

Stress Analysis Exam

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A	B	C	D	E	F	G
1	9	0	0	9	6	6

Question 1

$$\begin{aligned}\epsilon_x &= 26 \times 10^{-6} \\ \epsilon_y &= -16 \times 10^{-6}\end{aligned}$$

$$\begin{aligned}E &= 17 \times 10^9 \text{ N/m}^2 \\ \nu &= 0.28\end{aligned}$$

$$i) \sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_x = \frac{17 \times 10^9}{1-(0.28)^2} (26 \times 10^{-6} + 0.28(-16 \times 10^{-6}))$$

$$\sigma_x = +396.96 \times 10^3 \text{ Pa (tensile)}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\sigma_y = \frac{17 \times 10^9}{1-(0.28)^2} (-16 \times 10^{-6} + (0.28)(26 \times 10^{-6}))$$

$$\sigma_y = -160.85 \times 10^3 \text{ Pa (compressive)}$$

$$ii) \sigma_x = +86 \times 10^6 \text{ Pa}$$

$$\sigma_y = -46 \times 10^6 \text{ Pa}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

$$\epsilon_x = 5816.41 \times 10^{-6}$$

$$\epsilon_x = \frac{1}{17 \times 10^9} (86 \times 10^6 - 0.28(-46 \times 10^6))$$



$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x)$$

$$\epsilon_y = \frac{1}{17 \times 10^9} (-46 \times 10^6 - 0.28(86 \times 10^6))$$

$$\epsilon_y = -4122.35 \times 10^{-6}$$

QUESTION 2

$$\sigma_x = 109 \times 10^6 \text{ Pa}$$

$$\sigma_y = 206 \times 10^6 \text{ Pa}$$

$$\tau_{xy} = 125 \times 10^6 \text{ Pa}$$

i) ~~stress~~

$$\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_1 = \frac{109 \times 10^6 + 206 \times 10^6}{2} + \frac{\sqrt{(109 \times 10^6 - 206 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 291.58 \times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_2 = \frac{109 \times 10^6 + 206 \times 10^6}{2} - \frac{\sqrt{(109 \times 10^6 - 206 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_2 = 23.42 \times 10^6 \text{ Pa}$$

$$ii) \tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{291.58 \times 10^6 - 23.42 \times 10^6}{2}$$

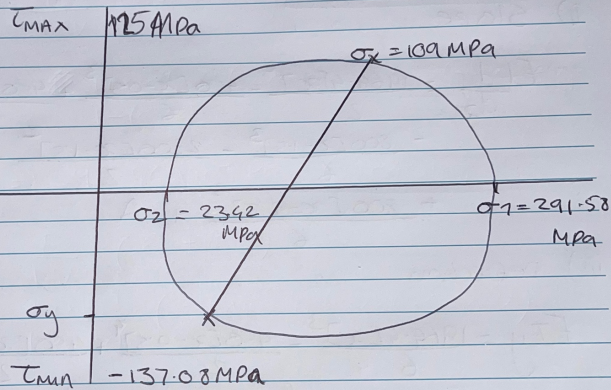
$$\tau_{max} = 137.08 \times 10^6 \text{ Pa}$$

$$\tau_{min} = - \left[\frac{\sigma_1 - \sigma_2}{2} \right]$$

$$\tau_{min} = - \left[\frac{291.58 \times 10^6 - 23.42 \times 10^6}{2} \right]$$

$$\tau_{min} = -137.08 \times 10^6 \text{ Pa}$$

$$iii) \tau_{max} = 137.08 \text{ MPa}$$



QUESTION 3.

$$R_A = 1969.4 \text{ N}$$

$$R_B = 2580.6 \text{ N}$$

i) Bending moment

$$EI \frac{d^2 y}{dx^2} = +1969.4[x] - \frac{450[x-0.5]^2}{2} + \frac{450[x-1.5]^2}{2} - \frac{800[x-1.5]^2}{2} - \frac{2300[x-1.5]^3}{6} - \frac{1000[x-2.5]^3}{6} + \frac{800[x-2.5]^2}{2}$$

ii) Slope

$$EI \frac{dy}{dx} = \frac{1969.4[x]^2}{2} - \frac{450[x-0.5]^3}{6} + \frac{450[x-1.5]^3}{6} - \frac{800[x-1.5]^3}{6} - \frac{2300[x-1.5]^2}{2} - \frac{1000[x-2.5]^2}{2} + \frac{800[x-2.5]^3}{6} + A$$

