

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

When using critical path analysis, it is assumed that reasonably accurate information is available on the duration and cost of activities. In efficient companies this should be true for the majority of activities. However, there may be circumstances when enough information is not available. This will be so for several reasons, such as when we have additional variables that we can't foresee (e.g. hard weather conditions affecting) or unique construction methods are used. It will also be so when networks are being used for projects involving research and development work. In these cases PERT can be used, as it takes account of uncertainty.

Uncertainty

For activities that are not on the critical path, where a considerable amount float is available, uncertainty may cause problems in scheduling labour, plant, material deliveries and subcontractor start times. For activities on the critical path, uncertainty will also result in the project duration being less clearly defined.

To allow for uncertainty, three estimates are used for activity duration:

Optimistic duration (d_o): This is defined as the minimum duration if everything goes well (differentiate it from the crash duration, they are not the same).

Most likely duration (d_m): this is based on analysis of work on the previous projects, experience and judgement.

Pessimistic duration (d_p): this is defined as the maximum time if everything goes wrong.

Activity probability distribution

The distribution can be showed in either direction, and the range is roughly determined by d_o and d_p . It is assumed that there is only a 1% chance of d_o or d_p to be exceeded.

With these estimates we can calculate the expected mean duration and uncertainty:

$$\text{expected mean duration, } d_e = \frac{d_o + 4d_m + d_p}{6}$$

$$\text{standard deviation, } \sigma d_e = \frac{d_p - d_o}{6}$$

$$\text{variance, } v d_e = \left(\frac{d_p - d_o}{6} \right)^2$$

If we represent graphically these values we will understand their meaning better, and we can see the activity probability distribution.

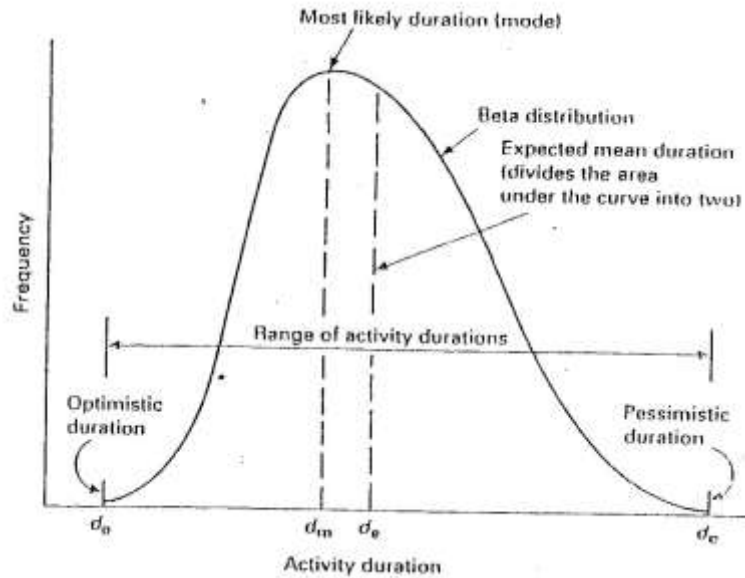


Figure 1. Activity probability distribution.

Critical path calculations

We can calculate the critical path using the expected mean duration for every activity. The standard deviation and the variance should also be calculated.

Event variance

Event variance based on earliest event times is the sum of the variances up to the event being considered following the critical path from the start of the project. For non-critical events, it is the sum of the variances at each event, following the path that determined the earliest event time.

Event variance based on latest event times is the sum of variances up to the event being considered following the critical path from the expected completion date. This variance gives the uncertainty remaining from the event being considered up to the end of the project measured along the critical path. For non-critical events it is the sum of the variances of each event, following the path that determined the latest event times.

Multiple critical paths

Where there is more than one critical path, the one with the maximum variance is used.

Meeting specific event schedule times

Event completion times are assumed to have a normal probability distribution with a mean value of T_x , the variance calculated as the event variance, and from this the standard deviation can be calculated. This assumption is strictly correct only for an infinite series, but is assumed to be accurate enough in practice for large projects. The event times in the example that follows are assumed to have normal distributions. To calculate the probability of achieving a specific event time, the area cut off by the specific time is calculated as a percentage of the total area beneath the curve.

Standard probability tables can be used for normal distribution functions, and a summarised table is given in the following text. Standard probability tables can be used for normal distribution functions. You can use the following table.

The factor Z is calculated thus:

$$z = \frac{T_s - T_x}{\sigma T_x}$$

Using this value the probability of meeting the specific event scheduled time (Ts) can be read off from the table, interpolating as necessary.

