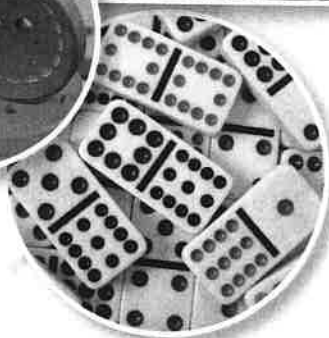


Improving Primary Mathematics Teaching and Learning



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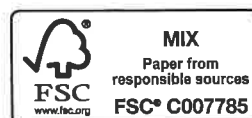
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8 Highly able children in ordinary classrooms

Lynne McClure

Introduction

Every so often we meet a child whose mathematical behaviour seems significantly more advanced than that of those we usually teach. Depending on whether or not we are confident in our own mathematics, these children can present a major challenge or threat. Whichever, we probably remember them long after they've left our care. In this chapter, we'll look at what we mean by 'advanced ability' in mathematics, how we can tell which children exhibit 'gifted' behaviour and what we might and should do to meet their classroom needs.

Chapter aims

The aims of this chapter are to:

- Examine the characteristics of 'gifted' behaviour and ways to identify them
- Provide examples of this behaviour in the ordinary classroom
- Present possible strategies for enabling, supporting and promoting mathematical thinking both within and outside the school environment
- Indicate where you might find support and resources for the children in your own class.

The mathematics curriculum and children's learning

A word about language

The UK literature on this topic uses a variety of vocabulary: gifted, talented, precocious, promising, exceptionally able, etc., and these labels

are usually attached to individual children. In the official literature, gifted and talented children are

'those with an ability to develop to a level significantly ahead of their year group (or with the potential to develop those abilities)'. In particular 'gifted' learners are those who have abilities in one or more academic subjects, like maths and English, while 'talented' learners are those who have practical skills in areas like sport, music, design or creative and performing arts.

(Directgov 2012)

Now this rather supposes a model where each person's mathematical ability is fixed, that is, we each possess our own particular information-processing or problem-solving capacity that remains fairly stable throughout our lives and is associated with traditional intelligence or IQ tests (Nieser et al. 1996). There is, however, an alternative view to this: that intelligence is malleable and can be improved or altered by attitudes, behaviour and contexts.

What you believe affects what you do. Dweck (2006) and others found that students who believed that intelligence is malleable (a 'growth mindset') actually learned better than students who believed that intelligence is fixed and unchangeable (a 'fixed mindset'). Students who learned to rely on their ability, whether labelled high or low, reacted differently from those who believed that with effort they could improve what they could do. So let us start by distinguishing between *identifying* aspects of high ability in your pupils, and using that information to *label* the children in your class. The former allows us to make a tailored appropriate response to each pupil, while the latter can lead to a 'one size fits all' approach that may not be suitable for any single individual who has been so labelled. Although the title of this chapter includes the phrase 'highly able children', I'm not advocating that they are a static group. Some will find difficult mathematics easy, others will struggle. The important thing is that if we focus on what they do and the effort they put in, rather than labelling them, we can help them all to become better mathematicians, and what they can do may surprise us if they are given the chance.

Why this is important

high attaining pupils require greater challenge in lessons: many primary teachers need stronger subject knowledge to do this well... It is of vital importance for pupils of all abilities to shift teaching and learning in mathematics away from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding.

(Ofsted 2008)

There are three main reasons for considering why we should pay particular attention to children who exhibit sophisticated mathematical behaviour:

- *Equal opportunities*: all children have the right to achieve as highly as possible. Children who find difficult mathematics easy need to be challenged to work hard to achieve even more, and they need teacher support to do this. 'Able' children are just as entitled to teacher time as those who struggle.
- *Institutional standards*: if we keep our eye on what we know is sophisticated mathematical behaviour, we find ourselves looking for it in *all* our children. In this current climate of questing for ever higher targets, teachers who develop a deeper understanding of what it is to think like a mathematician, and encourage it in all their pupils, raise the standards of the whole class (Her Majesty's Inspectorate (HMI) 1992: viii).
- *National need*: many children who find mathematics easy also find it boring (Nardi and Steward 2003) which means they are unlikely to choose to study it at a higher level. We desperately need top class mathematicians (National Numeracy 2012) for our future prosperity, and the responsibility for inculcating the enthusiasm for that begins in the primary school.

Conceptual and theoretical perspectives: what do we mean by 'highly able' behaviour in mathematics?

