitch offset (rake angle). These results confirm that while the model effectively captures pitch trends, refinements in weight transfer equations and aerodynamic modeling could further improve accuracy.

3.2.3 Damper Velocities - Tyre Loads

The RMSE values for damper velocities and tyre loads are significantly higher than those for pitch and roll angles, yet they remain consistent across sessions, indicating a high precision but lower accuracy in the 7DOF model. This suggests that while the model reliably predicts

trends and variations, there is a systematic offset or deviation from the actual values observed in MoTeC data.

For damper velocities, the RMSE averages around 27-29, with a noticeable peak in early sessions, likely due to variations in suspension damping settings. Despite the high numerical values, the trend consistency confirms that the model effectively captures relative changes in damper response but may require refined damping coefficients to improve absolute accuracy.

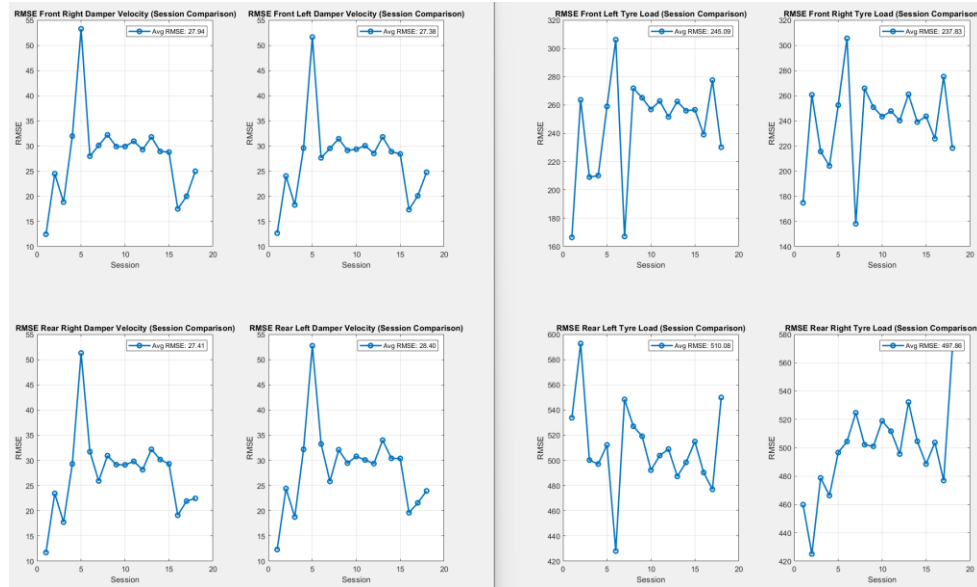


Figure 33 RMSE Damper Velocities - Tyre Loads for all session

For tyre loads, RMSE values fluctuate between 200-600 N, reinforcing the idea that load transfer effects are being captured but not perfectly scaled. The consistency in RMSE across sessions further supports the high precision, low accuracy characteristic of the model, meaning systematic refinements in weight transfer calculations, tyre modeling, and suspension compliance effects could improve correlation.

4. Conclusion

The development and validation of the 7 Degrees of Freedom (7DOF) vehicle dynamics model provided a comprehensive analysis of the transient response behavior of a Formula Ford race car. The model successfully captured key dynamic behaviors, including roll, pitch, heave, damper velocities, and tire loads, and was validated against MoTeC simulation data. Through direct data correlation and Root Mean Square Error (RMSE) analysis, the study demonstrated that the 7DOF model effectively follows trends observed in real-world simulations, with minor deviations due to assumptions and simplifications in suspension compliance, aerodynamic effects, and weight transfer modeling.

While the model showed high precision but lower accuracy, particularly in damper velocities and tire loads, the consistency of the deviations suggests that systematic refinements in parameter tuning could improve overall accuracy. The absence of anti-roll bar modeling was identified as a significant factor in roll angle overestimations, while the lack of a static rake angle (suspension geometry effects) contributed to a pitch offset in all sessions. Despite these limitations, the model accurately represented transient responses, proving useful for evaluating vehicle behavior under dynamic conditions.

To enhance the model's predictive capability, a sensitivity analysis was conducted on suspension damping, revealing its critical role in influencing vehicle stability and transient response. The study confirmed that higher damping values reduce oscillations, improving stability, while lower damping increases oscillatory behavior, affecting handling and ride comfort.

Looking forward, higher-order models such as 10DOF and 14DOF systems could further refine accuracy by incorporating detailed suspension kinematics, compliance, nonlinear tire models (e.g., Pacejka's Magic Formula), and aerodynamic effects. These refinements would provide a more realistic simulation environment, particularly for motorsport applications where even minor deviations can impact vehicle performance. Additionally, integrating multi-body dynamics (MBD) or Finite Element Models (FEM) would allow for chassis flex analysis, further improving load transfer predictions.

In conclusion, while the 7DOF model provides a solid foundation for transient vehicle response modeling, future enhancements in tire modeling, aerodynamic simulations, and higher DoF implementations will be necessary to match real-world behavior with greater accuracy.

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