

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD

MAY 2019

CYFNOD ARHOLIADAU

MAI 2019

MODULE CODE:

M6X00420: Advanced Stress & FEA (PART TIME)

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

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A formula sheet is provided for reference.

University approved calculators are permitted.

Assume 'g' = 9.81 m/s².

Question 1

a.

Whilst Aluminium offers no advantage in terms of tensile loading, due to its lower density, larger sections can be used which offer benefits in terms of bending and buckling due to the increased second moment of area when compared to steel alternatives.

bi.

Complete section:

$$\begin{aligned}
 I_{xx} &= 2 \times I_{xx(\text{section})} + 2 \times I_{xx(\text{plate})} \\
 &= 2 \times 1.5 \times 10^{16} + 2(I_G + Ah^2) \\
 &= 2 \times 1.5 \times 10^{16} + 2\left(\frac{bd^3}{12} + Ah^2\right) \\
 &= 2 \times 1.5 \times 10^{16} + 2\left(\frac{150 \times 2^3}{12} + 150 \times 2 \times 74^2\right) \\
 &= 3 \times 10^{16} + 3.29 \times 10^6 \\
 &= 6.29 \times 10^6 \text{ mm}^4
 \end{aligned}$$

$$\begin{aligned}
 I_{yy} &= 2 \times I_{yy(\text{section})} + 2 \times I_{yy(\text{plate})} \\
 &= 2(I_G + Ah^2) + 2 \times I_{yy(\text{plate})} \\
 &= 2 \times (5.5 \times 10^5 + 384 \times 60^2) + 2 \times \frac{2 \times 150^3}{12} \\
 &= 3.86 \times 10^6 + 1.13 \times 10^6 \\
 &= 4.99 \times 10^6 \text{ mm}^4
 \end{aligned}$$

So, $I_{xx} > I_{yy}$ so it will buckle about the yy axis.

bii.

Radius of Gyration

$$k = \sqrt{\frac{I_{yy}}{A}}$$

$$\begin{aligned}
 \text{Where;} \\
 A &= 2(2 \times 150 \times 384) \\
 &= 1368 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore k &= \sqrt{\frac{4.99 \times 10^6}{1368}} \\
 &= 60.4 \text{ mm}
 \end{aligned}$$

biii.

Slenderness Ratio

$$= \frac{l}{k} = \frac{1}{60.4 \times 10^{-3}} = 16.6$$

biv.

Euler, fixed end/free end;

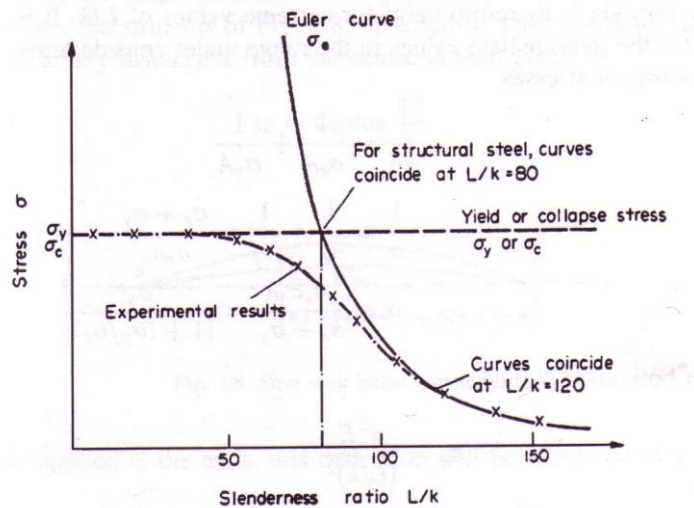
$$\begin{aligned}
 P_e &= \frac{\pi^2 EI}{4L^2} \\
 &= \frac{\pi^2 \times 200 \times 10^9 \times 4.99 \times 10^{-6}}{4 \times 1^2} \\
 &= 2.46 \times 10^6 \text{ N}
 \end{aligned}$$

bv.

