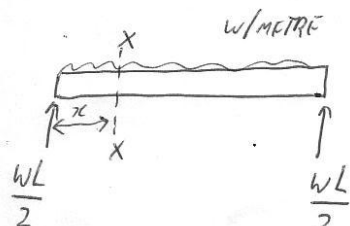




EXAMINATION SOLUTIONS: SESSION 2010/11

School:	LOGISTICS & MANUFACTURING ENGINEERING		
Programme:		Year:	
Module/Unit:	STRESS ANALYSIS & DYNAMICS		
Author:	K. LAKE	Page:	1 of 17

Question No	Question	Marks
1(a)	 $M_x = \frac{WLx}{2} - w \cdot x \cdot \frac{x}{2} = \frac{WLx}{2} - \frac{wx^2}{2}$ $\frac{WLx}{2} - \frac{wx^2}{2} = EI \frac{d^2y}{dx^2}$ INTEGRATING ONCE: $EI \frac{dy}{dx} = \frac{WLx^2}{4} - \frac{wx^3}{6} + A \quad (1)$ INTEGRATING TWICE $EIy = \frac{WLx^3}{12} - \frac{wx^4}{24} + Ax + B \quad (2)$	1 1 1 1/2 1 1/2

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Question No	Question	Marks
	BOUNDARY CONDITIONS: WHEN $x=0$, $y=0$ FROM (2) : <u>$B=0$</u> WHEN $x=L$, $y=0$ $\therefore$ FROM (2) $0 = \frac{WL^4}{12} - \frac{WL^4}{24} + AL$ $\therefore AL = -\frac{WL^4}{24}$ $\therefore A = -\frac{WL^3}{24}$ $\therefore$ FROM (2) $EI y = \frac{WLx^3}{12} - \frac{WLx^4}{24} - \frac{WL^3x}{24}$ (b) $EI = \frac{1}{y} \left[\frac{WLx^3}{12} - \frac{WLx^4}{24} - \frac{WL^3x}{24} \right]$ $= \frac{1}{-2 \times 10^{-3}} \left[\frac{200 \times 5 \times 2.5^3}{12} - \frac{200 \times 2.5^4}{24} - \frac{200 \times 5^3 \times 2.5}{24} \right]$ $= \frac{1}{-2 \times 10^{-3}} \left[1302.0 - 20833.3 - 2604.2 \right]$ $= \underline{\underline{69.87 \times 10^6 \text{ Pa m}^4}}$	(1) (1) (2) (1) (5)

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Question No	Question	Marks
(ii)	$EI = 69870000$ $\therefore I = \frac{69870000}{69 \times 10^9}$ $= \underline{\underline{1.01 \times 10^{-3} \text{ m}^4}}$	(1)
(iii)	$I = 1.01 \times 10^{-3} \text{ m}^4$ $= (1.01 \times 10^{-3}) \times 10^8$ $= \underline{\underline{101273 \text{ cm}^4}}$ $\therefore \text{LIGHTEST BEAM} = D \times B = 617.5 \times 307.0$ $I_{xx} = 151,631 \text{ cm}^4$ $\text{WEIGHT} = 113 \text{ Kg/m}$ $\therefore \text{WEIGHT OF BEAM} = 113 \times 5 = \underline{\underline{565 \text{ Kg}}}$	(4) 20