Determining a scheduled time based on a specific probability

The scheduled time for a specific probability can be established by transporting the formula as follows:

$$T_s = T_x + z\sigma T_x$$

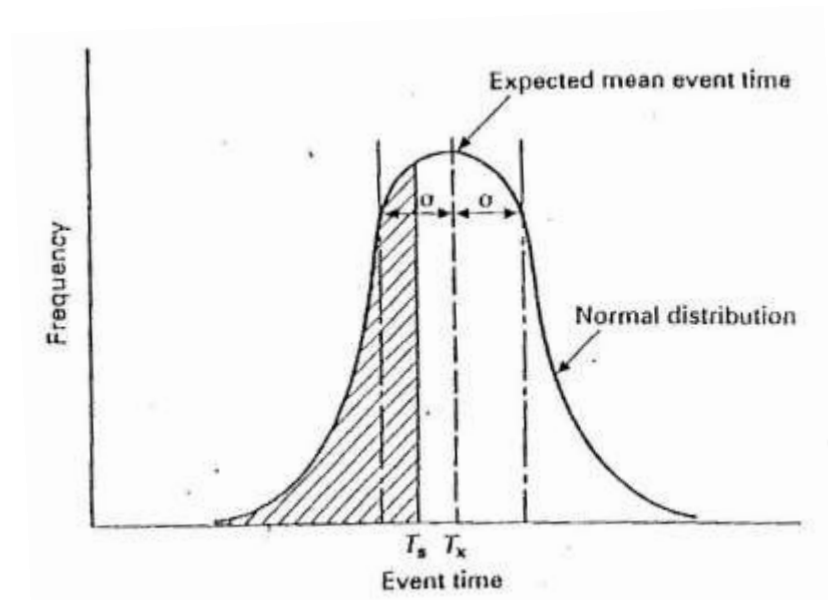


Figure 2. Calculation of the probability of achieving a specific event time.

z	area	z	area
-3.5	0.00023	0.0	0.5000
-3.4	0.00034	0.1	0.5398
-3.3	0.00048	0.2	0.5793
-3.2	0.00069	0.3	0.6179
-3.1	0.00097	0.4	0.6554
-3.0	0.00135	0.5	0.6915
-2.9	0.0019	0.6	0.7257
-2.8	0.0026	0.7	0.7580
-2.7	0.0035	0.8	0.7881
-2.6	0.0047	0.9	0.8159
-2.5	0.0062	1.0	0.8413
-2.4	0.0082	1.1	0.8643
-2.3	0.0107	1.2	0.8849
-2.2	0.0139	1.3	0.9032
-2.1	0.0179	1.4	0.9192
-2.0	0.0228	1.5	0.9332
-1.9	0.0287	1.6	0.9452
-1.8	0.0359	1.7	0.9554
-1.7	0.0446	1.8	0.9641
-1.6	0.0548	1.9	0.9713
-1.5	0.0668	2.0	0.9773
-1.4	0.0808	2.1	0.9821
-1.3	0.0968	2.2	0.9861
-1.2	0.1151	2.3	0.9893
-1.1	0.1357	2.4	0.9918
-1.0	0.1587	2.5	0.9938
-0.9	0.1841	2.6	0.9953
-0.8	0.2119	2.7	0.9965
-0.7	0.2420	2.8	0.9974
-0.6	0.2743	2.9	0.9981
-0.5	0.3085	3.0	0.99865
-0.4	0.3446	3.1	0.99903
-0.3	0.3821	3.2	0.99931
-0.2	0.4207	3.3	0.99952
-0.1	0.4602	3.4	0.99966
-0.0	0.5000	3.5	0.99977

Table 1. Standard normal distribution.

Example. Applying PERT to a construction project

Figure 3 shows a building project, and Table 2 shows the three estimated activity durations for each activity, together with the calculated expected durations (d_e), standard deviation (σd_e) and variance (vd_e). The method of analysis is as follows.

- 1.- Identify the critical path of this construction project.
- 2.- Calculate the earliest event times using the expected mean duration (d_e) by working through the diagram selecting the longest path (critical path). For that, you sum the time for every previous activity in this path.
- 2.- Calculate the latest event times, again using d_e , working backwards through the diagram selecting the longest path (critical path). For that, you subtract the time for every later activity in this path.
- 3.- Calculate the variance of each event from the first event using the variance of each activity (vd_e) by working through the diagram on the paths that determine the earliest event times. For that, you sum the variances for every previous activity in this path.
- 4.- Calculate the variance of each event from the last event, again using vd_e and following the paths that determine the latest event times. For that, you subtract the variances for every later activity in this path.

