



12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	PRE08094825
2	Title of Examination Paper	Mathematics Test 2
3	Module Code	GOEN4001
4	Date and Time of Examination	19 th May 2025 14/05/2025
5	Examination Venue	~~Gower Ty Coch~~ Main College Hall
6	Seat Number	H7
7	Number of Answer Books handed in	1

[illegible]

(Gweler yr ochr arall am y fersiwn Cymraeg)

Complete, fold and seal
(see note 2 below)

SURNAME: Prescott

FORENAME(S): Kieran

(a)(i) Det A

$$\begin{array}{ccc|ccc}
 & & & 12 & & \\
 2 & 0 & 3 & 2 & 0 & \\
 1 & 8 & 3 & 1 & 8 & \\
 0 & 2 & 0 & 0 & 2 & \\
 & & & 0 & 0 & 6
 \end{array}$$

$6 - (12) = -6$

Det A = -6

(4)

(ii)

Det C Not possible

✓ (1)

(iii) Det D

$$\begin{array}{cc}
 2 & 4 \\
 -3 & 8
 \end{array}$$

$16 - (-12) = 28$

Det D = 28

✓ (2)

(iv) A · B

$$\begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix}$$

=

Answer

$$\begin{bmatrix} 9 & 9 & 22 \\ 22 & 49 & 20 \\ 4 & 10 & 0 \end{bmatrix}$$

(4)

$$2(3) + 0(2) + 3(1) = 9$$

$$2(0) + 0(5) + 3(3) = 9$$

$$2(2) + 0(0) + 3(6) = 22$$

$$1(3) + 8(2) + 3(1) = 22$$

$$1(0) + 8(5) + 3(3) = 49$$

$$1(2) + 8(0) + 3(6) = 20$$

$$0(3) + 2(2) + 0(1) = 4$$

$$0(0) + 2(5) + 0(3) = 10$$

$$0(2) + 2(0) + 0(6) = 0$$

Q1

33

34

A | 16

6 | 9

9 | 8

(V) B.C

Answer

$$\begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 14 \\ 12 & -17 \\ 25 & -5 \end{bmatrix}$$

$$3(1) + 0(2) + 2(3) = 9$$

$$3(4) + 0(-5) + 2(1) = 14$$

$$2(1) + 5(2) + 0(3) = 12$$

$$2(4) + 5(-5) + 0(1) = -17$$

$$1(1) + 3(2) + 6(3) = 25$$

$$1(4) + 3(-5) + 6(1) = -5$$

(vi) D.C Not possible

(vii) A+D Not Possible

b

Inverse

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 4 & 7 \\ 0 & 5 & 2 \end{bmatrix} \begin{matrix} 70 & 8:78 \\ 2 & 4 \\ 1 & 4 \\ 0 & 5 \\ 2 & 1 \end{matrix} \quad A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$16 \quad 0 \quad 5:21 \quad A = -57$$

$$\begin{bmatrix} 4 & 7 \\ 5 & 2 \\ 8-35=-27 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ 0 & 2 \\ 2-0=2 \end{bmatrix} \begin{bmatrix} 14 \\ 05 \\ 5-0=5 \end{bmatrix} \begin{matrix} -27 & 2 & 5 \\ 3 & 4 & 10 \\ 24 & 13 & 4 \end{matrix}$$

$$\begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 8-5=3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 4-0=4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 5 \\ 10-0=10 \end{bmatrix} \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$\begin{bmatrix} 4 & 1 \\ 4 & 7 \\ 28-4=24 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 7 \\ 14-1=13 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 4 \\ 8-4=4 \end{bmatrix} \begin{matrix} -27 & -2 & 5 \\ -3 & 4 & -10 \\ 24 & -13 & 4 \end{matrix}$$

Q2

$$\begin{array}{r} 31 \\ 34 \end{array}$$

$$\begin{array}{r} a | 15 \\ b | 9 \\ c | 7 \end{array}$$

$$A^{-1} \frac{1}{-57} = \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & -10 & 4 \end{bmatrix} \checkmark$$

$$\frac{A \Delta A}{A} = \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & 10 & 4 \end{bmatrix} \text{ ? } \times$$