Continuation Sheet

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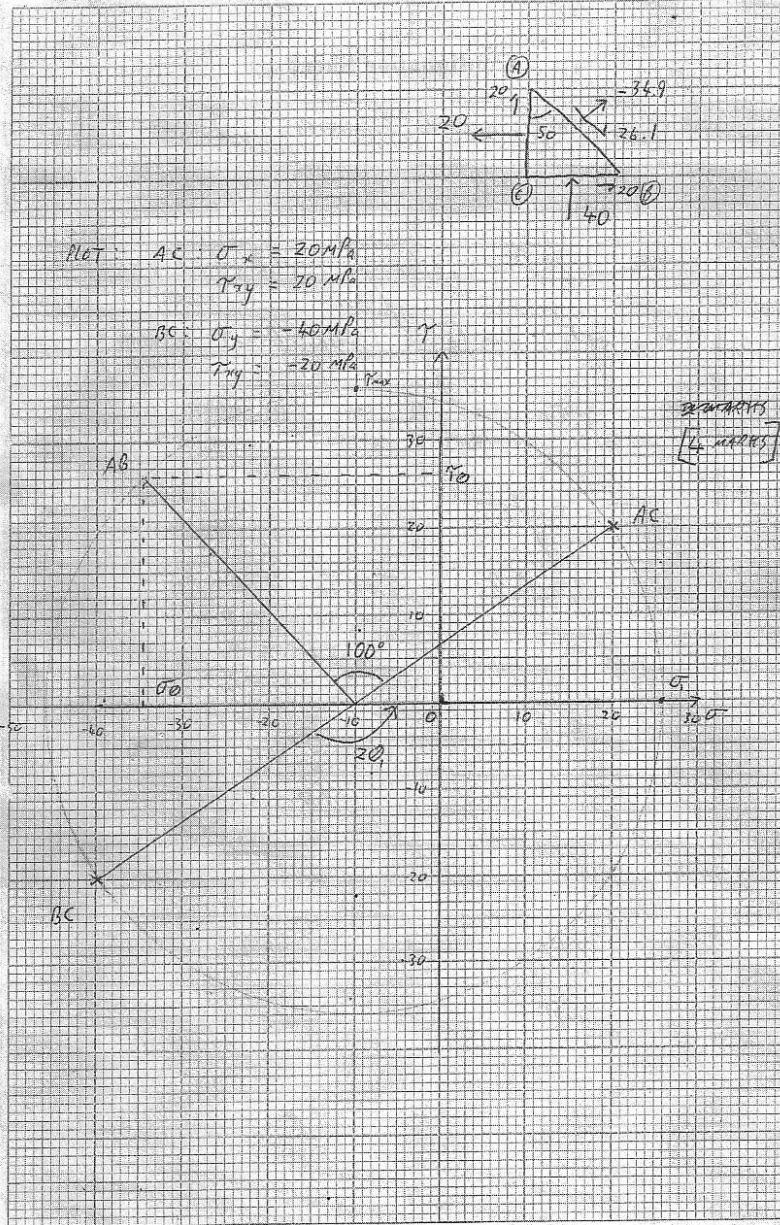
Question No	Question	Marks
(2)	$\theta = 90 - 40 = 50^\circ$ $\sigma_x = 20 \text{ MPa}$ $\sigma_y = -40 \text{ MPa}$ $\tau_{xy} = -20 \text{ MPa}$	1
	$\therefore \sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$ $= \frac{1}{2}(20 - 40) + \frac{1}{2}(20 + 40) \cos 100 - 20 \sin 100$ $= -34.9 \text{ MPa}$	2
	$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$ $= \frac{1}{2}(20 + 40) \sin 100 + 20 \cos 100$ $= 26.1 \text{ MPa}$	2
(6)	$\sigma_1 = \frac{1}{2}[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}]$ $= \frac{1}{2}[(20 - 40) + \sqrt{(20 + 40)^2 + 4(-20)^2}]$ $= \frac{1}{2}(-20 + \sqrt{3600 + 1600})$	2

$$= 26.06 \text{ MPa}$$

Continuation Sheet

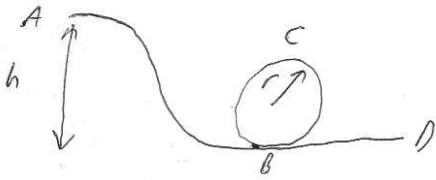
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Question No	Question	Marks
(20)	$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ $= \frac{1}{2} \sqrt{(20+40)^2 + 4(-20)^2}$ $= \underline{\underline{36.06 \text{ MPa}}}$	2
(21)	SEE MOMENTS CIRCLE OVER PAGE	(4)
	TAKING MEASUREMENTS: $\sigma_0 \approx \underline{\underline{-34.5 \text{ MPa}}}$	1
	$\tau_0 \approx \underline{\underline{26 \text{ MPa}}}$	1
	$\sigma_1 \approx \underline{\underline{26 \text{ MPa}}}$	1
	$\tau_{max} \approx \underline{\underline{36 \text{ MPa}}}$	1
(22)	BY MEASUREMENT $2\theta_1 \approx 146^\circ$ $\therefore \theta_1 \approx \underline{\underline{73^\circ}}$	2
(23)	(21) IS APPROXIMATE AS THEY ARE MEASURED.	1
		20