Reflection

You might like to think for a moment about a child you are teaching, or have taught recently, whom you would consider to be highly able in mathematics. What is it that they do, or say, that leads you to believe they are better at mathematics than the other children in your class?

Did you use any of the following criteria?

- They always achieve high scores on assessments.
- They appear to learn mathematical content easily and are very accurate in their work.
- They can solve more complicated problems than other children.
- They know far more about a mathematical topic than the other children in the class even before you've introduced it.

Definitions of ability are tied up with our ways of identification and there are probably as many different definitions of high ability in mathematics as there are researchers. The four criteria above all refer to

achieving an end product – the solution of a particular problem or a high score on a test. You may have found them useful, but most theorists in this area prefer instead to consider the *way* in which such children solve problems; and what kinds of behaviour they exhibit.

Straker (1983) looked at the mathematical behaviour of very young children. In particular she noted the tendency to impose order or pattern, to systemize. For example she describes small children who sort their toys according to unexpected criteria, who seek to impose some pattern or symmetry on the arrangement and who enjoy explaining and justifying their decisions. Straker (1983) suggests that children who exhibit these sorts of early behaviours later develop sophisticated mathematical thinking.

Krutetskii is one of the best known researchers in this field. He wanted to identify aspects of mathematical ability 'in which differences between pupils capable of learning mathematics, and pupils less capable, would be the most striking' (Krutetskii 1976: 84). He began his studies by observing and questioning young secondary aged pupils while they were solving problems. The characteristics he identified are included in Table 8.1.

Krutetskii (1976) distinguished between what he saw as the three stages of problem solving: interpreting, processing and retaining information. He concluded that more able pupils often ended up with a more elegant way of working than others, and this was because they could think flexibly about problems, drawing on their understanding and previous experience of appropriate approaches. How did they do this? Krutetskii (1976) suggested that these pupils saw problems in a different way: he called it 'a mathematical cast of mind' through which they saw the structure of a problem as a logical series of steps or perhaps an image to be manipulated. And because they remembered generalized approaches and chains of reasoning rather than, or as well as, detailed factual knowledge, they were in a good position to apply these in a variety of situations. It is interesting that Krutetskii was not really interested in how well these pupils could remember facts and did not recognize that as indicative of high ability.

Krutetskii (1976) then explored whether able children younger than these had the same characteristic ways of working. He found that children as young as 8 were beginning to work in these ways and that between 8 and 10 there was a noticeable development, especially in generalizing. At this sort of age, able children's mathematics is characterized by moving from describing objects to noticing the relationships between them, and from noticing specific cases towards noticing a general rule. However, Krutetskii did not find much evidence of children at this age remembering generalized approaches or chains of reasoning, or independently using more elegant or shorter ways of working, or working flexibly. Krutetskii (1976: 339) said that young children 'remember the general and the particular, the relevant and the irrelevant, the necessary and the unnecessary', and with maturity this is refined.

Table 8.1 Variation in outcomes from assessments

	High test scores	Lower test scores
Highly able	Usual	Reinterpret the question to fit their own interest. Answers do not fit marking rubric Work exceptionally quickly but not carefully Reading difficulty Uninterested Purposeful failure in order not to appear exceptional; perhaps an effect of peer pressure
Not highly able	Achieved with practice and if test emphasizes procedures and factual recall	Usual

Source: Adapted from Krutetskii (1976)

Other influential researchers in this field who are concerned with high ability generally (rather than specifically in mathematics) include Bloom (1985). Through studying very successful adults, he suggested a three stage development of high ability. In the first 'playful' stage, the individual finds the area of study fascinating and becomes immersed in it. The second 'mastery' phase is characterized by a more ordered exploration in which the individual seeks to make sense of the domain, to clarify the rules. And in the third 'creative' or 'personal style' phase, the individual uses their knowledge of the rules to create something new or different. Gardiner (2002) found that in primary school, most able children are in the first two phases and are unlikely to produce anything unique until they are older.

How can we identify mathematical behaviour?

Summative assessment, i.e. mathematics tests

In the previous section we considered what children say or do that leads us to believe they are able in mathematics. One of the criteria I offered was achieving high scores on assessments. Since formal testing is very common (partly because it is perceived as one of the easier forms of identification), it is worth exploring it first.

The correlation between ability and achievement is, usually, pretty high but we do need to look at those situations when it is not. Children

who appear to learn mathematical content easily and are very accurate in their work usually do well in test situations especially where the questions are of a predictable form and can be practised. But do these children exhibit the characteristics identified by researchers such as Krutetskii? Examination of recent Key Stage 2 National Curriculum questions indicates that only occasionally (but more in recent years than previously) are there opportunities to show, for example, chains of reasoning or the use of different strategies to solve a problem. Mostly, the questions are about remembering factual information and applying it (Figures 8.1 and 8.2). So high scores on tests can be achieved by children who are careful and consistent and who have good memories for facts and procedures.

