

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
MAY 2020

CYFNOD ARHOLIADAU
MAI 2020

MODULE CODE:
M6X00420: Advanced Stress & FEA

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

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University approved calculators are permitted.

Assume 'g' = 9.81 m/s².

Question 1

Your choice of elements will greatly influence your results in any analysis that you carry out in FEA software, as such;

- a) Discuss how you would choose the element type and what physical property information would be needed for each type.

[6 marks]

Element type will probably be dictated by the geometry. Elements can be either 1D, 2D, 3D (or 0D). You would generally use the simplest element type that is appropriate in order to reduce the run-time. Structures that may be considered and the required physical property information is as follows;

- 1D structure: Such as a beam/rod will probably use 1D elements where section information such as second moment of area, polar second moment of area and orientation are needed in the physical property.
- 2D structure: Such as a piece of sheet metal will probably use 2D elements where the section information needed is the thickness of the 2D structure. You would generally use this when the smallest dimension is $\leq 1/10$ of the next smallest dimension.
- 3D structure: Such as a casting will probably use 3D elements where the physical property information is dictated by the element geometry.

- b) Discuss 4 ways that you could use to help you decide what element size you should use.

[4 marks]

1. Use experience based on modelling this kind of structure/loading previously.
2. Carry out a subjective check. Assess the model using an arbitrary element size (such as the suggested default) and any areas where there is a large jump in the output being measured (for example stress) from one element to the next suggests that refinement is needed in that location. Refine the mesh and then reassess.
3. Carry out a quantitative check. Check the 'error norm' that puts a number on the check carried out in 3.
4. Plot the output being measured against the element size in that location and when this is converging on a solution this suggests that the element size is acceptable.

Refining the mesh unnecessarily has a computational penalty.

- c) Discuss how you would choose element order.

[4 marks]

Generally the higher the order of an element, the more accurate the results are since you are calculating stress etc in more locations. By default, software such as Solidworks uses second order tets and calls first order tets 'draft' quality elements. Choosing a higher order element has a computational cost in the same way that choosing a small mesh size increases the run-time. The exception to the above is when tools such as topology optimisation are used where the results are based on the element size rather than the position of the nodes. As such, for a certain number of nodes it is better to have smaller first order elements rather than larger, higher order elements.

- d) Discuss what 'h-adaptive' and 'p-adaptive' meshes are.

[4 marks]

H-adaptive meshes are automated meshes that are generated to reduce the error norm to a specified level by carrying out an analysis, measuring this, refining the mesh in areas of high error norm and then measuring this again. This iterative approach to refining the mesh stops when the error norm converges within an acceptable tolerance. This obviously has a computational cost.

P-adaptive meshes are similar to h-adaptive meshes but rather than refining the mesh, the order of the elements used are increased to reduce the error norm.

Q2(a)(i)

$$\text{Shear Modulus, } G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\gamma}$$

$$\text{but, } \tau = \frac{\text{Shear load}}{\text{C.S.A.}} = \frac{Q}{A}$$

$$\therefore G = \frac{Q}{A \cdot \gamma}$$

$$\text{So, shear strain } \gamma = \frac{Q}{A \cdot G} \quad (2 \text{ marks})$$

(a)(ii) Knowing, strain energy = work done, and the area under load-strain graph, so in a small element;

$$U = \frac{1}{2} \gamma \cdot Q = \frac{1}{2} \left(\frac{Q}{A \cdot G} \right) \cdot Q$$

$$\therefore U = \frac{1}{2} \cdot \frac{Q^2}{A \cdot G}$$

$$\text{So, total strain energy, } U = \int_0^L \frac{Q^2}{2AG} \, ds$$

$$\therefore U = \frac{Q^2 L}{2 \cdot A \cdot G}$$

(4 marks)

(b)(i) ~~$U = \frac{P^2 L}{2AG}$~~ where $Q = P$ & $A = \text{C.S.A.}$

$$\text{Using for tensile loading, } U = \frac{P^2 L}{2AE}, \text{ where } P = \sigma \cdot A$$

$$\therefore P = 480 \times 10^6 \cdot ((0.03 \times 0.02) - (0.025 \times 0.015))$$
$$= 108,000 \text{ N}$$

$$\therefore U = \frac{(108,000)^2 \times 2.4}{2(2.25 \times 10^{-4})(190 \times 10^9)} = 327.41 \text{ J} \quad (3 \text{ marks})$$

