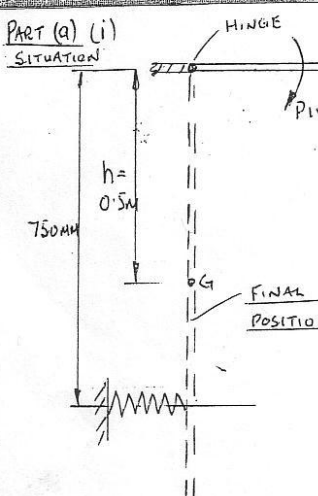
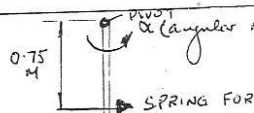




EXAMINATION SOLUTIONS : SESSION 2010 / 11

Programme:	BENG MANUF. SYSTEMS ENGINEERING	Year:	LEVEL 5
Module/Unit:	STRESS ANALYSIS + DYNAMICS		
Author:	R.E. WILLIAMS (50%)	Page:	1 of 7

Question No	Question	Marks
1	PART (a) (i)  1m long x 30kg beam CoM at mid-length. For Beam $I = \frac{ML^3}{3}$ (as given) $I = 30 \times \frac{1^3}{3} = 10 \text{ kgm}^2$ Potential Energy of Beam at Initial Position Relative to Final Position = mgh $= 30 \times 9.81 \times 0.5$ $= 147.15 \text{ J}$ This energy will have been absorbed by spring at instant it comes to rest, where 'strain energy' in spring = $\frac{1}{2} kx^2$ $\therefore$ Given $x(\text{max}) = 40 \text{ mm}$, and assuming no losses, $\frac{1}{2} k \times 0.04^2 = 147.15$ $\therefore k = \frac{147.15}{8 \times 10^{-4}} = 184 \times 10^3 \text{ Nm}^{-1} \text{ (184 kNm}^{-1}\text{)}$ (ii) As beam comes to rest have situation: 	2 (including interpretation of situation) 1 1 2 1

Continuation Sheet

Module/Unit:	STRESS ANALYSIS + DYNAMICS	Page:	2 of 7
--------------	----------------------------	-------	--------

Question No	Question	Marks
1	Spring Force $F = kx = (184 \times 10^3) \times 0.04$ $= 7360 \text{ N}$ Equating moments about pivot at 'point of maximum spring compression': $F \times 0.75 = I \alpha \quad (\text{where } I = 10 \text{ kgm}^2 \text{ as calculated})$ $\therefore 7360 \times 0.75 = 10 \alpha$ Giving $\alpha = (-) \frac{7360 \times 0.75}{10} = \underline{552 \text{ rad s}^{-2}}$ Ans <u>PART (b) (i)</u> At Maximum velocity Traction (F) = Resistance (R) S/T 10 Marks and $R \propto v^2$ as stated $\therefore R = kv^2$ Power = Traction $\times$ velocity $\therefore$ At maximum speed $P = R \times v = (kv^2) \times v$ Giving $k = \frac{P}{v^3} = \frac{80 \times 10^3}{50^3} = 0.64$ Thus, generally at velocity v: $R = 0.64 v^2$ (as given) When Accelerating at velocity $v \text{ ms}^{-1}$ $F_{\text{ACCEL}} = F_{\text{TRACTION}} - R$ At 50 ms^{-1} (maximum vel. at full power), $F_{\text{TRACTION}} = \frac{P}{v}$ i.e. $F_{\text{TRACTION}} = \frac{80 \times 10^3}{50} = 1600 \text{ N}$ Hence $F_{\text{ACCEL}} = m \frac{dv}{dt} = 1600 - 0.64 v^2$ (at full throttle and assuming constant tractive force, as stated) i.e. $900 \frac{dv}{dt} = 1600 - 0.64 v^2$ <u>Separating Variables and Integrating between Stated Velocity limits:</u> $\int_0^t dt = \int_{20}^{30} \frac{900 dv}{1600 - 0.64 v^2}$	1 2 1 2 1 1 1

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	3 of 7
--------------	----------------------------	-------	--------

Question No	Question	Marks
1	$\therefore t \text{ (acc. time)} = \int_{20}^{30} \frac{1406.25}{50^2 - v^2} dv$ (Dividing 'top + bottom' by 0.64 to get 'standard form') Giving $t = \frac{1406.25}{2 \times 50} \left[\ln \frac{50+v}{50-v} \right]_{20}^{30}$ (Applying 'standard integral' given) $= 14.0625 \left\{ \ln \left(\frac{50+30}{50-30} \right) - \ln \left(\frac{50+20}{50-20} \right) \right\}$ $= 14.0625 (1.386 - 0.847) \approx \underline{7.58 \text{ s}}$ S/T Question Total	1 2 10 marks 20 marks