7

Calculate $336 - 192$

8

Each of these four squares has been cut into two new shapes.

Write the letters of all the new shapes that are hexagons.

Write the letters of all the new shapes that are pentagons.

Figure 8.1 Memory of factual information

The opposite is also occasionally true – highly able children sometimes do not do well in tests for a variety of reasons, including misinterpreting the question to make it more challenging!

Formative assessment, i.e. observation, conversation, annotation

Formative assessment is intended to inform future actions and provide detailed information about what a child can and cannot do. Of course that information can be obtained from a test, but tests do not reveal the

4 Liam, Sarah and Amy buy lunch at a salad bar.

salad bar			
Salads		Desserts	
cheese	£1.20	banana	25p
egg	90p	apple pie	50p
tuna	£1.60	yogurt	35p

Liam has £2.50 to spend.
He buys a tuna salad and an apple pie.

How much money has he got left?

p

Sarah buys a cheese salad and a yogurt.
Amy buys an egg salad.

How much more does Sarah pay than Amy?

Show your working. You may get 2 marks.

Figure 8.2 Using chains of reasoning

richness of a child's ways of working and why they choose certain strategies. The usefulness of formative assessment may, however, be limited by the sort of activity the child is invited to do. If it is a consolidation worksheet, an observant teacher might discover where a child makes mistakes, or how quickly and carefully they work, but it will not tell much about those chains of reasoning, or thinking flexibly, or the other Krutetskii hallmarks of high ability.

Let us go back to that list of characteristics. We could further refine the sort of task we offer, or the context in which we offer it, by choosing those that specifically target the sort of behaviour we're looking for (Table 8.2).

Most of these characteristics can *only* be identified through problem-solving or investigative activities, and therefore an effective way of identifying high ability is to observe children during lots of opportunities to solve problems and do investigations.

Polya (1990) is known for his seminal work on mathematical problem solving. Figure 8.3 is an adaptation of Polya's problem-solving cycle with Krutetskii's hallmark characteristics inserted. This emphasizes even further how children's work in problem solving and investigation is an essential component in the identification of ability.

Ideally these sorts of activities should be accessible to all children in the class, so that they have opportunities to use more and more sophisticated thinking and perhaps surprise you. At the NRICH project in Cambridge, these are known as 'low threshold high ceiling' activities; we will return to these in the next section to learn more about what useful tools they are in the teacher's toolbox.

In this section, we have explored strategies to identify the mathematical behaviour of highly able pupils. There are of course also batteries of general tests for cognitive ability and non-verbal reasoning, and while a discussion of their relative merits is beyond the scope of this chapter, it is worth finding out more about them (see for example www.psychometric-success.com/faq/faq-cognitive-ability-test.htm). 

Models of classroom provision to support mathematical behaviour

Identification in itself is only useful if some appropriate action follows. What strategies might we use to meet the needs of pupils who would benefit from greater challenge? Freeman (1998) identifies two different models – the 'diagnose and treat' model and the 'sports' model – underpinned by opposing philosophies.

In the 'diagnose and treat' model, a definition for high ability is adopted, children who fulfil that definition are identified and labelled as

Table 8.2 Opportunities for identifying mathematical behaviour

Definition or characteristic of mathematical behaviour	Opportunities for encouraging, supporting, identifying
*A liking for numbers	Informal exposure to numbers of different types, including very large, fractions, negative numbers and zero Free play with groups of regular and irregular objects
*Pattern-making revealing balance or symmetry	
*Precision in setting out toys, e.g. teddies in order or size, cars according to colour	
*Using sophisticated criteria for sorting	Free play with objects that could be grouped in lots of different ways, containers for sorting Sharing their personal mathematics Judicious questioning: Why did you? How did you know?
*Enthusiasm for mathematics	
*Using connectives such as 'if', 'because' to question and reason	
*Describing, explaining and justifying their method (chains of reasoning)	Opportunities to refine and rehearse their explanations
**Can understand the 'big picture' of the problem and the necessary processes	Offered problems for which the route is unclear, multistep problems without signposts
**Recognize problems which have similar problem-solving approaches	Offered problems which have the same underlying structure, opportunities to talk about their methods of working
**Have a variety of different methods for solving problems and switch between them	Offered problems for which the route is unclear, multistep problems without signposts
**Work backwards and forwards to solve problems	Offered problems for which the route is unclear, multistep problems without signposts
**Reason logically and link arguments together in chains	Offered problems for which the route is unclear, multistep problems without signposts
**Leave out steps and abbreviate their recording	Given choice of recording media and format
** Use mathematical symbols as part of the thinking process	Encouraged to use their own recording and be introduced to accepted symbolic forms when appropriate

