



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Examiner Only

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2420823
2	Title of Examination Paper	Mathematics Paper 2
3	Module Code	ACDA 4008
4	Date and Time of Examination	7/12/25 9:00
5	Examination Venue	IQ 310
6	Seat Number	062
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1	28
Q2	25
Q3	29
Q4	9
Total	91

9120

Very good

✓ AT Sep 25

i) i) $y = x + 3$ $\frac{dy}{dx} = 1$ ✓

ii) $y = 4x^3 + 10x^{1/2}$ $\frac{dy}{dx} = 12x^2 + 5x^{-1/2}$ ✓

iii) $y = x^{-5} + x^3$ $\frac{dy}{dx} = -5x^{-6} + 3x^2$ ✓

iv) $y = \sin(8x)$ $\frac{dy}{dx} = 8\cos(8x)$ ✓

v) $y = x \cos x$ let $u = x$ and $v = \cos x$ so that $y = uv$

$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = x(-\sin x) + \cos x$ (1)

$= \cos x - x \sin x$ ✓

vi) $y = (2x-3)^{10}$ let $u = 2x-3$ so that $y = u^{10}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $\frac{du}{dx} = 2$ $\frac{dy}{du} = 10u^9$

$\frac{dy}{dx} = 10u^9 \times 2 = 20u^9 = 20(2x-3)^9$ ✓

vii) $y = 3x + \tan x$ $\frac{dy}{dx} = 3 + \sec^2 x$ ✓

b) $y = \frac{1}{x} + \cos(2x) = x^{-1} + \cos(2x)$

$\frac{dy}{dx} = -x^{-2} + 2(-\sin(2x)) = -x^{-2} - 2\sin(2x)$

$\frac{d^2y}{dx^2} = -(-2x^{-3}) - 2(2\cos(2x)) = 3x^{-3} - 4\cos(2x)$

Cancels here!

$\frac{d^3y}{dx^3} = -(-2x^{-4}) - 4(-2\sin(2x)) = 8x^{-4} + 8\sin(2x)$ ✓

1) c) $y = \frac{x-1}{4+x^2}$ let $u = x-1$ and $v = 4+x^2$ so that $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \frac{du}{dx} = 1 \quad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{(4+x^2)(1) - (x-1)(2x)}{(4+x^2)^2}$$

$$= \frac{4+x^2 - (2x^2 - 2x)}{(4+x^2)^2}$$

$$= \frac{4+x^2 - 2x^2 + 2x}{(4+x^2)^2}$$

$$\frac{dy}{dx} = \frac{4+2x-x^2}{(4+x^2)^2}$$

at $x=0$, $\frac{dy}{dx} = \frac{4+2(0)-(0)^2}{(4+(0)^2)^2}$

$$= \frac{4+0-0}{(4+0)^2}$$

$$= \frac{4}{16}$$

$$= 1/4$$

at $x=0$, $\frac{dy}{dx} = 1/4$

6

✓

23

$$2) \text{ i) } \int (10x + x^3) dx = 5x^2 + \frac{x^4}{4} + C$$

$$\text{ii) } \int (x - 3e^x) dx = \frac{x^2}{2} - 3e^x + C$$

$$\text{iii) } \int \left(\frac{1}{x} + 4\right) dx = \int x^{-1} + 4 dx = \ln(x) + 4x + C$$

$$\text{iv) } \int (2x(3x+4)) dx = \int (6x^2 + 8x) dx = 2x^3 + 4x^2 + C$$

$$\text{v) } \int_0^2 (3x^2 + 2x) dx = \left[x^3 + x^2 \right]_0^2$$

$$= (2^3 + 2^2) - (0^3 + 0^2) = (8 + 4) - 0 = 12$$

$$\text{b) } \int_0^1 \frac{x}{\sqrt{x^2+1}} dx = \int_0^1 x(x^2+1)^{-1/2} dx$$

$$\text{let } x^2+1 = u \quad \frac{du}{dx} = 2x \quad dx = \frac{du}{2x} = \frac{du}{2(u-1)^{1/2}}$$