Continuation Sheet

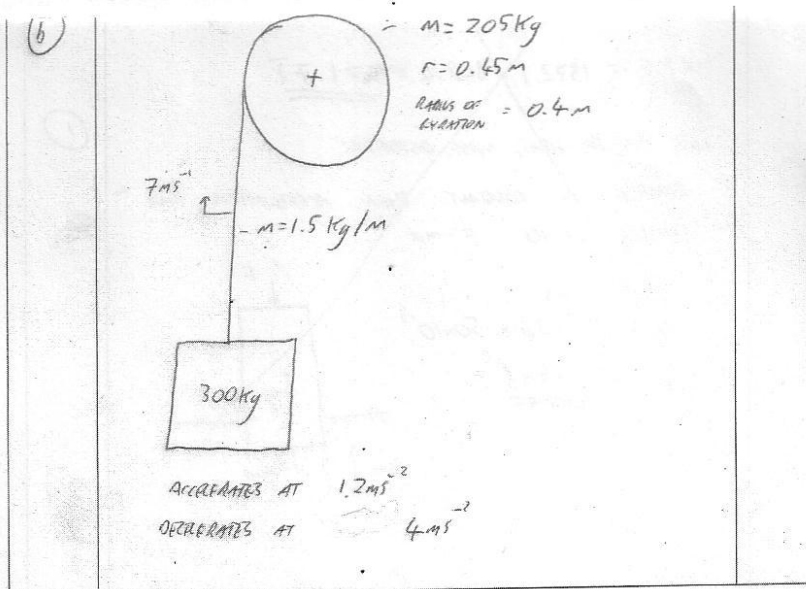
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Question No	Question	Marks
3a;	 <u>A → C:</u> LOSS OF P.E. = GAIN IN K.E. $mg(h-2r) = \frac{1}{2}mv^2$ $\therefore v^2 = 2g(h-2r) \quad *$ FOR THE CAR TO REMAIN IN CONTACT THE UPWARD CENTRIFUGAL FORCE MUST BE GREATER OR EQUAL TO THE WEIGHT $\frac{mv^2}{r} \geq mg$ FROM * $\frac{2mg(h-2r)}{r} \geq mg$ $2(h-2r) \geq r$ $2h-4r \geq r$ $2h \geq 5r$ $\therefore \underline{h \geq 2.5r}$	(5)

Continuation Sheet

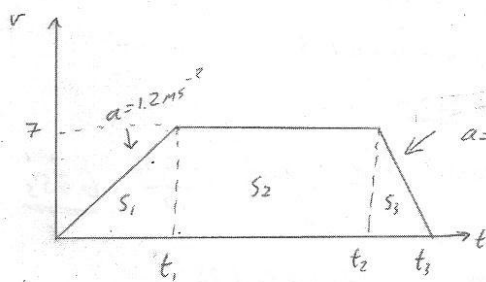
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Question No	Question	Marks
(i)	 $S_{TOTAL} = 80m$ <u>PHASE 1 ACCELERATION</u> $u=0, v=7, a=1.2, t=?, s=?$ $v = u + at$ $v^2 = u^2 + 2as$ $t_1 = \frac{v-u}{a}$ $s = \frac{v^2 - u^2}{2a}$ $= \frac{7-0}{1.2}$ $= \frac{7^2 - 0^2}{2 \times 1.2}$ $= 5.83s$ $= 29.4m$ <u>PHASE 2 DECELERATION</u> $u=7, v=0, a=-4, t=?, s=?$ $t = \frac{v-u}{a}$ $s = \frac{v^2 - u^2}{2a}$ $= \frac{0-7}{-4}$ $= \frac{0^2 - 7^2}{2 \times -4}$ $= 1.75s$ $= 6.125m$	(1) (1) (1)

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Question No	Question	Marks
(1)	$S_{TOTAL} = S_1 + S_2 + S_3$ $80 = 29.4 + S_2 + 6.125$ $\therefore S_2 = \underline{44.5m}$ CONSTANT VELOCITY $v = \frac{d}{t} \therefore t = \frac{d}{v} = \frac{44.5}{7} = \underline{6.35s}$ $\therefore TOTAL TIME = 5.83 + 1.75 + 6.35$ $= \underline{13.9s}$	(2)
(b) ii	ROPE TENSION AT DRUM = $M_{TOT} \times a_{TOT}$ $= (M_{CAGE} + M_{ROPE}) \times (a + g)$ $= (300 + 80 \times 1.5) \times (1.2 + 9.81)$ $= \underline{4624.2N}$ TORQUE TO ACCELERATE CAGE + DRUM = 4624.2×0.45 $= \underline{2080.9Nm}$ HOWEVER TORQUE IS ALSO NEEDED TO ROTATE DRUM ITSELF $T_{DRUM} = I_{DRUM} \times \alpha_{DRUM}$ NOW $\alpha_{DRUM} = \frac{a}{r} = \frac{1.2}{0.45} = \underline{2.67 rad s^{-2}}$	(2)