iii) Deflection

$$EI y = \frac{1969.4[x]^3}{6} - \frac{450[x-0.5]^4}{24} + \frac{450[x-1.5]^4}{24} - \frac{800[x-1.5]^4}{24} - \frac{2300[x-1.5]^3}{6} - \frac{1000[x-2.5]^3}{6} + \frac{800[x-2.5]^4}{24} + Ax + B$$

iv) $y=0$, $x=0$

$$0 = 0 - 0 + 0 - 0 - 0 - 0 + 0 + A(0) + B$$

$$B = 0$$



$$y=0, \quad x=3.1 \text{ m}$$

$$0 = \frac{1969.4 [3.1]^3}{6} - \frac{450 [2.8]^4}{24} + \frac{450 [1.6]^4}{24} - \frac{800 [1.6]^4}{24} - \frac{2300 [1.6]^3}{6} - \frac{1000 [0.6]^3}{6} + \frac{800 [0.6]^3}{24} + 3.1 A$$

$$A = -2331.31$$

$$v) EI = 1.96 \times 10^6 \quad x = 1.55 \text{ m}$$

$$1.96 \times 10^6 = \frac{1969.4 [1.55]^3}{6} - \frac{450 [1.05]^4}{24} + \frac{450 [0.05]^4}{24} - \frac{800 [0.05]^4}{24} - \frac{2300 [0.05]^3}{6} - \frac{1000 [0.05]^3}{6} + \frac{800 [0.05]^3}{24}$$

$$(-2331.31)(1.55)$$

$$y = 1.23 \times 10^{-3} \text{ m}$$



QUESTION 4

$$d = 0.28 \text{ m}$$

$$l = 0.6 \text{ m}$$

$$\eta_L = 0.56$$

$$\eta_C = 0.66$$

$$P = 14.9 \times 10^5 \text{ Pa}$$

$$\sigma_{\text{MAX}} = 150 \times 10^6 \text{ Pa}$$

$$\text{i) } \sigma_C = \frac{PD}{2t\eta_L} \Rightarrow \sigma_C \times 2t\eta_L = PD$$

$$2t = \frac{PD}{\sigma_C \eta_L} = \frac{PD}{2\sigma_C \eta_L}$$

$$t = \frac{14.9 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.56}$$

$$t = 2.48 \times 10^{-3} \text{ m}$$

$$\text{ii) } \sigma_L = \frac{PD}{4t\eta_C} \Rightarrow t = \frac{PD}{4\sigma_L \eta_C}$$

$$t = \frac{14.9 \times 10^5 \times 0.28}{4 \times 150 \times 10^6 \times 0.66} = 1.05 \times 10^{-3} \text{ m}$$

$$\text{iii) } 1.05 \times 10^{-3} \text{ m} \quad \text{min thickness}$$

QUESTION 5

$$\text{i) } I = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$I = \frac{\pi}{64} \times (19.05 \times 10^{-3})^4 - (16.65 \times 10^{-3})^4$$

$$I = 2.69 \times 10^{-9} \text{ m}^4$$

$$\text{ii) } A = 0.25$$



$$\text{iii) } F_{cr} = \frac{n \pi E I}{L^2}$$

$$F_{cr} = \frac{0.25 \times \pi^2 \times 200 \times 10^9 \times \cancel{1.36^2} \times 2.69 \times 10^{-9}}{1.36^2}$$

$$F_{cr} = 717.70 \text{ N}$$

$$\text{iv) } SR = \frac{L}{R} \quad SR = \frac{L}{1/A}$$

$$R = \frac{1}{A}$$

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$A = \frac{\pi}{4} (19.05 \times 10^{-3}^2 - 16.65 \times 10^{-3}^2)$$

$$A = 6.73 \times 10^{-5} \text{ m}^2$$

$$K = \frac{2.69 \times 10^{-9}}{6.73 \times 10^{-5}} = 39.97 \times 10^6$$

$$SR = \frac{L}{R} = \frac{1.36}{39.97 \times 10^6} = 34025.52$$

$$34025.52 > 180 \therefore \text{Strut.}$$

