

Thermal Physics:

(1)

1) The Specific Heat, C , of a material is defined as the energy required to raise unit mass by 1 degree

$$C \quad \text{units} \quad \text{J kg}^{-1} \text{K}^{-1}$$

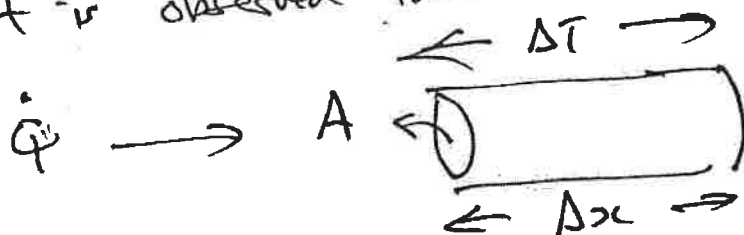
$\therefore$ from this definition, by simple proportionality:

Energy Q to raise m kg through ΔT degrees is

$$Q = C m \Delta T$$

(2) Fourier's law of heat conduction:

Empirically it is observed that:



the rate of heat flowing through a body is

$$\frac{dQ}{dt} \propto \Delta T$$

$$\frac{dQ}{dt} \propto A$$

$$\frac{dQ}{dt} \propto \frac{1}{\Delta x}$$

$$\therefore \frac{dQ}{dt} = k A \frac{dT}{dx} \quad k \text{ thermal conductivity}$$

$$\text{or } \frac{dQ}{dt} = k A \nabla T \quad \text{using del notation}$$

Units of thermal conductivity, k : $W K^{-1} m^{-1}$ (2)

Thermal Diffusivity, α

$$\alpha = \frac{k}{\rho C}$$

Power and Energy

Power is the rate of change of energy

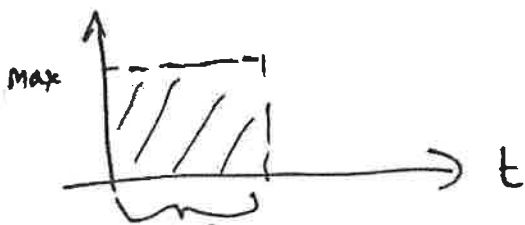
$$P = \frac{dE}{dt}$$

Alternatively :

$$E = \int P(t) dt$$

Example :

A uniform laser pulse of total energy $10 J$ is applied for a duration of $1 \mu s$



Area under curve is
i.e.

$$E = P_{max} \times 10^{-6}$$

$$P_{max} = \frac{10}{10^{-6}} \text{ Watts}$$

$$P = 10 \text{ MW} !!$$

Derivation of Fourier's Transient Thermal Diffusivity Equation ⁽³⁾

Recall two equations:

$$Q = C m T$$

Calorimetric Equation

Q is the internal heat energy at a temperature T .

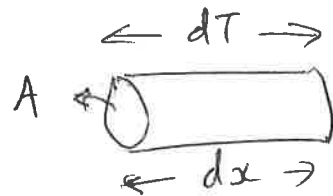
C specific heat of mass m

Alternatively, the change in internal energy dQ of a mass experiencing a change of temperature dT is

$$dQ = C m (dT) \quad \text{--- (1)}$$

Secondly recall Fourier's law of heat conduction:

In 1D there is heat flowing through surface A , $\frac{dQ}{dt}$, is

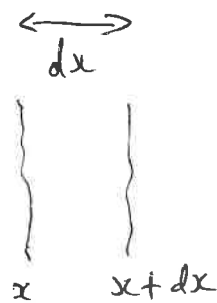


$$\frac{dQ}{dt} = - k A \frac{dT}{dx} \quad \text{--- (2)}$$

Note the minus sign. Physically, this is explained by the fact that, if $\frac{dQ}{dt}$ is positive i.e. heat flux increasing with time over face, then the temperature on LHS face would be always higher than the RHS face i.e. temperature decreases with increasing x i.e. negative gradient. Rewrite (2) to make dQ the subject

$$dQ = - dt k A \frac{dT}{dx}$$

Now consider a very thin slice of material of density, ρ ,⁽⁴⁾
in 1D:



From Conservation of Energy Law:

Change in Internal Heat energy = Heat in from LHS
over time interval dt - Heat out of RHS

For LHS:

Apply the calorimetric equation twice: at time $t+dt$ and then at time t

$$\begin{aligned}
 & C(\rho A dx) T(x, t+dt) \\
 & - C(\rho A dx) T(x, t) = d\Phi|_{LHS} - d\Phi|_{RHS} \\
 & = -\cancel{dt} k A \frac{dT}{dx} \Big|_x - \left(-\cancel{dt} k A \frac{dT}{dx} \right) \Big|_{x+dx}
 \end{aligned}$$