Continuation Sheet

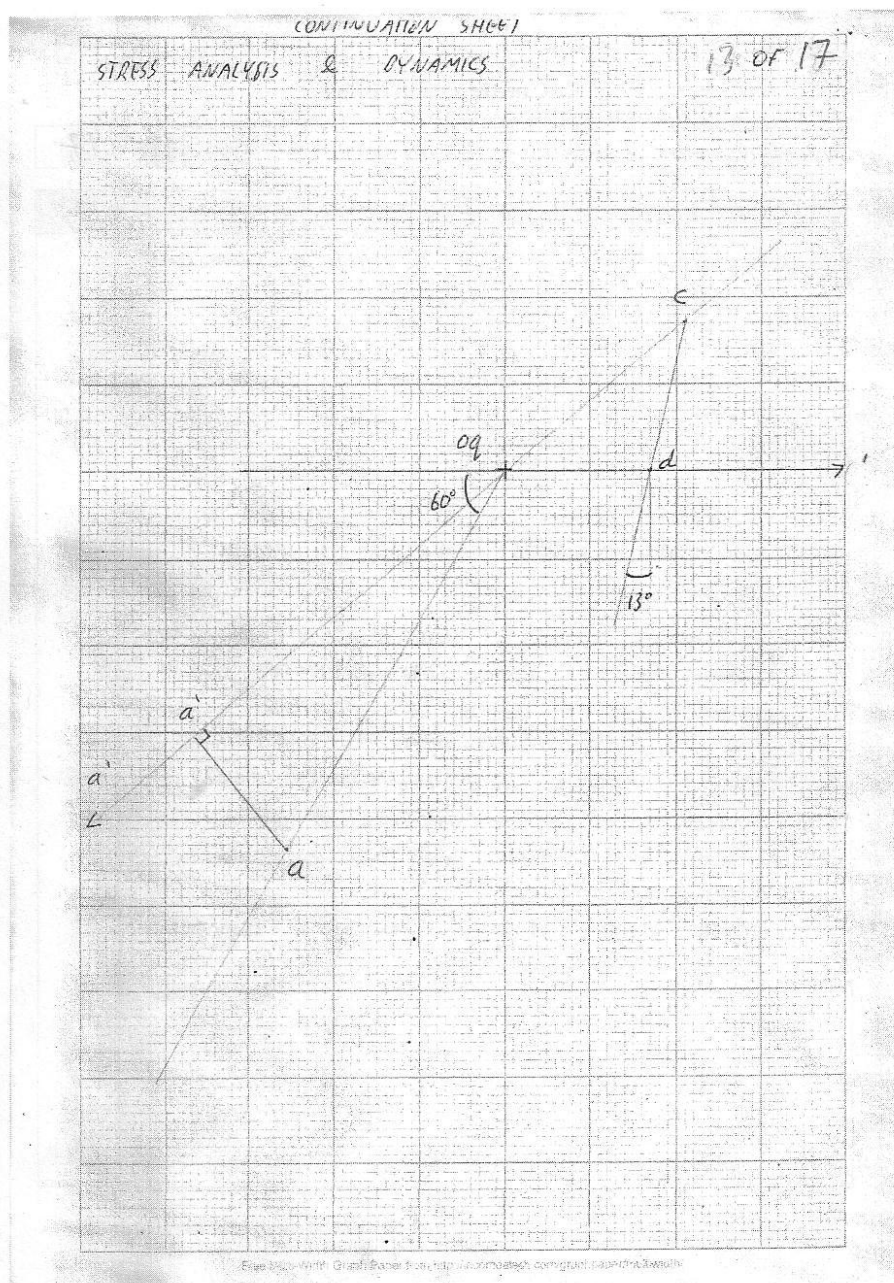
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Question No	Question	Marks
	$I_{\text{drum}} = m k^2$ $= 205 \times 0.4^2$ $= \underline{\underline{32.8 \text{ kgm}^2}}$	(2)
	$\therefore T = I \alpha$ $= 32.8 \times 2.67$ $= \underline{\underline{87.6 \text{ Nm}}}$	(1)
	$\therefore \text{TOTAL TORQUE} = 2080.9 + 87.6$ $= \underline{\underline{2168.5 \text{ Nm}}}$	
(iii)	ASSUMPTIONS: - THE MOTOR DRIVING THE DRUM CHANGES SPEED INSTANTLY - THERE IS ZERO FRICTION BETWEEN THE ROPE & THE DRUM - THERE IS ZERO AIR RESISTANCE ETC	(3)