Table 8.2 (Continued)

Definition or characteristic of mathematical behaviour	Opportunities for encouraging, supporting, identifying
**Can draw out a generalization, pattern or rule, from several examples **Remember generalizations rather than specific examples	Investigations which have an underlying generalization which is not evident at the start Opportunities to work in depth on longer examples as well as consolidation through shorter examples

Sources: *Straker (1983), **Krutetskii (1976). Adapted from Eyre and McClure (2001: 68)

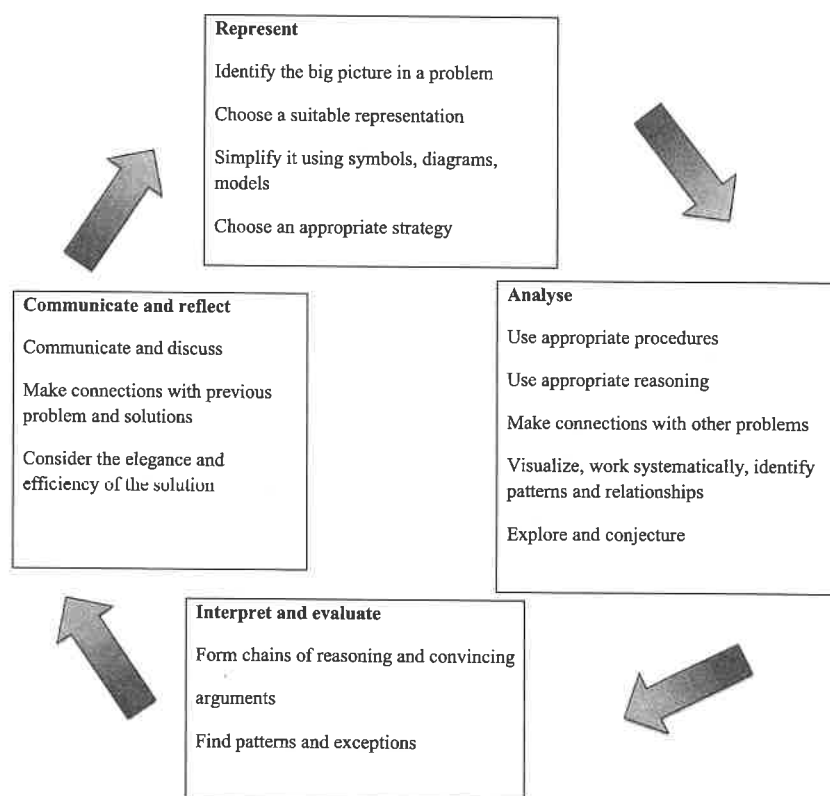


Figure 8.3 Problem solving Cycle 9 (adapted from Piggott (2010) Rich Tasks and Contexts, available at <http://nrich.maths.org/5662>)

'gifted', 'highly able', 'bright', etc., and are then given a specially devised programme of study. The underlying philosophy here is Dweck's (2004) 'fixed mindset' described at the beginning of this chapter. Schools and teachers who hold this philosophy tend to practise an exclusive model of provision. What might this look like?

Acceleration

The most prevalent 'diagnose and treat' strategy is acceleration. Children who are accelerated in mathematics do the same work as every other child but do it more quickly and therefore meet it earlier. Typically they work with the materials, books and resources usually offered to older children. They may be doing this independently, working alone or in a small group within their usual class. Many older pupils report that they remember primary school as a time where they taught themselves mathematics (with the teacher suggesting they ask for help if they get stuck) but otherwise they plough through the exercises with little intervention. Sometimes such children will be moved into the next class to work with older pupils – this solves the immediate problem for their own teacher. However, if this is a policy in the school, these children are unlikely to be working with the most able as they too will have moved to the next class, and so they will be working with children in the middle of the older ability range.

However, the main problem with acceleration is that it sets up future problems. Gardiner (2002) suggests there are three conditions that should be in place before acceleration is even discussed:

- Children should have absolute mastery of their age-appropriate curriculum.
- Children should be emotionally mature enough to cope with advanced ideas
- A long-term plan for the child's mathematical education should be in place, i.e. they are not accelerated to an abrupt stop.