EXAMINATION SOLUTIONS : SESSION 2000 / 01

Programme:	BENG MANUF. SYSTEMS ENGINEERING	Year:	LEVEL 5
Module/Unit:	STRESS ANALYSIS + DYNAMICS		
Author:	R.E. WILLIAMS (50%)	Page:	4 of 7

Question No	Question	Marks
2	<u>Part (a)</u> Must consider engagement of gears A/B and gears C/D separately, with information used in A/B engagement applied in C/D engagement:- <u>A/B Engagement</u> : $\frac{N_A}{N_B} = \frac{T_B}{T_A} = \frac{1440}{630} = \frac{48}{21}$ (exactly) $\therefore$ Suitable teeth $N^{\circ}s$ are $\frac{T_A}{T_B} = \frac{21}{48}$ } Ans. and $\frac{T_B}{T_A} = \frac{48}{21}$ Must also consider shaft centre distance using above to proceed: Shaft Centre Distance = $\frac{M_1(T_A + T_B)}{2} = \frac{8(21+48)}{2}$ (either engagement) = 276 mm Now for C/D Engagement : $\frac{M_2(T_C + T_D)}{2} = 276$ (also) $\therefore \frac{6(T_C + T_D)}{2} = 276$, giving $T_C + T_D = \frac{276}{3}$ = 92 (exactly) <u>C/D Speed Ratio</u>: $\frac{N_C}{N_D} = \frac{T_D}{T_C} \approx \frac{1440}{520}$ (as close as possible) $\therefore T_D \approx 2.769 T_C$ (2)	2 2 2 1

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	5 of 7
--------------	----------------------------	-------	--------

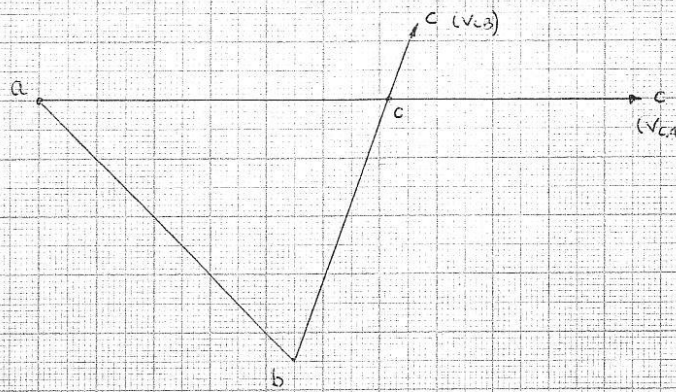
Question No.	Question	Marks
2	Combining (1) and (2): $T_c + 2.769 T_c \approx 92$ $\therefore T_c \approx \frac{92}{3.769} \approx 24.41 \text{ teeth} \rightarrow \underline{24 \text{ teeth}}$ Thus, if $T_c = 24 \text{ teeth}$, $T_D = 92 - 24 = \underline{68 \text{ teeth}}$ (Actual lower speed = $\frac{24}{68} \times 1440 = \underline{508.2 \text{ rpm}}$) <u>PART (b)</u> <u>VELOCITY DIAGRAM</u> (See 'Page 3') V_{BA} (Tangential vel. at B) = $AB \times \omega_{AB} = 0.1 \times \left(\frac{2\pi \times 300}{60} \right)$ $= \underline{3.14 \text{ m s}^{-1}}$ <u>Constructional Step:</u> $\vec{ab}$ Drawn $\frac{3.14}{0.5 \text{ (scale)}}$ = 6.28 cm at 45° angle from vertical in 'SE' sense. <u>Constructional Step:</u> linear velocity of C relative to A represented by horizontal through a, <u>magnitude unknown</u> <u>Constructional Step:</u> V_{CB} (Tangential vel. at C) drawn perpendicular to axis of BC i.e. at 20° angle from vertical. Intersection with horizontal fixes c. Velocity of slider $\equiv \vec{ac} = 6.2 \text{ cm} = 6.2 \times 0.5$ $\approx \underline{3.1 \text{ m s}^{-1}}$ in sense $a \rightarrow c$ (moving to right) <u>ACCELERATION DIAGRAM</u> (See 'Page 4') Centripetal Accelⁿ. $B \rightarrow A = AB \times \omega_{AB}^2 = \frac{V_{BA}^2}{AB}$ $= \frac{3.14^2}{0.1} = \underline{986 \text{ m s}^{-2}}$ <u>Constructional Step:</u> $\vec{ab}$ drawn at 45° angle of link $\frac{986}{10 \text{ (scale)}}$ $\approx 9.9 \text{ cm}$ long in sense $B \rightarrow A$. <u>Linear Accelⁿ of C relative to A</u> will be horizontal. <u>Constructional Step:</u> $\vec{ac}$ drawn, <u>magnitude unknown</u>	Ans - 3 S/- 10 Marks $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1