Continuation Sheet

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Question No	Question	Marks
(4) (1)	O & Q DO NOT MOVE RELATIVE TO EACH OTHER VELOCITY OF A RELATIVE TO O $V_a = \omega \cdot OA$ $= 120 \times \frac{2\pi}{60} \times 0.2$ $= \underline{2.515 \text{ m/s}}$ VELOCITY OF A, V_a, IS $\perp$ TO OA i.e. 60° TO THE HORIZ, HENCE oa IS DRAWN $(2.515 \times 4 = 10.06 \text{ cm})$ RELATIVE TO Q, D HAS TO SLIDE HORIZONTALLY, HENCE $q\vec{d}$ IS DRAWN WITH INDETERMINATE LENGTH VELOCITY OF A', $V_{a'}$, IS $\perp$ TO QA' i.e. 40° TO THE HORIZ MEASURED FROM THE DIAGRAM. HENCE $q\vec{a'}$ IS DRAWN WITH INDETERMINATE LENGTH VELOCITY OF A IS PARALLEL TO QA i.e. $\perp$ TO $q\vec{a}$, HENCE $a'\vec{a}$ CAN BE DRAWN, FIXING a' NOW $\frac{V_{a'q}}{V_{qc}} = \frac{A'Q}{QC}$ FROM DIAGRAM $V_{aq} = 2.35 \text{ m/s}$ $A'Q = 264 \text{ mm}$ $QC = 150 \text{ mm}$ $\frac{2.35}{V_{qc}} = \frac{264}{150}$ $V_{qc} = 1.34 \text{ ms}^{-1} \quad (= 5.36 \text{ cm/s})$ HENCE $q\vec{a'}$ IS EXTENDED TO $q\vec{c}$	(1) (1) (1) (1) (1) (1) (1)