Rankine Gordon

$$\begin{aligned}
 P_R &= \frac{\sigma_y A}{1 + a \left(\frac{l}{k} \right)^2} \\
 &= \frac{320 \times 10^6 \times 1.368 \times 10^{-3}}{1 + \frac{1}{7500} \times 16.6^2} \\
 &= \frac{437760}{1.00035} \\
 &= 437322.4 \text{ N} \\
 &= 0.437 \text{ MN}
 \end{aligned}$$

bvi.



Euler is only valid for large slenderness ratios. For smaller values Rankine Gordon should be used, in this case this is likely to be more accurate.

Question 2

a.

$$\frac{\sigma}{y} = \frac{M}{I} \quad \therefore \sigma = \frac{My}{I}$$

Now

$$I = \frac{\pi d^4}{64}$$

Therefore

$$\begin{aligned}\sigma &= \frac{64My}{\pi d^4} \\ &= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4} \\ &= 155 \text{ MPa}\end{aligned}$$

b.

$$\frac{T}{J} = \frac{\tau}{R} \quad \therefore \tau = \frac{TR}{J}$$

Now

$$J = \frac{\pi d^4}{32}$$

Therefore

$$\begin{aligned}\tau &= \frac{32TR}{\pi d^4} \\ &= \frac{32T \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4} \\ &= 40.7 \times 10^3 T\end{aligned}$$

ci.

Maximum stress theory;

$$\sigma_1 - \sigma_3 = \sigma_{yield}$$

From complex stress theory;

$$\sigma_1 = \frac{1}{2} \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

$$\text{Let } a = (\sigma_x + \sigma_y) \text{ and } b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_1 = \frac{1}{2}(a + b) = \frac{1}{2}a + \frac{1}{2}b$$

$$\sigma_3 = \frac{1}{2} \left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

$$\sigma_3 = \frac{1}{2}(a - b) = \frac{1}{2}a - \frac{1}{2}b$$

Now;

$$\begin{aligned}\sigma_{yield} &= \sigma_1 - \sigma_3 \\ &= \frac{1}{2}a + \frac{1}{2}b - \left(\frac{1}{2}a - \frac{1}{2}b\right) \\ &= b \\ &= \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\end{aligned}$$

Since $\sigma_y = 0$

$$\begin{aligned}\sigma_{yield} &= \sqrt{\sigma_x^2 + 4\tau_{xy}^2} \\ \therefore \tau_{xy} &= \sqrt{\frac{\sigma_{yield}^2 - \sigma_x^2}{4}} \\ &= \sqrt{\frac{(200 \times 10^6)^2 - (155 \times 10^6)^2}{4}} \\ &= 63.2 \times 10^6 \text{ Pa}\end{aligned}$$

Since

$$\begin{aligned}\tau &= 40.7 \times 10^3 T \\ 63.2 \times 10^6 &= 40.2 \times 10^3 T \\ \therefore T &= 1.55 \text{ kNm}\end{aligned}$$

cii.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

Now,

$$\sigma_1 = \frac{1}{2}(a + b) = \frac{1}{2}a + \frac{1}{2}b$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{1}{2}(a - b) = \frac{1}{2}a - \frac{1}{2}b$$

$$\left(\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(-\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a - \frac{1}{2}b\right)^2 = 2\sigma_y^2$$

$$\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a + \frac{1}{2}b\right) + \left(-\frac{1}{2}a + \frac{1}{2}b\right)\left(-\frac{1}{2}a + \frac{1}{2}b\right) + b^2 = 2\sigma_y^2$$

$$\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{4}b^2 + \frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}ab + \frac{1}{4}b^2 + b^2 = 2\sigma_y^2$$

$$\frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$$

Now

$$a = (\sigma_x + \sigma_y) = \sigma_x \text{ since } \sigma_y = 0$$

$$b = \sqrt{(\sigma_x)^2 + 4\tau_{xy}^2}$$

$$\therefore \frac{1}{2}a^2 + \frac{3}{2}b^2 = 2\sigma_y^2$$

$$\frac{1}{2}\sigma_x^2 + \frac{3}{2}(\sigma_x^2 + 4\tau_{xy}^2) = 2\sigma_y^2$$

$$\frac{1}{2}\sigma_x^2 + \frac{3}{2}\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$$

$$2\sigma_x^2 + 6\tau_{xy}^2 = 2\sigma_y^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$$

Now

$$\sigma_x = 155MPa$$

$$\tau_{xy} = 40.7 \times 10^3 TPa = 40.7 \times 10^{-3} TPa$$

$$\therefore 155^2 + 3(40.7 \times 10^{-3})^2 T^2 = 200^2$$

$$(40.7 \times 10^{-3})^2 T^2 = \frac{200^2 - 155^2}{3}$$

$$\therefore T^2 = \frac{5325}{(40.7 \times 10^{-3})^2} \therefore T = 1.79 kNm$$

Question 3

a.