Continuation Sheet

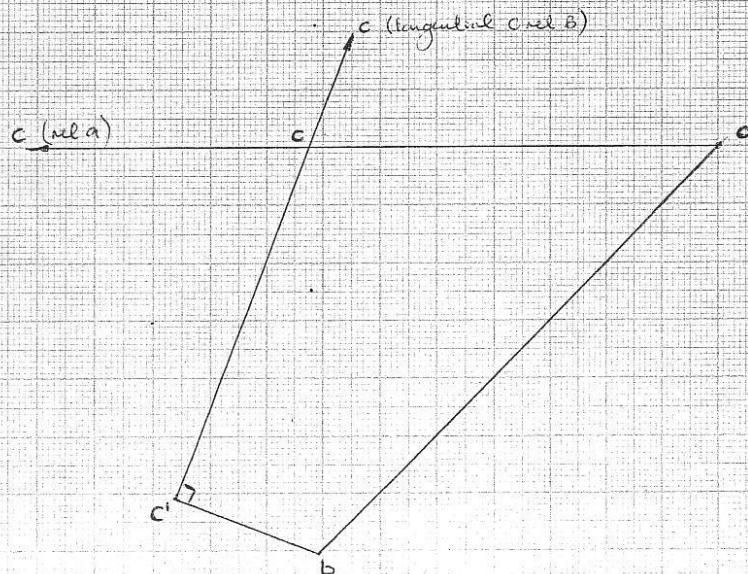
Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	6 of 7
--------------	----------------------------	-------	--------

Question No.	Question	Marks
2	<u>Centripetal Accelⁿ C → B</u> = $\frac{V_{CB}^2}{BC}$, where $V_{CB} \equiv \vec{BC}$ on V.D $\therefore \text{C.P.}_{C \rightarrow B} = \frac{(4.8 \times 0.5 \text{ (scale)})^2}{0.21} = 27.4 \text{ m s}^{-2}$ — 1½ <u>Constructional Steps</u>: bc' drawn at 20° angle of BC in sense $C \rightarrow B$, $\frac{27.4}{10} \approx 2.74 \text{ cm long}$ <u>Tangential Accelⁿ of C relative to B</u> drawn through c' at 90° angle to bc', magnitude unknown — 1 <u>Intersection with $\vec{ac}$ fixes C</u> (Construction Complete) <u>Accelⁿ of Slider</u> $\equiv \vec{ac} = 72 \text{ cm (measured)}$ <math>\therefore \text{Accelⁿ of slider} = 7.2 \times 10 \text{ (scale)} = 72 \text{ m s}^{-2}</math> — 1 in sense $a \rightarrow c$, which is opposite to instantaneous velocity, $\therefore$ <u>Decelerating.</u> (NOTE: Details of 'Constructional Steps' are not expected)	8/T 10 marks 20 MARKS
	Question Total	

VELOCITY DIAGRAM : Scale $1\text{ cm} = 0.5\text{ m s}^{-1}$



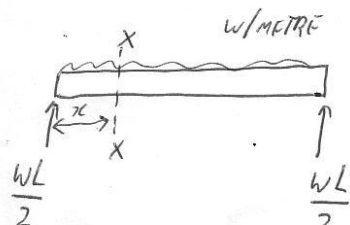
ACCELERATION DIAGRAM : Scale $1\text{ cm} = 10\text{ m s}^{-2}$





EXAMINATION SOLUTIONS: SESSION 2010/11

School:	LOGISTICS & MANUFACTURING ENGINEERING		
Programme:		Year:	
Module/Unit:	STRESS ANALYSIS & DYNAMICS		
Author:	K. LAKE	Page:	1 of 5

Question No	Question	Marks
3 a)	 $M_x = \frac{WL}{2}x - w \cdot x \cdot \frac{x}{2} = \frac{WLx}{2} - \frac{wx^2}{2}$ $\frac{WLx}{2} - \frac{wx^2}{2} = EI \frac{d^2y}{dx^2}$ INTEGRATING ONCE: $EI \frac{dy}{dx} = \frac{WLx^2}{4} - \frac{wx^3}{6} + A \quad (1)$ INTEGRATING TWICE $EIy = \frac{WLx^3}{12} - \frac{wx^4}{24} + Ax + B \quad (2)$	① ① ①½ ①½