C

Cramer's Rule

$$3x - 2y + 2z = 2$$

$$x - 3y + 2z = 1$$

$$2x + y - 3z = -5$$

(8)

$$\Delta = \begin{vmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{vmatrix} \quad \begin{matrix} x = 2 \\ y = 1 \\ z = -5 \end{matrix} \quad \begin{vmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{vmatrix} \begin{matrix} 3 & -2 \\ 1 & -3 \\ 2 & 1 \end{matrix}$$

$20 - (6) = 14$
 $27 - 8 = 19$
 $1 = 20$

$$\Delta x = \begin{vmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ -5 & 1 & -3 \end{vmatrix} \begin{matrix} 2 & -2 \\ 1 & -3 \\ -5 & 1 \end{matrix}$$

$15 \quad 4 \quad 6 = 25$
 $39 - (25) = 14$
 $18 \quad 20 \quad 1 = 39$

$$\Delta = 14$$

$$\Delta x = 14$$

$$\Delta y = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & -5 & -3 \end{vmatrix} \begin{matrix} 3 & 2 \\ 1 & 1 \\ 2 & -5 \end{matrix}$$

$2 \quad -30 \quad -6 = -34$
 $-6 - (-34) = 28$
 $-9 \quad 8 \quad -5 = -6$

$$\Delta y = 28$$

$$\Delta z = \begin{vmatrix} 3 & -2 & 2 \\ 1 & -3 & 1 \\ 2 & 1 & -5 \end{vmatrix} \begin{matrix} 3 & -2 \\ 1 & -3 \\ 2 & 1 \end{matrix}$$

$-12 \quad 3 \quad 10 = 1$
 $43 - (1) = 42$
 $45 \quad -4 \quad 2 = 43$

$$\Delta z = 42$$

$$\frac{\Delta x}{\Delta} = \frac{14}{14} = 1 \checkmark$$

$$\frac{\Delta y}{\Delta} = \frac{28}{14} = 2 \checkmark$$

$$\frac{\Delta z}{\Delta} = \frac{42}{14} = 3 \checkmark$$

Q2

(a) (i) $z_1 + z_2 = (3+4j) + (5+3j) = (8+7j)$ ①

(ii) $2z_2 + 3z_3 = 2(5+3j) + 3(2+6j)$
 $10+6j + 6+18j$
 $= 16+24j$ ✓ ②

(iii) $z_1^2 + z_3 = (3+4j)(3+4j) + (2+6j)$
 $9+12j+12j-16$
 $-7+24j+2+6j$
 $= -5+30j$ ③

(iv) $z_1 \cdot z_2 \cdot z_3 = (3+4j)(5+3j)(2+6j)$
 $15+9j+20j-12(2+6j)$
 $(3+29j)(2+6j)$
 $6+18j+58j-174$
 $-168+76j$ ✓ ④

(v) ~~$z_1 \cdot z_2 \cdot z_3$~~ $z_1 \cdot z_2 \cdot z_3 = -168+76j$
 ~~$(3+4j)(5+3j)(2+6j)$~~
 ~~$15+9j+20j-12(2+6j)$~~
 ~~$(3+29j)(2+6j)$~~
 ~~$6+18j+58j-174$~~
 ~~$-168+76j$~~ ✗

$$z_1 \cdot z_2 \cdot z_3 = -168+76j$$

(V1)

$$\frac{Z_1}{Z_2} = \frac{3+4j}{5+3j} = \frac{3+4j}{5+3j} \times \frac{5-3j}{5-3j}$$

$$\frac{15-9j+20j+12}{25-15j+15j+9} = \frac{27+11j}{34+0j}$$

$$\frac{Z_1}{Z_2} = \text{Answer} \quad \frac{27}{34} + \frac{11}{34}j$$

$$b/ \quad 5x^2 - 8x + 5 = 0$$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(5)}}{2(5)} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(5)}}{2(5)}$$

