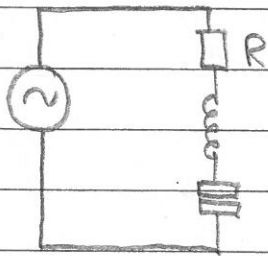


16th December 2019

Resonance in AC Circuits

We introduce the effect of frequency variation in an RLC circuit. Let's consider the RLC circuit below and relevant phasor diagram.

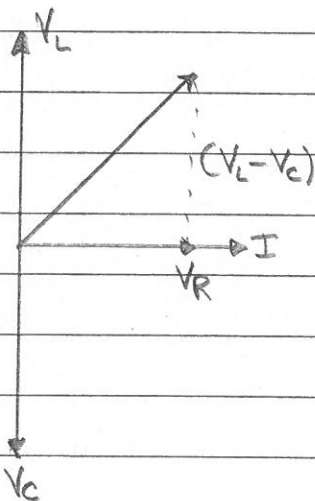
We assume that the capacitive is less than the inductive reactance



The impedance (Z) of the circuit is given by

$$Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$



By convention we consider inductive reactance to be positive and capacitive reactance negative

As the frequency increases X_L increases in direct proportion, while X_C decreases inversely for resonance

$$V_L = V_C$$

$$IX_L = IX_C$$

$$X_L = X_C$$

therefore $2\pi fL = \frac{1}{2\pi fC}$ at resonance

$$f = \frac{1}{2\pi\sqrt{LC}}$$

This selection of frequency can also be used in radio and television receivers and is the process of tuning for example radio 4. The degree of predomination is given by the Q factor.

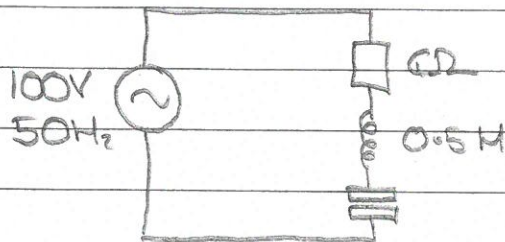
$$Q \text{ factor} = \frac{I^2 X_L}{I^2 R} \text{ or } \frac{I^2 X_C}{I^2 R}$$

$$Q \text{ factor} = \frac{\omega_{\text{resonance}} \times L}{R}$$

Worked example

A circuit having a resistance 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V 50Hz supply.

- (a) Calculate the capacitance for resonance
- (b) The voltages across the inductor & capacitor
- (c) The Q factor of the circuit



at resonance

$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{(2\pi f)^2 \times L}$$

$$C = \frac{1}{(2\pi \times 50)^2 \times 0.5} = 69.36802701$$

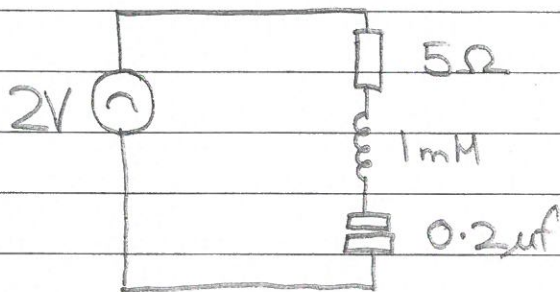
$$C = 20.3 \times 10^{-6} \text{ F}$$

$$C = 20.3 \mu\text{f}$$

at resonance $I = \frac{V}{R} = \frac{100}{4} = 25\text{A}$

Worked example

A coil of resistance 5Ω and inductance 1mH is connected in series to a $0.2\mu\text{F}$ capacitor. The circuit is connected to a 2V variable frequency supply. Calculate the frequency at which resonance occurs, the voltage across the coil and the capacitor at this frequency, and the Q factor of the circuit.



using

$$f_{\text{resonance}} = \frac{1}{2\pi\sqrt{LC}}$$

$$\begin{aligned} f_{\text{resonance}} &= \frac{1}{2\pi\sqrt{(1 \times 10^{-3}) \times 0.2 \times 10^{-6}}} \\ &= 11253.95 \text{ Hz} \\ &= 11.25 \text{ kHz} \end{aligned}$$

$$I = \frac{V}{R} = \frac{2}{5} = 0.4 \text{ A}$$

Summary

for a series RLC Circuit

$$X_L = X_C$$

$$\omega = 2\pi f$$

$$f_{\text{resonance}} = \frac{1}{2\pi \sqrt{LC}}$$

$$Q \text{ factor} = \frac{\omega_r L}{R}$$

Worked example

Two capacitors, each with a capacitance of $10\mu\text{f}$, are connected together in series. The capacitors are then connected across the terminals of a 110 Vrms 60 Hz supply. Calculate the current flowing in the circuit?

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\text{total}} = 5 \times 10^{-6} \text{ f}$$

using $X_c = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 60 \times 5 \times 10^{-6}}$$

$$X_c = 530.51 \Omega$$

$$X_c = \frac{V}{I}$$

$$I = \frac{V}{X_c}$$

$$= \frac{110}{530.51}$$

$$= 0.2 \text{ A}$$

$$= 0.2 \text{ A}$$

$$= 0.2 \text{ A}$$