5.- It can be seen from Figure 3 that the project duration is 278.67 days, and the variance of the last event is 35.93. With this variance, we calculate a standard deviation of 5.99.

The probability of completing the project in 278.67 days is 50% (0.5), as it corresponds to the expected mean durations and event times are assumed to be normal distributions.

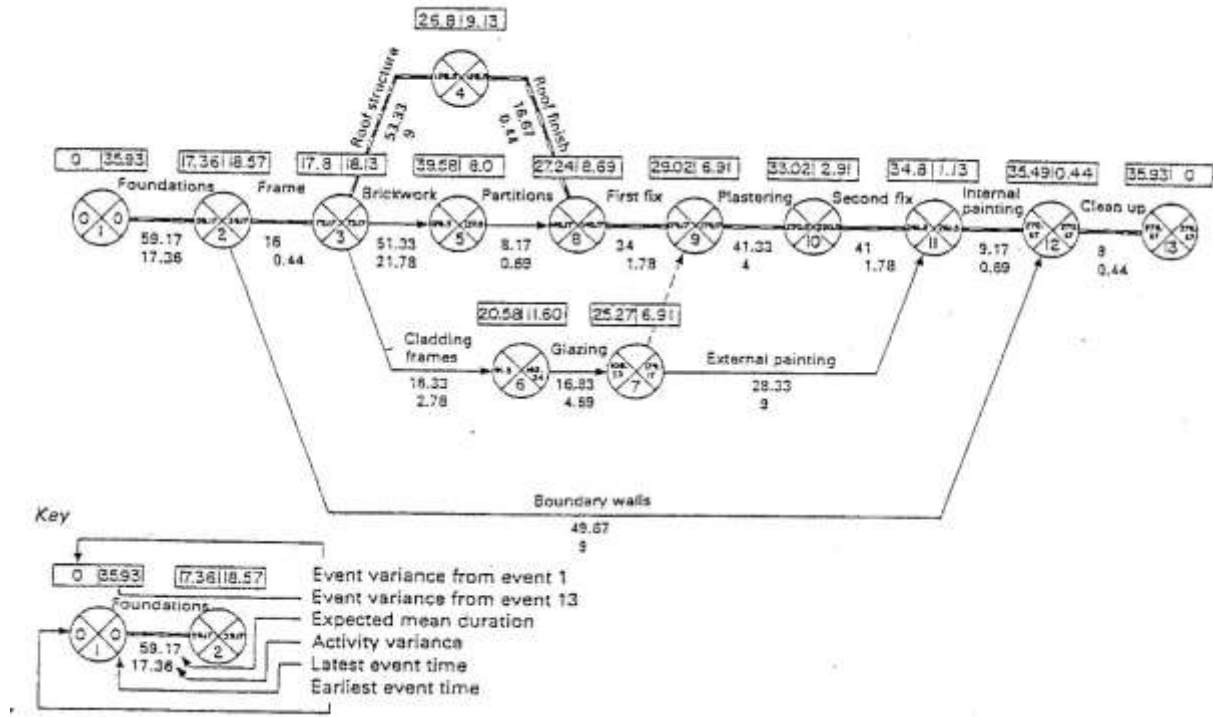


Figure 3. Arrow diagram of a building project example.

Activity	d_o	d_m	d_p	d_e	V_{de}	σ_{de}
Foundations	45	60	70	59.17	17.36	4.17
Frame	14	16	18	16.00	0.44	0.67
Roof structure	47	52	65	53.33	9.00	3.00
Roof finish	14	17	18	16.67	0.44	0.67
Brickwork	42	49	70	51.33	21.78	4.67
Partitions	69	8	11	8.17	0.69	0.83
First fix	30	34	38	34.00	1.78	1.33
Plastering	38	40	50	41.33	4.00	2.00
Second fix	37	41	45	41.00	1.78	1.33
Internal painting	7	9	12	9.17	0.69	0.83
Cladding frames	12	16	22	16.33	2.78	1.67
Glazing	12	16	25	16.83	4.69	2.17
External painting	22	27	40	28.33	9.00	3.00
Brickwork to boundary walls	42	49	60	49.67	9.00	3.00
Clear up	6	8	10	8.00	0.41	0.67

Table 2. Estimated activity durations for building project example.

To calculate the probability of completing the project in 270 days, first calculate the factor z:

$$z = \frac{270 - 278.67}{5.99} = -1.45$$

From Table 1, probability is 0.07 or 7%.

To calculate the project duration if it is required to be 90% certain of completion on time, first look up the factor z for 90% probability in Table 1. With it, you substitute in the formula and calculate the duration.

$$\frac{x - 278.67}{5.99} = 1.3$$

$$x = 286.46 \text{ days}$$