$$x^2 = u-1 \\ x = \sqrt{u-1} = (u-1)^{1/2} \text{ at } x=$$

$$\text{at } x=1, u = (1)^2 + 1 = 2$$

$$\text{at } x=0, u = (0)^2 + 1 = 1$$

$$\therefore \int_0^1 x(x^2+1)^{-1/2} dx = \int_1^2 (u-1)^{1/2} (u)^{-1/2} \frac{du}{2(u-1)^{1/2}}$$

$$= \int_1^2 \frac{(u-1)^{1/2} (u)^{-1/2}}{2(u-1)^{1/2}} du$$

$$\text{let } u = v \\ u = v+1$$

$$= \int_1^2 \frac{(u^2 - u)^{1/2}}{2(u-1)^{1/2}} du$$

2b reduce later on.

$$3) a) i) (3+2i)(1-i) = 3 - 3i + 2i - 2i^2 \\ = 3 - i - 2(-1) \\ = 3 - i + 2 \\ = 5 - i \quad \text{where } a=5 \text{ and } b=-1$$

$$ii) \frac{3}{2+i} \times \frac{(2-i)}{(2-i)} = \frac{6-3i}{4+2i-2i-i^2} = \frac{6-3i}{4-(-1)} = \frac{6-3i}{5}$$

$$= \frac{6}{5} - \frac{3i}{5} \quad \text{where } a=6/5 \text{ and } b=-3/5$$

$$iii) \frac{4}{i^5} = \frac{4}{i^4 i} = \frac{4}{(1)i} = \frac{4}{i} \cdot \frac{i}{i}$$

$$\frac{4}{i} \times \frac{-i}{-i} = \frac{-4i}{-i^2} = \frac{-4i}{-(-1)} = \frac{-4i}{1} = -4i$$

$$= -4i \quad \text{where } a=0 \text{ and } b=-4$$

$$b) Z_1 = \frac{2-j}{2+j} \times \frac{2-j}{2-j} = \frac{4-2j-2j+j^2}{4-2j+2j-j^2}$$

$$= \frac{4-4j+j^2}{4-j^2} = \frac{4-4j-1}{4-(-1)} = \frac{3-4j}{5}$$

$$Z_2 = \frac{3+j}{3-j} \times \frac{3+j}{3+j} = \frac{9+3j+3j+j^2}{9+3j-3j-j^2}$$

$$= \frac{9+6j+(-1)}{9-j^2} = \frac{8+6j}{9-(-1)} = \frac{8+6j}{10} = \frac{4+3j}{5}$$

$$Z_1 = \frac{3}{5} - \frac{4j}{5}, \quad Z_2 = \frac{4}{5} + \frac{3j}{5}$$

$$c) i) p = 2z_1 + z_2$$

$$= 2\left(\frac{3}{5} - \frac{4j}{5}\right) + \left(\frac{4}{5} + \frac{3j}{5}\right)$$