$$a=5 \quad b=-8 \quad c=5$$

$$x_1 = \frac{4+3j}{5}$$

$$x_2 = \frac{4-3j}{5}$$

$$x_1 = 0.8 + 0.6j$$

$$x_2 = 0.8 - 0.6j$$

$$r = \sqrt{0.8^2 + 0.6^2}$$

$$\theta = \tan^{-1} \frac{0.6}{0.8} = 36.87^\circ$$

$$360 - 36.87 = 323.1^\circ$$

$$r < \theta$$

$$x_1 = 0.8 + 0.6j$$

$$x_2 = 0.8 - 0.6j$$

$$= 1 \angle 36.87^\circ$$

$$= 1 \angle 323.1^\circ$$

de Moivre's $(3+4j)^4$

$$r = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \tan^{-1} \frac{4}{3} = 53.13^\circ$$

De Moivre's theorem

$$r^n (\cos n\theta + j \sin n\theta)$$

$$5^4 (\cos 4 \times 53.13 + j \sin 4 \times 53.13)$$

$$625 (\cos 212.52 + j \sin 212.52)$$

$$625 (-0.843 - 0.537j)$$

$$(3+4j)^4 = -526.87 - 335.62j$$

Q3

a (i) $\int (25x^4 + 12x^2 + 6x - 10) dx$

$$= \frac{25x^5}{5} + \frac{12x^3}{3} + \frac{6x^2}{2} - 10x + c$$

(ii) $\int (\sin 7x + 6 \cos 3x + 5e^{5x}) dx$

$$= -\frac{1}{7} \cos 7x + \frac{6}{3} \sin 3x + \frac{5}{5} e^{5x} + c$$

(iii) $\int \frac{10\theta^4 + 2}{2\theta^5 + 2\theta + \pi} d\theta$

$$= \ln(x) 2\theta^5 + 2\theta + \pi + c$$

Q3

$$\frac{22}{32}$$

a	11
b	6
c	5

$$(iv) \int \left(\frac{16}{5} + \ln(e^2) + 3 \right) dx$$

$$= \int \ln(x) + \frac{1}{x} e^2 + 3x + c \quad \times$$

$$(v) \int_1^5 (2x^3 + 5x^2 + 3x - 9) dx$$

$$\frac{2x^4}{4} + \frac{5x^3}{3} + \frac{3x^2}{2} - 9x + c$$

$$\left(\frac{2(5)^4}{4} + \frac{5(5)^3}{3} + \frac{3(5)^2}{2} - 9(5) \right) - \left(\frac{2(1)^4}{4} + \frac{5(1)^3}{3} + \frac{3(1)^2}{2} - 9(1) \right)$$

$$513.33 - (-5.333) \quad \textcircled{4}$$

$$(vi) \int_0^4 (2e^{2x} + \sin \frac{x}{2}) dx \quad 518.663$$

~~$$\frac{2e^{2x}}{2} - \frac{1}{2} \cos \frac{x}{2} dx$$~~

~~$$\frac{2e^8}{2} - \frac{1}{2} \cos \frac{4}{2} + c \quad \times$$~~

~~$$\int x^2 (4x^3 + 9)^5 dx$$~~

$$u = 4x^3 + 9 \quad \frac{du}{dx} = 12x^2 \quad du = 12x^2 dx \quad dx = \frac{du}{12x^2}$$

$$\int x^2 (4x^3 + 9)^5 dx = \int x^2 u^5 \frac{du}{12x^2}$$

$$\frac{1}{12} \int u^5 du = \frac{1}{12} \frac{u^6}{6} du$$

$$\text{Answer} = \frac{1}{72} (4x^3 + 9)^6 + c$$

$$c) \int 5x e^{5x} dx$$

$$u = 5x \quad \frac{du}{dx} = 5 \quad du = 5 dx \quad (2)$$

$$dv = e^{5x} dx \quad v = \int \frac{1}{5} e^{5x} \quad (3)$$

$$\int 5x e^{5x} dx = 5x e^{5x} dx =$$

$$5x \frac{1}{5} e^{5x} - \int \frac{1}{5} e^{5x} 5 \quad ?$$