The problem of maintaining continuity and progression is solvable perhaps within a single institution but can have really damaging effects if primary and secondary schools do not communicate about individual children. There is a whole discussion to be had here about the acceleration of Year 6 children in order to take level 6 tests (recently reintroduced as a result of the Bew (2011) inquiry into Key Stage 2 assessment) and how primary and secondary schools will use the results to respond to individual students' needs.

In Freeman's (1998) alternative 'sports model', teachers offer children activities that require more and more sophisticated thinking – a bit like raising the bar in a high jump. This is indicative of Dweck's (2006) 'growth

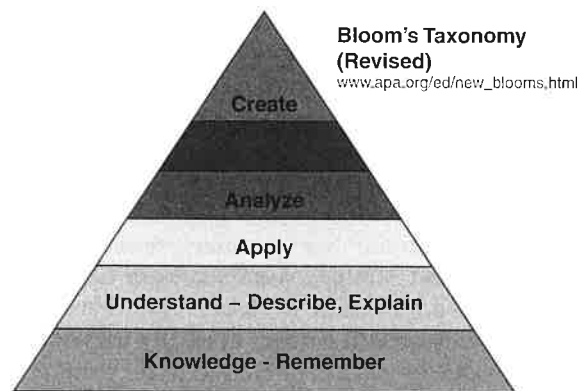


Figure 8.4 Bloom's taxonomy of thinking (Based on an APA adaptation of Anderson and Krathwold (2001))

mindset': we can know what children can do only if we offer them opportunities to show us. This philosophy usually leads to an inclusive curriculum model. In classroom terms what does this mean?

Deepening understanding

Bloom's (1956) taxonomy (Figure 8.4) is often used by teachers as a planning tool. We can also link Bloom's taxonomy back to the problem-solving cycle, and the characteristics of highly mathematical behaviour. By starting with an activity and then extending it to include the higher orders of thinking of analysis, evaluation and synthesis (or creation) teachers are able to offer more sophisticated versions of tasks to children who need more challenge. The interesting thing here is that in order to do this we rarely need to teach children additional content, which means that most children in the class can begin the task but some (perhaps not previously identified) will engage with it at a much higher level – hence an inclusive strategy. This is the origin of the term 'low threshold high ceiling'.

At the NRICH project in Cambridge, we specialize in devising rich tasks, many of which are low threshold high ceiling. We'll explore one of these, 'Magic Vs', in detail later. Low threshold high ceiling tasks:

- are accessible to a wide range of learners
- offer different levels of challenge for all learners
- allow for different methods and different responses (different starting points, different middles and different ends)
- engage the learner in the mathematics either because the starting point is intriguing or the mathematics that emerges is intriguing

- offer opportunities to identify elegant or efficient solutions
- allow learners to pose their own problems
- have the potential to broaden students' skills and/or deepen and broaden mathematical content knowledge
- encourage creativity and imaginative application of knowledge
- have the potential for revealing patterns or lead to generalizations or unexpected results
- have the potential to reveal underlying principles or make connections between areas of mathematics
- encourage collaboration and discussion
- encourage learners to develop confidence and independence as well as to become critical thinkers.

(Adapted from Piggott 2010)

Enriching the curriculum

The word 'enrichment' is used in different ways in the literature. Here I am using it to mean the additional mathematics that children are offered that is outside the core curriculum. This may be through linking mathematics with another subject, such as looking at the history of a particular piece of mathematics, considering the symmetry in the art of the Aztecs or looking for number patterns in musical composition, or alternatively finding out about some mathematics which is of interest but not central to the curriculum, for example the mathematics of codes and ciphers. It would be lovely to think that all children would have the opportunity to study these aspects of mathematics but we know that the curriculum is already too full for most children. Realistically, then, enrichment is usually an exclusive if worthy model.

Reality in the classroom

Of the three models above, we're going to focus here on deepening understanding. By now it should be evident that helping them to think mathematically is important for all children and is a way of increasing challenge. It is of course also central to the current curriculum but is an aspect of mathematics that many teachers find the most difficult. What exactly does 'thinking mathematically' mean?

Cuoco et al. (1997) talk about mathematical 'habits of mind'. They say that children should learn to be 'pattern sniffers, experimenters, tinkerers, visualizers, conjecturers, guessers' and to that we could add 'systematic workers' and 'provers'. Let us explore what this looks like in the classroom.