$$= \frac{6}{5} - \frac{8j}{5} + \frac{4}{5} + \frac{3j}{5}$$

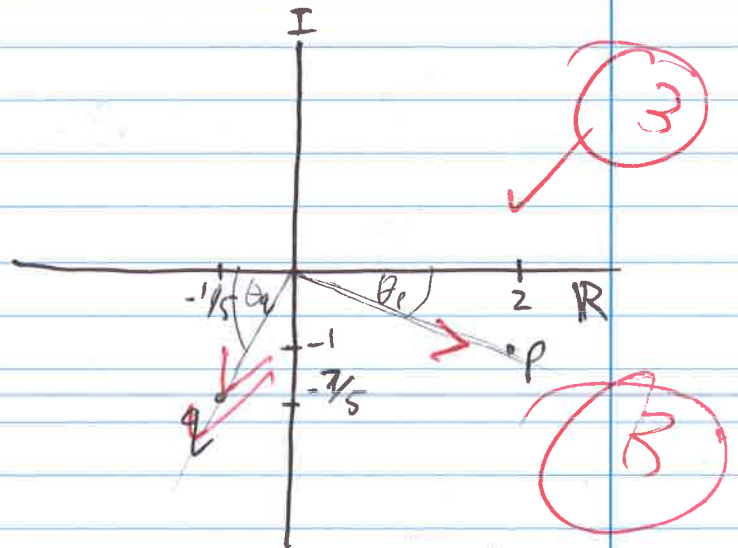
$$p = \frac{10}{5} - \frac{5j}{5} = 2 - j$$

$$q = z_1 - z_2$$

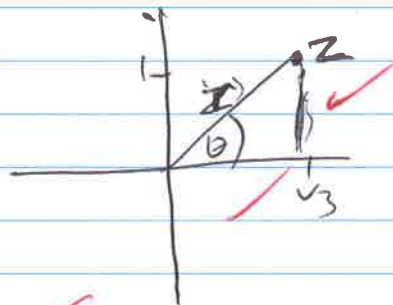
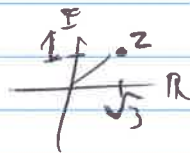
$$= \left(\frac{3}{5} - \frac{4j}{5}\right) - \left(\frac{4}{5} + \frac{3j}{5}\right)$$

$$= \frac{3}{5} - \frac{4j}{5} - \frac{4}{5} - \frac{3j}{5}$$

$$q = -\frac{1}{5} - \frac{7j}{5}$$



$$d) z = \sqrt{3} + j$$



$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

$$r = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$$

$$z = 2\left(\sqrt{3} \cos(30^\circ) + j \sin(30^\circ)\right)$$

$$= 2\left(j \sin(30^\circ) + \sqrt{3} \cos(30^\circ)\right)$$

6

$$3) e) \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

$$+i$$

$$r=1 \quad \theta=90^\circ$$

$$= 1(\cos 90^\circ + i \sin 90^\circ) = (\cos 90^\circ + i \sin 90^\circ)$$

$$= 1(0 + i \sin 90^\circ)$$

$$= 1(i \sin 90^\circ)$$

$$= i(\sin 90^\circ)$$

$$= i(1)$$

$$= i$$

5

29 ✓

$$4) \text{ i) } 3^{2x} = 2^{x+1} \quad \log$$

$$\log 3^{2x} = \log 2^{x+1} \quad \checkmark$$

$$2x \log 3 = (x+1) \log 2 \quad \checkmark$$

$$2x \log 3 = x \log 2 + \log 2 \quad \checkmark$$

$$2x \log 3 = x \log 2 + \log 2 \quad \checkmark$$

$$x(2 \log 3 - \log 2) = \log 2$$

$$x = \frac{\log 2}{2 \log 3 - \log 2}$$

$$2 \log 3 - \log 2$$

2dp1

3

$$\text{ii) } 2 \ln x = \ln(5x-6) \quad \checkmark$$

$$\ln x^2 = \ln(5x-6) \quad \checkmark$$

$$\therefore x^2 = 5x-6 \quad \checkmark$$

$$x^2 - 5x + 6 = 0 \quad \checkmark$$

$$(x-3)(x-2) = 0 \quad \checkmark$$

$$x=3 \text{ or } x=2 \quad \checkmark$$

2

$$\begin{aligned} 4) \text{ h) i) } \log(x^6 y z) &= \log x^6 + \log y + \log z \\ &= 6 \log x + \log y + \log z \end{aligned}$$

2

$$\begin{aligned} \text{ii) } \ln \frac{x^3 y}{z} &= \ln x^3 + \ln y - \ln z \\ &= 3 \log_e x + \log_e y - \log_e z \end{aligned}$$