Discussion such as;

FEA is only an approximation of the real problem, where assumptions are made from one point to the next, from node to node. As such, the mesh will significantly influence the results. Checks that should be carried out include;

1. **Element size.** This is always a trade off between runtime and accuracy. Generally, the smaller the mesh the more accurate the results are but the longer the analysis will take.
2. **Element quality.** The assumptions that are made from node to node across the elements are only valid if the elements maintain a 'perfect' shape. If the elements deviate from this, then the results will be questionable. Checks could include Aspect Ratio and Jacobian etc. Elements deviate from a perfect shape due to the nature of the geometry of the part being considered and trying to fit elements to this.
3. **Element duplicates.** These can occur depending on how the CAD has been generated, for example there may be two overlapping surfaces on the geometry, when obviously in reality only one surface exists. This if both of these surfaces are meshed then the part will obviously be stiffer in the FE model than in reality.
4. **Element free edges.** These can also occur depending on how the CAD has been generated, for example when two edges are butted up against each other in CAD but are not 'stitched' together. If these surfaces are simply meshed and the nodes between the two meshes are not connected, then the part will behave as if these surfaces are independent of each other, even if in fact on the real part they are connected.
5. **Element Normals.** The element normal is dependent on the order that the nodes are generated (dependent on the CAD and the meshing algorithm used). This can influence contacts in non linear analysis (where to save on computational time, often only one side of the element is checked in the contact). This will also influence any stresses etc seen if these are plotted on the top/bottom surface.

b.

Discussion such as;

The easiest way to correlate a model is by comparing the displacements in the physical tests to those seen in the FE model. This can be done using simple measuring devices such as DTI gauges, or more advanced measuring devices such as strain gauges or Photo-Elastic techniques. Other ways that may be used to correlate a model are by measuring the forces in certain locations using load cells, using accelerometers or by comparing the FE model to results that have obtained using traditional mechanics of materials type hand calculations.

Question 4

ai.

In order for analysis to be considered to be linear static the following conditions must be met.

- The force is static, that is to say that there is no variation of the force with respect to time ($\frac{dF}{dt} = 0$)
- The part is in equilibrium, i.e. $\Sigma Forces = 0$ and $\Sigma Moments = 0$

aii.

Analysis may be considered to be non-linear for the following reasons,

Geometric Non-linearity. This may be caused by large deformations in the model due to large forces relative to the stiffness of the model.

Material Non-linearity. This may be due to the following reasons.

- The material has exceeded its linear region – for example when metals exceed yield.
- The material behaves in a non-linear fashion. For example rubbers.
- Creep. Over time, the behaviour of the structure changes.

Contact. If anything in the model contacts something else then the response must be non-linear, (for example the structure would have one stiffness prior to contact and another after contact). This may be modelled using GAP elements or by adding contacts to the FE model.

b.

Other forms of analysis include;

1. **Dynamic analysis.** When either the loads or the response is a function of time.
2. **Buckling analysis.** Used for slender beams where the part is loaded in compression and the bending stiffness is much less than the axial stiffness. This results in a large lateral deformation.
3. **Thermal analysis.** This may consider conduction, convection or radiation.
4. **Fatigue analysis.** The fatigue of a component is considered using SN curves when the part is subjected to cyclic loading.
5. **Optimisation.** This may include techniques such as topology, topography, topometry etc
6. **CFD analysis.** This can consider Hydrostatics as well as fluid dynamics.
7. **Crash analysis.** This may be used to develop vehicle crashworthiness, drop tests or occupant safety
8. **NVH analysis.** These may consider sound radiation or vibroacoustic problems.