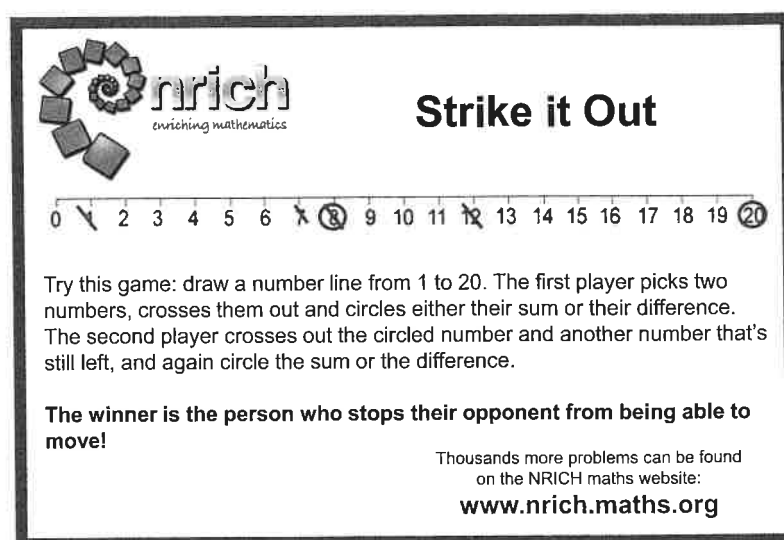


Figure 8.5 Strike it Out

Choosing tasks carefully

Closed procedural tasks rarely allow children to think mathematically. Low threshold high ceiling (LTHC) tasks as described above are those that allow children to move up through Bloom's hierarchy.

Some of these offer consolidation of basic skills or knowledge – which all children need – and combine this with opportunities to develop a strategy or proof, both of which involve working systematically and developing chains of reasoning. For example Strike it Out (Figure 8.5) consolidates addition and subtraction to 20. For many children this will be sufficiently engaging, but those who need more challenge can work towards the longest possible chain of calculations and then be asked whether it is possible to cross all the numbers out – and justify their answer.

Similarly Dotty Six (Figure 8.6) consolidates number bonds to 6, and allows children to develop a winning strategy. With this activity children can also choose to change the rules in some way to make their own game. Making up a good game is actually quite a sophisticated piece of thinking, involving analysis and synthesis, and then evaluation – those higher order skills of Bloom.

Giving children choices in what they do in mathematics is another way of supporting children's mathematical behaviour. Joseph Renzulli is known for his work on the Schoolwide Enrichment Model (Renzulli and Reis 1976), in which he suggests that for children to achieve highly, they

Dotty Six

Here's a game to play with a friend, a 3 by 3 grid and a six sided dice.

Take turns to throw a dice, then draw that many dots in one of the boxes on the grid. You can't split them up and a box can't have more than six dots in.

The person who completes a line of three sixes wins!



•••		
	•	•••••

Thousands more problems can be found on the NRICH maths website:
www.nrich.maths.org

Figure 8.6 Dotty Six

need to be of above average ability, be creative, and be committed to what they want to do, that is, work hard. Renzulli's curriculum model (Renzulli and Reis 1976), used in many North American schools, offers children choices about what they want to study so that they are more likely to be engaged with it.

Longer tasks can be a suitable vehicle for children to gain some independence in mathematics. Most activities in school are started and completed within a short time, but real mathematicians often spend long periods working on one idea or problem. Combining an element of choice with a longer 'simmering' activity can often be very motivating. Dotty Six (Figure 8.6) is a good example of an activity that the children could take home to work on, or pursue in their free time within the school day.

Children who enjoy mathematics do sometimes feel a little left out in the classroom, especially if the ethos is not one of excellence, or if they are significantly more able than anyone else in the class. Part of the role of being a classroom teacher also involves being a 'champion' for children with minority interests or abilities and communicating with them about, for example, mathematics in the news, competitions, and appropriate activities or visits outside school.

Ways of working in the classroom

Facilitating purposeful talk

Any task is only as good as the way it is presented and the context in which it is undertaken. Vygotskian social constructivism suggests that

meaning is constructed through interaction with people and through purposeful talk.

What the research shows consistently is that if you face children with intellectual challenges and then help them talk through the problems towards a solution, then you almost literally stretch their minds. They become cleverer, not only in the particular topic, but across the curriculum.

(Adey 2001)

Recognizing and modelling mathematical processes

In conversations with all children it is important to use mathematical vocabulary to describe not only the content, but also the processes being used (e.g. Cuoco's habits of mind) and to use this terminology when modelling behaviour.