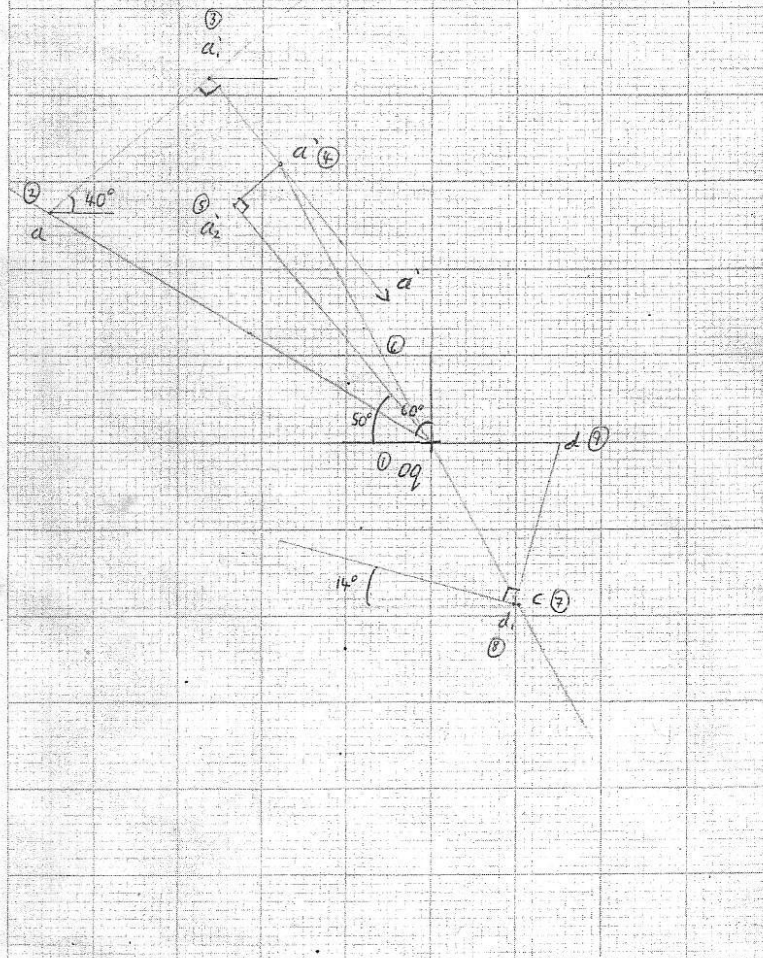


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Question No	Question	Marks
	• VELOCITY OF D RELATIVE TO C, V_{dc}, IS 13° FROM VERTICAL (MEASURED FROM DIAGRAM) HENCE $\vec{CD}$ CAN BE DRAWN, FIXING α. BY MEASUREMENT $V_{dc} = 0.8 \text{ ms}^{-1}$	① ①
⑥	① THE CENTRIPETAL ACCELERATION OF A RELATIVE TO O IS GIVEN BY: $a = \omega^2 OA$ $= \left(\frac{120 \times 2\pi}{60} \right)^2 \times 0.2$ $= 31.6 \text{ ms}^{-2}$ ② THIS ACTS TOWARDS O, HENCE $\vec{OA}$ CAN BE DRAWN. SINCE THE CRAWK IS CONSTANT VELOCITY THERE IS NO TANGENTIAL COMPONENT ($\alpha = 0$) • THE ACCELERATION OF A' RELATIVE TO A HAS TWO COMPONENTS TANGENTIAL COMPONENT $= \alpha r + 2V_{a'o} \times \frac{V_{a'q}}{AQ}$ $= 0 + 2 \times V_{a'o} \times \frac{V_{a'q}}{AQ}$ FROM VELOCITY DIAGRAM $V_{a'o} = 0.8$ $V_{a'q} = 2.37$ $AQ = 0.26$ TANGENTIAL COMPONENT $= 2 \times 0.8 \times \frac{2.37}{0.26}$ $= 14.6 \text{ ms}^{-2}$	① ①

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Question No	Question	Marks
③	THIS IS $\perp$ TO AXIS QB, FROM THE DIAGRAM THIS IS 40° FROM THE HORIZ. HENCE $\vec{a}_a$ IS DRAWN	①
④	THE RADIAL ACCELERATION OF A RELATIVE TO A IS PARALLEL TO QB ($\perp$ TO TANGENTIAL A COMPONENT) BUT OF UNKNOWN LENGTH. THEREFORE $\vec{a}_a$ IS DRAWN OF INDETERMINATE LENGTH	①
	THE ACCELERATION OF A' RELATIVE TO Q ALSO HAS TWO COMPONENTS:	
⑤	THE CENTRIFUGAL COMPONENT = $\frac{v_{a'q}^2}{A'Q}$ $= \frac{2.37^2}{0.26}$ FROM VELOCITY DIAGRAM $= 21.6 \text{ ms}^{-2}$	①
	THIS THIS IS DIRECTLY TOWARDS Q ALONG THE AXIS A'Q FROM THE DIAGRAM, THIS IS 50° TO THE HORIZONTAL. $\therefore$ VECTOR $\vec{q}a_2$ CAN BE DRAWN.	①
	THE TANGENTIAL COMPONENT IS $\perp$ TO THE CENTRIFUGAL COMPONENT, AND PASSES THROUGH a_2 WHERE THIS CROSSES $\vec{a}_a$ FINDS POINT a'	①
⑥	$\vec{a'o}$ CAN THEN BE DRAWN	①

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Question No	Question	Marks
	$\vec{a}'_q$ IS EXTENDED TO FIND C, SUCH THAT $\frac{\vec{a}'_q}{q_c} = \frac{A'Q}{QC}$ FROM ACCELERATION DIAGRAM $\frac{22.2}{q_c} = \frac{0.26}{0.15}$ $\therefore q_c = \underline{12.8 \text{ ms}^{-2}}$ ② THE CENTRIFUGAL ACCELERATION OF D RELATIVE TO C IS GIVEN BY $\frac{v_{dc}^2}{BC} = \frac{0.9^2}{0.5}$ ← FROM A VELOCITY DIAGRAM $\therefore = \underline{1.62 \text{ ms}^{-2}}$ ③ THIS IS DIRECTED TOWARDS C i.e. 14° FROM THE HORIZONTAL $\therefore \vec{CD}$ IS DRAWN ④ THE TANGENTIAL ACCELERATION OF D RELATIVE TO C IS $\perp$ TO CD, AND IS REPRESENTED BY $\vec{a}_{cd}$ D HAS TO MOVE & HORIZONTALLY RELATIVE TO Q, THUS, FIXING d. THE ACCELERATION OF D RELATIVE TO O IS THEN MEASURED OFF OF THE DIAGRAM $a_d = o_d = \underline{10 \text{ ms}^{-2}}$	① ① ① ①