Q2(b)(ii)

Assume all Potential Energy is converted into strain energy when mass is dropped.

$$\text{So, } mgh = \frac{P^2 L}{2AE} \left(\times \frac{A}{A}, \text{ where } AL = V \right)$$

$$\text{then, } mgh = \frac{\sigma^2 V}{2E} \quad (mg = W)$$

$$\therefore h = \frac{\sigma^2 V}{2EW} \quad (3 \text{ marks}).$$

$$(b)(iii) \text{ Now, } \sigma = \sqrt{\frac{2EWh}{V}} = \sqrt{\frac{2 \times 190 \times 10^9 \times 20 \times 9.81 \times 0.45}{((0.03 \times 0.02) - (0.025 \times 0.015)) \times 24}}$$

$$\therefore \sigma = \sqrt{\frac{2 \times 190 \times 10^9 \times 20 \times 9.81 \times 0.45}{(2.25 \times 10^{-4}) \times 2.4}}$$

$$\sigma = 249,258,901.5 \text{ N/m}^2 \quad (249.26 \text{ MN/m}^2)$$

$$(b)(iv) \text{ Knowing, } A = 2.25 \times 10^{-4} \text{ m}^2$$

$$\text{where, for hollow pipe, } A = \frac{\pi}{4} (D^2 - d^2)$$

$$\therefore d = \sqrt{D^2 - \frac{4A}{\pi}} \quad (D = 0.02 \text{ m})$$

$$d = 0.01066 \text{ m}$$

$$\text{Now, wall thickness, } t = \frac{0.02 - 0.01066}{2}$$

$$\therefore t = 0.00467 \text{ m} \quad (4.67 \text{ mm})$$

(5 marks).

Q3(a)

Principal stresses are; $\sigma_1 = \tau_{xy}$
 $\sigma_2 = 0$

$$\sigma_3 = -\tau_{xy}$$

From simple theory of elastic torsion;

$$\frac{T}{J} = \frac{\tau}{R} \quad \therefore \tau = \frac{T \cdot R}{J}$$

where, polar moment of inertia, $J = \frac{\pi}{32} \cdot (D^4 - d^4)$

$$\therefore J = \frac{\pi}{32} (0.075^4 - 0.065^4) = 1.354 \times 10^{-6} \text{ m}^4$$

$$\text{So, } \tau = \frac{T(0.0375)}{1.354 \times 10^{-6}} = 27699.2 \cdot T \text{ N/m}^2$$

Applying maximum shear stress theory; (TRESCA)

where, factor of safety, $n = 6$

elastic limit, $\sigma_y = 325 \text{ MN/m}^2$

$$\text{Now, } \sigma_1 - \sigma_3 = \frac{\sigma_y}{n}$$

$$\therefore \tau_{xy} - (-\tau_{xy}) = \frac{325 \times 10^6}{6}$$

$$\text{So, } \tau_{xy} = \frac{325 \times 10^6}{2 \times 6} = 27083333.3 \text{ N/m}^2$$

Knowing $\tau = 27699.2 \cdot T$

$$\therefore T = \frac{27083333.3}{27699.2}$$

$$\underline{T = 977.8 \text{ Nm (TRESCA) (8 marks)}}$$

Q3(b) For Von-Mises failure criterion;

$$\frac{2\sigma_y^2}{n^2} = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$5.8681 \times 10^{15} = [(\tau_{xy} - 0)^2 + (0 - (-\tau_{xy}))^2 + (-\tau_{xy} - \tau_{xy})^2]$$

$$\text{So, } 5.8681 \times 10^{15} = 6 \tau_{xy}^2 = 6. (27699.2 T)^2$$

$$\therefore T = \sqrt{\frac{5.8681 \times 10^{15}}{6 \times 27699.2^2}}$$

$$T = 1129.03 \text{ Nm (4 marks)} \quad \text{(12 TOTAL marks)}$$

(c) From Von-Mises failure criterion;

$$\frac{2\sigma_y^2}{n^2} = 6 \tau_{xy}^2 \quad \text{where, } \tau = \frac{T.R}{J}$$

$$\text{So, } \tau = \frac{T.(0.025)}{\frac{\pi}{32} (0.05^4 - 0.044^4)} = 101781.65.T$$

$$\therefore \frac{2(880 \times 10^6)^2}{2.75^2} = 6.(101781.65.T)^2$$

$$2.048 \times 10^{17} = (6.2157025 \times 10^{10}).T^2$$

$$\therefore T = \sqrt{\frac{2.048 \times 10^{17}}{6.2157 \times 10^{10}}} = 1815.18 \text{ Nm (6 marks)}$$