Opportunities to explore

The most usual way of introducing new mathematical content in primary classrooms is through a teacher demonstration, followed by some examples that the children then work through. Ruthven (1989) offers a different structure in which children are given opportunities to explore a new situation – a bit like Bloom's playful phase. This is followed by a teacher-led discussion during which the learning that has taken place is formalized (codified) through sharing and questioning. Then the children work through some examples to consolidate their learning. This cycle of exploration–codification–consolidation requires deep teacher knowledge and a positive and safe classroom climate so that children can take risks in the way they work. It is suitable for all children but especially engaging for those who find mathematics easy.

Asking good questions

Watson and Mason (2004) suggest ideas for opening up questions in order to promote mathematical thinking. Closed questions can confirm what children do or do not know, while open questions help to understand how they think.

Magic Vs

Let us now explore some of the ways in which a classroom activity, Magic Vs, can provide means of supporting gifted and talented learners. A description of the task is punctuated with opportunities for reflection.

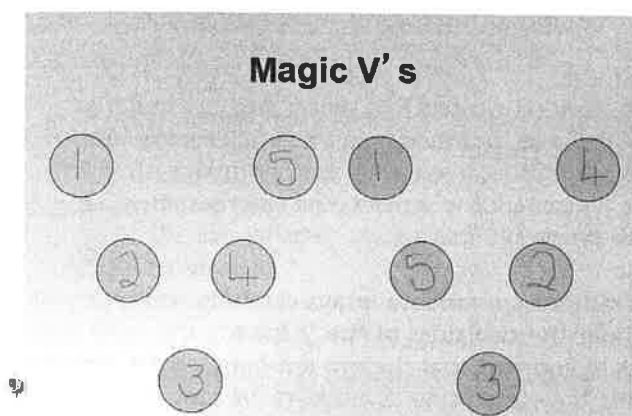


Figure 8.7 Magic Vs

Commentaries are drawn from experience of using this approach in a number of primary classrooms.

Magic Vs (nrich.maths.org/6274) is a typical LTHC task that can be used with children from age 5 upwards. The task begins with identifying what a magic V is, by comparing two Vs (Figure 8.7). The children are asked what is the same and what is different.

Reflection

Do this yourself: identify five similarities and five differences. How is this introduction inclusive?

Commentary

There would be very few children who couldn't identify something the same and different. Some will be simple, others more sophisticated. By listening to others' ideas they develop their own.

The teacher then focuses on the 'magic' V and ensures that everyone is clear about the definition of 'magic', that the two 'arms' add to the same total. Each pair of children are given 15 linking cubes and asked to make towers to show the numbers 1–5. Can they place them to make a different magic V?

Reflection

Do this yourself – if you don't have linked cubes, use other resources. Why have towers been chosen as a preferred resource?

Commentary

The choice of towers rather than random arrays carefully scaffolds children into thinking about the two sides of the V having the same total height. That simplifies to knowing that the two left-hand circles and two right-hand circles must have the same cumulative height, because the apex tower is common to both. The structure would be less clear with random collections and at this stage the activity is intended to be as inclusive as possible. Working in pairs promotes purposeful talk.

The teacher asks pairs if they have found a magic V. Can they find another with a different number at the bottom? How many do they think there are? How do they know they've found them all?

Reflection

How are these questions structured? What sort of responses would you expect? What other questions might you ask?

Commentary

The questions are structured to support increasingly high levels of thinking. Being able to find one magic V requires elementary counting and equivalence. Being able to find another pushes the children's idea of same and different, because many children will rearrange the towers on the arms without changing the bottom number. An idea of how many possible ones there are introduces the idea of working systematically, which is built on in the next question. Answering this requires significantly more sophisticated thinking. If you have done the activity yourself, you will realize that it is only possible to make a magic V with one of the 1, 3 or 5 towers at the apex. Knowing whether all have been found depends on being able to analyse why this is so and developing a convincing chain of reasoning.

The teacher allows the class to continue working until the majority have realized that 1, 3 and 5 have to be at the bottom, then brings the class together. How many magic Vs did we find altogether? Selected children come and record them on the board, with the teacher seeking agreement that each is different from the previous ones. The teacher facilitates a discussion about what constitutes same and different, and that only 1, 3 and 5 can be at the bottom. She asks whether anyone had 2 or 4 and whether it is possible, asking then why not.

Reflection

What is the teacher's role here? What would you do next?

Commentary

The teacher is formalizing the learning that has taken place. Those who are sure of their argument are asked to frame their explanation for others. By the end of this section of the lesson, all the children know that because the towers add up to an odd number there has to be an odd number at the bottom, leaving an even number to be divided between the two arms of the V. Some will have generalized this and others will need consolidation.

