



SURNAME: HOSKINS

FORENAME(S): TAYLOR

12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
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List the numbers of the questions you have attempted	Office Use
A	25
B 1	25.
3	25
4	20.
Total	95

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SECTION A

(a)

x	f	xf	$f(x-\bar{x})^2$	
6	23	138	176.477	$23(6-8.77)^2$
7	56	392	175.442	$56(7-8.77)^2$
8	65	520	38.539	$65(8-8.77)^2$
9	53	477	2.804	$53(9-8.77)^2$
10	47	470	71.106	$47(10-8.77)^2$
11	37	407	183.997	$37(11-8.77)^2$
12	19	228	198.225	$19(12-8.77)^2$
Total =	300	2632	846.59	

$$\begin{aligned}\bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{2632}{300} \\ &= \underline{8.77}\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} \\ &= \sqrt{\frac{846.59}{300}} \\ &= \underline{1.68}\end{aligned}$$

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$$\begin{aligned}(b)(i) P(X=3) &= \frac{e^{-2.25} 2.25^3}{3!} \\ &= \underline{0.2}\end{aligned}$$

$$(ii) P(X=0) = \frac{e^{-2.25} 2.25^0}{0!} = 0.1054$$

$$P(X=1) = \frac{e^{-2.25} 2.25^1}{1!} = 0.2371$$

$$\begin{aligned}P(\text{More than 1}) &= 1 - (0.2371 + 0.1054) \\ &= \underline{0.6575} \\ &= 0.66\end{aligned}$$

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25
25

SECTION B

$$(1) R = 30\Omega, C = \frac{1}{30} \text{ F} = 30e^{-3t} \quad I(0) = 0$$

$$E(t) = RI + \frac{1}{C} \int I dt$$

$$= 30I + 30 \int I dt = 30e^{-3t}$$

$$\text{diff} \quad = 30 \frac{dI}{dt} + 30I = -90e^{-3t} \quad \downarrow \div 30$$

$$\frac{dI}{dt} + I = -3e^{-3t}$$

$$P = 1, Q = -3e^{-3t}$$

$$\int P dx = \int 1 dx$$

$$= x$$

$$I.f = e^t$$

$$Ie^t = \int e^t \times -3e^{-3t} dx$$

$$= \int -3e^{-2t} dx$$

$$= \frac{-3}{-2} e^{-2t} + C$$

$$\text{gs. } Ie^t = 1.5e^{-2t} + C$$

$$\text{Put } I(0) = 0 \quad : \quad I=0, t=0 \quad (x=0, y=0)$$

$$0 = 1.5e^0 + C$$

$$C = -1.5$$

$$\underline{Ie^t = 1.5e^{-2t} - 1.5}$$

25
25

(3) $y'' + 4y' + 3y = e^{-2t}$ $y_0 = 0$, $y_1 = 3$, $t = 0$

$$(s^2 \bar{y} + s y_0 - y_1) + 4(s \bar{y} - y_0) + 3 \bar{y} = \frac{1}{s+2}$$

$$s^2 \bar{y} - s y_0 - y_1 + 4s \bar{y} - 4 y_0 + 3 \bar{y} = \frac{1}{s+2}$$

Sub in $y_0 = 0$, $y_1 = 3$

$$s^2 \bar{y} - s(0) - 3 + 4s \bar{y} - 4(0) + 3 \bar{y} = \frac{1}{s+2}$$

$$s^2 \bar{y} + 4s \bar{y} + 3 \bar{y} = \frac{1}{s+2} + 3$$

$$\bar{y}(s^2 + 4s + 3) = \frac{1 + 3(s+2)}{s+2} \quad 1+3s+6$$

$$\bar{y}(s+3)(s+1) = \frac{3s+7}{s+2}$$

$$\bar{y} = \frac{3s+7}{(s+1)(s+2)(s+3)}$$

PARTIAL FRACTIONS

$$3s+7 = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$3s+7 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

Sub in $s = -2$

$$3(-2)+7 = A(0) + B(-2+1)(-2+3) + C(0)$$

$$-1 = B(-1)(1)$$

$$\frac{1}{-1} = B$$

$$-1$$

$$\underline{\underline{B = -1}}$$

Sub in $s = -1$

$$3(-1)+7 = A(-1+2)(-1+3) + B(0) + C(0)$$

$$4 = A(1)(2)$$

$$\frac{4}{2} = A$$

$$2$$

$$A = 2$$

$$\underline{A = 2}$$

$$\text{Sub in } s = -3$$

$$3(-3) + 7 = A(0) + B(0) + C(-3+1)(-3+2)$$

$$-2 = C(-2)(-1)$$

$$\frac{-2}{2} = C$$

$$C = -1$$

$$\mathcal{L}^{-1} \left\{ \frac{2}{s+1} - \frac{1}{s+2} - \frac{1}{s+3} \right\}$$

$$\underline{y = 2e^{-t} - e^{-2t} - e^{-3t}}$$

$$\frac{25}{25}$$

$$(4) E(t) = 70V, R = 65\Omega, C = \frac{1}{50}, L = 3.5H$$

$$(0.02)$$

$$I(0) = 0, I'(0) = 25$$

$$E = RI + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

$$3.5 \frac{dI}{dt} + 65I + 50 \int I dt = 70V$$

$$\frac{dE}{dt} = 0 \text{ When constant}$$

$$3.5 \frac{d^2I}{dt^2} + 65 \frac{dI}{dt} + 50I = 0$$

$$am^2 + bm + c = 0$$

$$3.5m^2 + 65m + 50 = 0$$

$$m = \frac{-65 \pm \sqrt{65^2 - 4(3.5)(50)}}{2(3.5)}$$

$$= \frac{-65 \pm \sqrt{4225 - 700}}{7}$$

$$= \frac{-65 \pm \sqrt{3525}}{7}$$

$$m = \frac{-65 \pm \sqrt{3525}}{7}$$

$$m = -0.804 \text{ or } -17.767$$

Real and different

$$I = Ae^{-0.804t} + Be^{-17.767t} \quad (A) \quad \checkmark$$

put $I(0) = 0$ into equation

$$0 = Ae^0 + Be^0$$

$$0 = A + B \quad (1) \quad \checkmark \quad \times 0.804 \quad \checkmark$$

diff (A)

$$\frac{dI}{dt} = -0.804Ae^{-0.804t} - 17.767Be^{-17.767t} \quad (B) \quad \checkmark$$

put $I'(0) = 25$ into (B)

$$25 = -0.804Ae^0 - 17.767Be^0$$

$$25 = -0.804A - 17.767B \quad (2) \quad \checkmark$$

$$25 = -0.804A - 17.767B \quad +$$

$$\textcircled{0.804} = 0.804A + 0.804B$$

$$25.804 = -16.963B$$

$$B = \frac{25.804}{-16.963}$$

$$B = -1.52 \quad \checkmark$$

Sub B = -1.52 into (1)

$$0 = A - 1.52$$

$$A = 1.52 \quad \checkmark$$

$$\underline{\underline{I(t) = 1.52e^{-0.804t} - 1.52e^{-17.767t}}} \quad \checkmark$$

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25