(d) Percentage difference in transmitted torque;

$$\frac{(-1129.03 + 1815.18)}{\left(\frac{1129.03 + 1815.18}{2}\right)} \times 100 = 46.61\% \text{ diff. (2 marks)}$$

Q4(a).

$$I_{xx} = 2(I_{xx} \text{ Channels}) + 2(I_{xx} \text{ Plates})$$

$$= 2(26.2 \times 10^6) + 2(I_g + A \cdot h^2)$$

$$= (52.4 \times 10^6) + 2\left(\frac{300 \times 12^3}{12} + (300 \times 12 \times 106^2)\right)$$

$$\therefore I_{xx} = 133.3856 \times 10^6 \text{ mm}^4$$

$$\text{Now, } I_{yy} = 2(I_{yy} \text{ Channels}) + 2(I_{yy} \text{ Plates})$$

$$= 2(I_g + A h^2) + 2\left(\frac{b d^3}{12}\right)$$

$$= 2(1.57 \times 10^6 + (4400 \times 79^2)) + 2\left(\frac{12 \times 300^3}{12}\right)$$

$$\therefore I_{yy} = 112.0608 \times 10^6 \text{ mm}^4$$

Now, $I_{yy} < I_{xx}$ hence strut will buckle about the Y-axis. (7 marks).

(b) Radius of gyration about YY;

$$k = \sqrt{\frac{I_{yy}}{A}} \text{ where, } A \text{ is total cross-sectional area of the strut.}$$

$$\text{So, } A = 2(4400 + (300 \times 12)) = 16000 \text{ mm}^2 \quad (0.016 \text{ m}^2)$$

$$\therefore k = \sqrt{\frac{112.0608 \times 10^6}{16000}} = 83.69 \text{ mm} \quad (0.08369 \text{ m}) \quad (1 \text{ mark}).$$

(c) Slenderness ratio = $\frac{L}{k}$

$$\therefore \text{Slenderness ratio} = \frac{8}{0.08369} = 95.59 \quad (1 \text{ mark})$$

Q4(d). Euler buckling load for both pinned ends;

$$P_e = \frac{\pi^2 EI}{L^2} \quad (\text{Note: } 1 \text{ mm}^4 = 1 \times 10^{-12} \text{ m}^4).$$

$$\text{So, } P_e = \frac{\pi^2 \times 200 \times 10^9 \times (112.0608 \times 10^{-6})}{8^2}$$

$$\therefore P_e = 3456236.8 \text{ N} \quad (3.456 \text{ MN}) \quad (3 \text{ marks})$$

(e). Rankine-Gordon buckling load;

$$P_R = \frac{\sigma_y A}{1 + a \left(\frac{L}{k}\right)^2} \quad \text{where, } \sigma_y = 320 \times 10^6 \text{ N/m}^2$$

$$a = \frac{1}{7500}$$

$$= \frac{(320 \times 10^6)(0.016)}{1 + \frac{1}{7500} \cdot (95.59)^2}$$

$$\therefore P_R = 2308046.3 \text{ N} \quad (2.308 \text{ MN}) \quad (3 \text{ marks}).$$

(f). For new box section;

$$\text{For vertical steel plates, } I_{xx} = 2 \cdot \left(\frac{6 \times 224^3}{12} \right) = 11239424 \text{ mm}^4$$

$$\text{and, } I_{yy} = 2 \left(\frac{224 \times 6^3}{12} + (224 \times 6 \times 153^2) \right) = 62931456 \text{ mm}^4$$

$$\text{So, } I_{xx}' = 133.3856 \times 10^6 + 11239424 = 144.625024 \times 10^6 \text{ mm}^4$$

$$\text{and, } I_{yy}' = 112.0608 \times 10^6 + 62931456 = 174.992 \times 10^6 \text{ mm}^4$$

hence will tend to buckle about x-axis;

$$\text{Now, } P_e = \frac{\pi^2 \times 200 \times 10^9 \times (144.625024 \times 10^{-6})}{8^2}$$

$$\therefore P_e = 4460599.3 \text{ N} \quad (4.461 \text{ MN}) \quad (5 \text{ marks}).$$