Rearranging gives:

$$\frac{T(x, t+dt) - T(x, t)}{dt} = \frac{k}{\rho C} \left[\frac{\frac{dT}{dx} \Big|_{x+dx} - \frac{dT}{dx} \Big|_x}{dx} \right]$$

i.e.

$$\frac{dT}{dt} = \frac{k}{\rho C} \frac{d^2 T}{dx^2}$$

(5)

The quantity $\frac{k}{\rho c}$ can be replaced by a constant α

$$\alpha = \frac{k}{\rho c} \quad \text{thermal diffusivity}$$

so that Fourier's time dependent thermal diffusion equation becomes

$$\frac{d^2 T}{dx^2} = \frac{1}{\alpha} \frac{dT}{dt}$$

In 3D it's easy to see that this equation becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

We usually use the symbol $\frac{\partial^2 T}{\partial x^2}$ rather than $\frac{d^2 T}{dx^2}$ to remind ourselves that T depends on more than one variable. The del² operator ∇^2 can be used to abbreviate:

$$\boxed{\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t}}$$

(6)

Stefan's Law

Power emitted by a body per unit surface area

$$\dot{q} = \sigma \epsilon T^4$$

 T in Kelvin σ = Boltzmann's constant

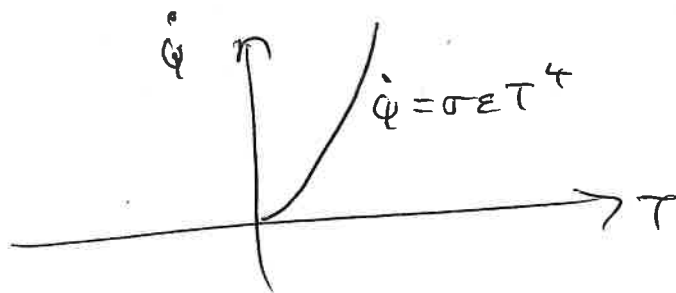
$$\sigma = 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

 ϵ = emissivity of surface

$$0 < \epsilon < 1$$

$\left[\frac{dQ}{dt} = \right] \dot{q}$ is the energy emitted over all wavelengths per unit time per unit area.

Note the 4th power. This implies that the power emitted by a body due to its temperature, increases dramatically with increasing temperature

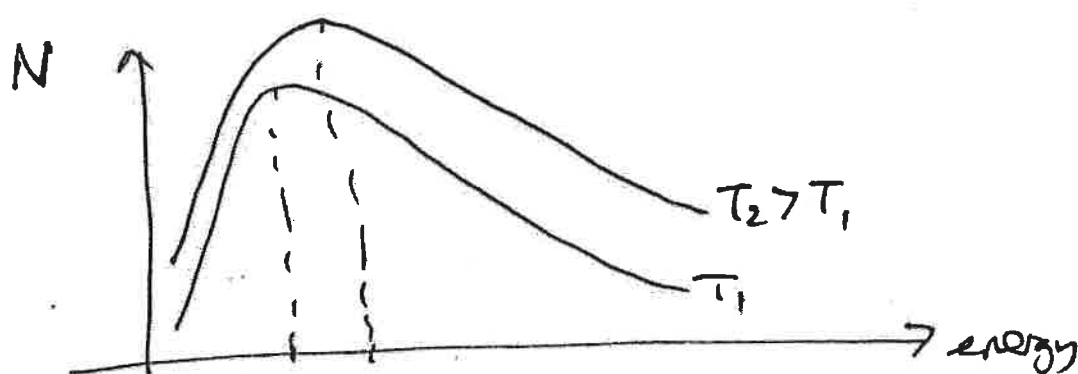


Zeroth Law of Thermodynamics:

"If two bodies are in thermal equilibrium with a third body, then those bodies are in thermal equilibrium with each other."

— this introduces the concept of temperature

When a body is in thermal equilibrium i.e. at a certain temperature, the atoms that make up that body have a range, or distribution, of energies, expressed as the Maxwell-Boltzmann distribution.



The average energy of atoms at T_2 is greater than the average energy at T_1 .

Coefficient of thermal expansion

(8)

In 1D:

A metal bar has length L_0 at temp $= T_0$
Due to thermal expansion it has length L_1 at temp T_1

$$L_1 = L_0 + \alpha L_0 (T_1 - T_0)$$

$$L_1 = L_0 (1 + \alpha \Delta T)$$

α is coefficient of linear expansion

T_0 is the reference temperature.