



EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BEng Motorcycle Engineering	Level:	5
Module:	Motorcycle Dynamics		
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Q	Question	Marks
1a	$M \frac{d^2x}{dt^2} + C \frac{dx}{dt} + Kx$ explaining velocity, acceleration	(2)
b	Define Ride frequency as the undamped natural frequency of the system $f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$	(3)
c	Define critical damping as the damping required to return a mass to rest in fastest time with no overshoot $C_{crit} = 2\sqrt{Km}$	(3)
d	The damping ratio is used as a measure of what percentage deviation from critical damping the system is at $\zeta = \frac{c}{C_{crit}}$	(2)
e	we get $\frac{d^2x}{dt^2} + 20 \frac{dx}{dt} + 64x = 0$ so $r^2 + 20r + 64 = 0$ $r = -4, -16$ $(r+4)(r+16) = 0$ so $x(t) = C_1 e^{-4t} + C_2 e^{-16t}$ So the system is overdamped	(5)



If $x(0) = 0$ so $C_1 + C_2 = 0$

Now differentiate:

$$x'(t) = -4C_1 e^{-4t} - 16C_2 e^{-16t}$$

$$x'(0) = -4C_1 - 16C_2 = 0.6$$

Since $C_2 = -C_1$

$$12C_1 = 0.6$$

$$C_1 = 0.05$$

Therefore

$$x = 0.05(e^{-4t} - e^{-16t})$$

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2a Description to cover :

Applied torque translated due to conservation of angular momentum. Resultant torque orthogonal to both Spin axis and applied torque axis.

(4)

b During high speed sections, the applied torque translates directly to roll angle due to gyroscopic effects. This is less pronounced at low speeds where more steer angle is used as the motorcycle tends towards the bicycle steering model.

(4)

c assume $\frac{1}{2}mr^2$ or mr^2 but justify

$$mr^2 = 6.19 \times 10^{-5} \text{ kgm}^2$$

(2)

d label as Rotation 1, 2, torque
or Torque, rotation, precession
or Spin, input, output etc

(1)

e Reasonable descriptive analysis of gyroscopic precession

(3)

f Clockwise as viewed from the precession axis

(2)

g $G_m = I\omega\Omega$ (include $\cos \theta$ if assuming an angle)

$$mgr = 6.19 \times 10^{-5} \times 890 \times \Omega$$

$$0.0882g \times 0.04 = 0.0547 \Omega$$

$$\Omega = 0.632 \text{ rad/sec}$$

$$\Rightarrow 9.94 \text{ Sec/rev.}$$

(4)



3a Determine Masses

$$\text{Total} = 232 + 75 = 307 \text{ kg}$$

$$\text{Sprung front Mass} = (0.5 \times 307) - 8 = 145.5 \text{ kg}$$

$$\text{Sprung Rear Mass} = (0.5 \times 307) - 11.2 = 142.3 \text{ kg}$$

using a ride frequency of 1.8 Hz for this type of bike

$$K' = (2\pi f)^2 \times m$$

$$\text{So } K'_f = (2\pi \times 1.8)^2 \times 145.5 = 18,610 \text{ N/m}$$

$$K_f = K'_f \cos^2 \epsilon = 18610.9 \times \cos^2(27.5)$$

$$K_f = 14,643 \text{ N/m}$$

So each fork spring would need to be approx 7.5 N/mm

for the Rear:

$$K'_r = (2\pi \times 1.8)^2 \times 142.3 = 18,201.6$$

$$K_r = K'_r \times LR^2 = 18201.6 \times 2.1^2$$

$$K_r = 80,269 \text{ N/m}$$

So a rear shock spring of 80 N/mm

b using velocity data as the start point, differentiate for acceleration, use $F=ma$ to give a resultant driving or braking force, then wheel load balance (N_f) equations to get wheel loads, resolve using rake angle for suspension loads and use $F=Kx$ to get displacement

(4)

c Velocity trace would be required from real data trace
Since $F=CV$. The simulated trace would be lacking the transient response from road inputs which would skew the real suspension travel histogram

(4)



4a	Students description of 'winglet' technology	(2)
b	Overloading of the front tyre in cornering would be the primary concern	(2)
c	Descriptive analysis of ~~recent~~ ^{recent} dynamical changes and resultant riding style and racing line changes to compensate / take advantage of new geometry and dynamics	(4)
d	Basic analysis of damper dyns chart needed	(2)
e	$\text{Max} \sim \frac{2000}{0.15} = \underline{13333.3 \text{ Ns/m}}$ $\text{Min} \sim \frac{1000}{0.1} = \underline{10000 \text{ Ns/m}}$	(3)
f	At the wheel $\sim$ $\text{Max} = \frac{13333}{2.7^2} = \underline{1828.9 \text{ Ns/m}}$ $\text{Min} = \frac{10000}{2.7^2} = \underline{1371.7 \text{ Ns/m}}$	(3)
g	$C_{crit} = 2\sqrt{kM} = 2 \times \sqrt{18000 \times 112} = 2839.7$ $\zeta = \frac{C}{C_{crit}} = \frac{C(\text{Average})}{C_{crit}}$ $\zeta = \frac{1600.3}{2839.7} = \underline{0.56}$	(4)
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