2

1
9

$$2b) \int_0^1 \frac{x}{(x^2+1)^{1/2}} dx$$

$$\text{let } u = (x^2+1)^{1/2}$$

$$u^2 = x^2 + 1$$

$$x = u^2 - 1$$

$$\frac{du}{dx}$$

$$\frac{du}{2} = x dx \text{ on top?}$$

$$2b) \text{ let } x^2 + 1 = u \quad \frac{du}{dx} = 2x \quad x = \frac{du}{2dx}$$

need to
change
limits
for
u!!

$$I = \int_0^1 \frac{x}{(x^2+1)^{1/2}} dx$$

$$I = \int_{x=0}^{x=1} \frac{du}{2dx} \cdot \frac{1}{u^{1/2}} dx$$

$$I = \int_{x=0}^{x=1} \frac{du}{2u^{1/2}}$$

$$= \int_{x=0}^{x=1} \frac{u^{-1/2}}{2} du$$

$$\text{at } x=1, u = (1)^2 + 1 = 2$$

$$\text{at } x=0, u = (0)^2 + 1 = 1$$

$$I = \int_{u=1}^{u=2} \frac{u^{-1/2}}{2} du$$

$$= \left[\frac{u^{1/2}}{2(1/2)} \right]_1^2 = \left[u^{1/2} \right]_1^2$$

$$= \left[(x^2+1)^{1/2} \right]_1^2 = (2)^{1/2} - (1)^{1/2} = \sqrt{2} - \sqrt{1} = \sqrt{2} - 1 \text{ in } u$$

$$x^2 + 1 = \sqrt{2} - 1 \quad x^2 = \sqrt{2} - 2$$

4

✓

$$2c) i) \int x \sin(2x) dx.$$

let ~~$u = x$~~ and ~~$\sin 2x$~~

let $\frac{dv}{dx} = x$ and $u = \sin(2x)$

$$v = \frac{x^2}{2} \quad \times \quad \frac{du}{dx} = 2\cos(2x) \quad \text{is bad}$$

$$I = \int \left(\frac{dv}{dx} u \right) dx.$$

$$= uv - \int v \frac{du}{dx} dx \quad \checkmark$$

$$= \sin(2x) \left(\frac{x^2}{2} \right) - \int \frac{x^2}{2} (2\cos(2x)) dx$$

$$= \frac{x^2 \sin(2x)}{2} - \int x^2 \cos(2x) dx$$

Other way
around!

Idea is
Correct!
though!

2



20) ii) $\int_1^e (x \ln x) dx$

let $u = \ln x$ and $\frac{dv}{dx} = x$

So that $I = \int_1^e u \frac{dv}{dx} dx$

$\frac{du}{dx} = \frac{1}{x}$

$v = \frac{x^2}{2}$

$= uv - \int \left(\frac{du}{dx} \right) v dx$

$= \left[\ln x \left(\frac{x^2}{2} \right) - \int_1^e \left(\frac{1}{x} \right) \left(\frac{x^2}{2} \right) dx \right]_1^e$

$= \left[\frac{x^2 \ln x}{2} - \int_1^e \frac{x}{2} dx \right]_1^e$

$= \left[\frac{x^2 \ln x}{2} - \frac{x^2}{4} \right]_1^e$

$= \left(\frac{e^2 \ln e}{2} - \frac{e^2}{4} \right) - \left(\frac{(1)^2 \ln(1)}{2} - \frac{(1)^2}{4} \right)$

$= \left(\frac{e^2(1)}{2} - \frac{e^2}{4} \right) - \left(\frac{\ln 1}{2} - \frac{1}{4} \right)$

$= \frac{e^2}{4} - \frac{\ln 1 + 1/4}{2} = \frac{e^2 - \ln(1) + 1}{4}$

$= \frac{e^2 + 1}{4}$

$= 2.097$

4

25

This is a very good effort
Well Done!

Arholwr yn unig