Pairs then return to work and given a choice of possible next tasks:

1. Swap towers for number cards and record how many different magic Vs they can make.
2. Predict then confirm, using towers (or number cards) 2, 3, 4, 5 and 6 to make as many different magic Vs as possible.
3. Predict then confirm, using towers (or number cards) 3, 4, 5, 6 and 7 to make as many different magic Vs as possible.
4. Predict then confirm, using towers (or number cards) 1, 2, 3, 4, 5, 6 and 7 to make as many different magic Vs as possible.
5. Be able to make a rule about what number will be at the bottom of any magic V.
6. Make up your own set of numbers for magic Vs.

Reflection

What is the teacher's role here? What sort of responses would identify what types of mathematical thinking?

Commentary

The tasks are designed to scaffold children's thinking. Each requires some additional sophistication. Giving choice allows children to opt in where they feel confident. The teacher's role is to provoke children to explain their thinking by asking questions that will draw them further up the thinking hierarchy.

1. Move from drawing as representation to using symbols, still counting and confirming: consolidation, symbolic representation.
2. Understand that an odd number goes at the bottom for 1–5 but may not know why, trial and improvement to find that an even number is at the bottom this time, try to explain this: consolidation, conjecturing, reasoning.
3. Make a reasoned prediction based on partial explanation, e.g. middle number of set is odd, therefore bottom number must be odd, or making pairs to same total by matching smallest to largest, leaving middle number as bottom: consolidation, conjecturing, reasoning.
4. Make a reasoned prediction based on full explanation: reasoning, conjecturing, generalizing.
5. Can say what numbers should go at the bottom for any consecutive set of numbers, by deciding whether total is odd or even or alternative explanation: reasoning, generalizing, analysing, justifying.
6. Choose a very large set of consecutive numbers, predict correct using argument similar to 5, or choose non-consecutive numbers, e.g. even numbers and predict characteristic of bottom number, or perhaps choose set of very large numbers and generalize without doing arithmetic: analysing, synthesizing, evaluating.

The teacher draws the activity to a close indicating the 'messy maths wall' where findings can be displayed.

Reflection

How might the teacher evaluate this lesson? What is the purpose of the display?

Commentary

The evaluation should be against the objectives of the lesson, which may be about children's increasing confidence working with even and odd numbers, and/or about the processes of generalizing, working systematically and justifying. It could include assessing children's explanatory language. It is a suitable activity to both identify able learners and challenge them, in all primary ages. Some 5 and 6 year olds are capable of very sophisticated thinking, although their recording and justifications may not be so clear.

The display will be added to over the next week, making this a simmering activity. Some children will be very engaged in it and want to continue exploring in their own time. At suitable junctures the teacher will comment on new additions and offer additional prompts so that the children realize that some mathematical problems can be extended to last a long time.

Summary

In this chapter we have examined what is meant by high ability in mathematics. We have considered ways of identifying the behaviour that mathematicians value and how we may be surprised by children if we offer them rich tasks to display their thinking. We have looked at the sort of tasks we might offer and the ways of working that run in parallel with them. As teachers we have a responsibility to all the children in our class and providing appropriately for the most able is important for the children themselves, for us as teachers, and for all our futures.

Further reading

McClure, L. (2011) *Low Threshold High Ceiling Tasks in Ordinary Classrooms*. Available at <http://nrich.maths.org/7701> (accessed 30 January 2012). This article picks out some favourite LTHC tasks which are suitable for teachers and pupils new to the idea.

Kennard, R. (2001) *Teaching Mathematically Able Children*. London: NACE/Fulton.

A summary of the research and implications for teaching across the age range. The following are also useful titles:

McClure, L. and Piggott, J. (2007) *Meeting the Needs of your Most Able Pupils: Mathematics*. Abingdon: Routledge.

Mason, J., Burton, L. and Stacey, K. (1985) *Thinking Mathematically*. London: Addison-Wesley.

Mislevy, R.J. (2012) *What You Test is What You Get*. College Park, MD: College of Education, University of Maryland. Available at www.education.umd.edu/news/news2008/news0803QATest.html (accessed 30 May 2012).

Useful websites

(All accessed 30 May 2012)

Government Information on Gifted and Talented Children: www.direct.gov.uk/en/Parents

National Association for Gifted Children: www.nagcbritain.org.uk

National Numeracy: www.nationalnumeracy.org.uk

NRICH: www.nrich.maths.org.uk

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