Continuation Sheet

Module/Unit:	STATICS ANALYSIS & DYNAMICS	Page:	2 of 5
--------------	-----------------------------	-------	--------

Question No	Question	Marks
	BOUNDARY CONDITIONS: WHEN $x=0$, $y=0$ FROM (2) : <u>$B=0$</u> WHEN $x=L$, $y=0$ $\therefore$ FROM (2) $0 = \frac{WL^4}{12} - \frac{WL^4}{24} + AL$ $\therefore AL = -\frac{WL^4}{24}$ $\therefore A = -\frac{WL^3}{24}$ $\therefore$ FROM (2) $EI y = \frac{WLx^3}{12} - \frac{WLx^4}{24} - \frac{WL^3x}{24}$ (b) $EI = \frac{1}{y} \left[\frac{WLx^3}{12} - \frac{WLx^4}{24} - \frac{WL^3x}{24} \right]$ $= \frac{1}{-2 \times 10^{-3}} \left[\frac{200 \times 5 \times 2.5^3}{12} - \frac{200 \times 2.5^4}{24} - \frac{200 \times 5^3 \times 2.5}{24} \right]$ $= \frac{1}{-2 \times 10^{-3}} \left[1302.0 - 20833.3 - 2604.2 \right]$ $= \underline{\underline{69.87 \times 10^6 \text{ Pa m}^4}}$	(1) (1) (2) (1) (5)

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	3 of 5
--------------	----------------------------	-------	--------

Question No	Question	Marks
(ii)	$EI = 69870000$ $\therefore I = \frac{69870000}{69 \times 10^9}$ $= \underline{\underline{1.01 \times 10^{-3} \text{ m}^4}}$	(1)
(iii)	$I = 1.01 \times 10^{-3} \text{ m}^4$ $= (1.01 \times 10^{-3}) \times 10^8$ $= \underline{\underline{101273 \text{ cm}^4}}$ LIGHTEST BEAM = D & B: 617.5 & 307.0 $I_{xx} = 151,631 \text{ cm}^4$ WEIGHT = 113 Kg/m $\therefore$ WEIGHT OF BEAM = $113 \times 5 = \underline{\underline{565 \text{ Kg}}}$	(4) 20

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	4 of 5
--------------	----------------------------	-------	--------

Question No	Question	Marks
4 a	$\theta = 90 - 40 = 50^\circ$ $\sigma_x = 20 \text{ MPa}$ $\sigma_y = -40 \text{ MPa}$ $\tau_{xy} = -20 \text{ MPa}$	1
	$\therefore \sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$ $= \frac{1}{2}(20 - 40) + \frac{1}{2}(20 + 40) \cos 100 - 20 \sin 100$ $= -34.9 \text{ MPa}$	2
	$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$ $= \frac{1}{2}(20 + 40) \sin 100 + 20 \cos 100$ $= 26.1 \text{ MPa}$	2
(b)	$\sigma_1 = \frac{1}{2}[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ $= \frac{1}{2}[(20 - 40) + \sqrt{(20 + 40)^2 + 4(-20)^2}]$ $= \frac{1}{2}(-20 + \sqrt{3600 + 1600})$	2

$$= 26.66 \text{ MPa}$$

Continuation Sheet

Module/Unit: STRESS ANALYSIS & DYNAMICS Page: 5 of 5

Question No	Question	Marks
(20)	$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ $= \frac{1}{2} \sqrt{(20 + 40)^2 + 4(-20)^2}$ $= \underline{\underline{36.06 \text{ MPa}}}$	2
(21)	SEE MOHR'S CIRCLE OVER PAGE	(4)
	TAKING MEASUREMENTS: $\sigma_0 \approx \underline{\underline{-34.5 \text{ MPa}}}$	1
	$\tau_0 \approx \underline{\underline{26 \text{ MPa}}}$	1
	$\sigma_1 \approx \underline{\underline{26 \text{ MPa}}}$	1
	$\tau_{max} \approx \underline{\underline{36 \text{ MPa}}}$	1
(22)	BY MEASUREMENT $2\theta_1 \approx 146^\circ$ $\therefore \theta_1 \approx \underline{\underline{73^\circ}}$	2
(23)	(21) IS APPROXIMATE AS THEY ARE MEASURED.	1
		20