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DEVELOPMENT OF A VEHICLE SETUP
CHARACTERISATION MODEL FOR THE
MCR SPORTS 2000 RACE CAR.

LEVEL SIX MAJOR PROJECT

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Abstract

Prior to this project there was no comprehensive way of recording setup changes throughout a race event. As a starting point, the author solved this issue and built upon it to create a transient quasi-semi state load transfer calculator, mathematically analysing loading and distributions throughout the vehicle to characterise handling. This model was developed in stages and validated against multiple scenarios to ensure accurate results. The model was completed successfully and produced results that reflected what was expected based on the author's research.

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This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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Nomenclature

Unless otherwise stated, the following notation corresponds to the given data throughout the project:

<u>Notation</u>	<u>Data</u>
δ	Vertical displacement
t	Track
K_w	Wheel rate
K_s	Spring rate
MR	Motion ratio
K_{ARB}	Anti-roll bar rate
$M\theta$	Anti-roll moment
$K\theta$	Roll stiffness
θ	Roll angle
M	Mass
M_s	Sprung mass
M_{us}	Unsprung mass
A_y	Lateral acceleration
A_x	Longitudinal acceleration
H	Centre of gravity (CoG) height above roll axis
h	Roll axis height at CoG
$H + h$	CoG height above road
WT_s	Elastic weight transfer
WT_{sg}	Sprung geometric weight transfer
WT_{usg}	Unsprung geometric weight transfer

Throughout this project, the phrases ‘load transfer’ and ‘weight transfer’ are used simultaneously. Both load and weight refer to the force of ($mass \cdot g$) acting on the contact patch, and how it changes as the vehicle moves; both are measured in Newtons and have exactly the same meaning.

1 Introduction

Vehicle setup is crucial in the world of racing; the right setup can give a huge advantage over competitors. Small adjustments to suspension components can fine-tune vehicle balance to exactly what the driver requires, giving them confidence to push the vehicle to its limit at all points on circuit (Milliken and Milliken, 1995). This project shall discuss the creation of a series of models that take basic load transfer equations and apply them in both a semi state and quasi-semi state fashion, to critically analyse vehicle balance and handling characteristics. These fundamental equations will be built upon by including transient data such as the effects of tyres and dampers within the vehicle for more valid, accurate results.

The University of Wales Trinity Saint David runs a Sports 2000 MCR race car consisting of a double wishbone, pushrod actuated suspension system with onboard Penske coil over dampers. Front and rear anti-roll bars (ARBs) can be adjusted onboard and the ride height, camber and toe is fully adjustable at each wheel. The chassis is a box tubular space frame construction with integral safety cage with strengthening side structures and a front-mounted aluminium crash box. The vehicle is equipped with a splitter and rear wing (Sports 2000, 2020). The models produced in this project focused mainly on this vehicle, however the author ensured every aspect was accessible by the user and could be easily changed so the model could be run for any vehicle, as long as certain data was available.

1.1 Project Aims and Relevancy

Currently, Team M-SportEng does not have a comprehensive setup sheet for the car. This would be a valuable tool for both setup prior to race weekends and for use at the track to efficiently log changes and how they affect vehicle balance. Therefore, the author created one. This was developed into a steady state load transfer calculator to analyse the balance and handling characteristics of the vehicle. This model gave an instantaneous overview of vehicle behaviour at any point around the circuit (Kadam and Chauhan, 2018). It also gave the author an opportunity to research the fundamental mathematics and principles behind developing a more comprehensive, transient load transfer model later in the project, which would analyse vehicle behaviour over a time period chosen by the user.

The Sports 2000 MCR has a large selection of setup options, therefore developing a tool that allows the team to confidently simulate and analyse setup changes at the track is advantageous as it could save large amounts of time and effort. This means the final outcome of this project has potential to lead to a more successful season for the team, as a greater understanding of the handling and balance characteristics would be achieved, encouraging a more competitive vehicle setup.

This project also formed a great way of consolidating a wide variety of skills learnt throughout the last three years of university, including practical elements such as data collection from the car, data analysis and using analytical software such as MoTeC, Microsoft Excel, MATLAB and Simulink.

2 Project Planning

2.1 Project Flowchart

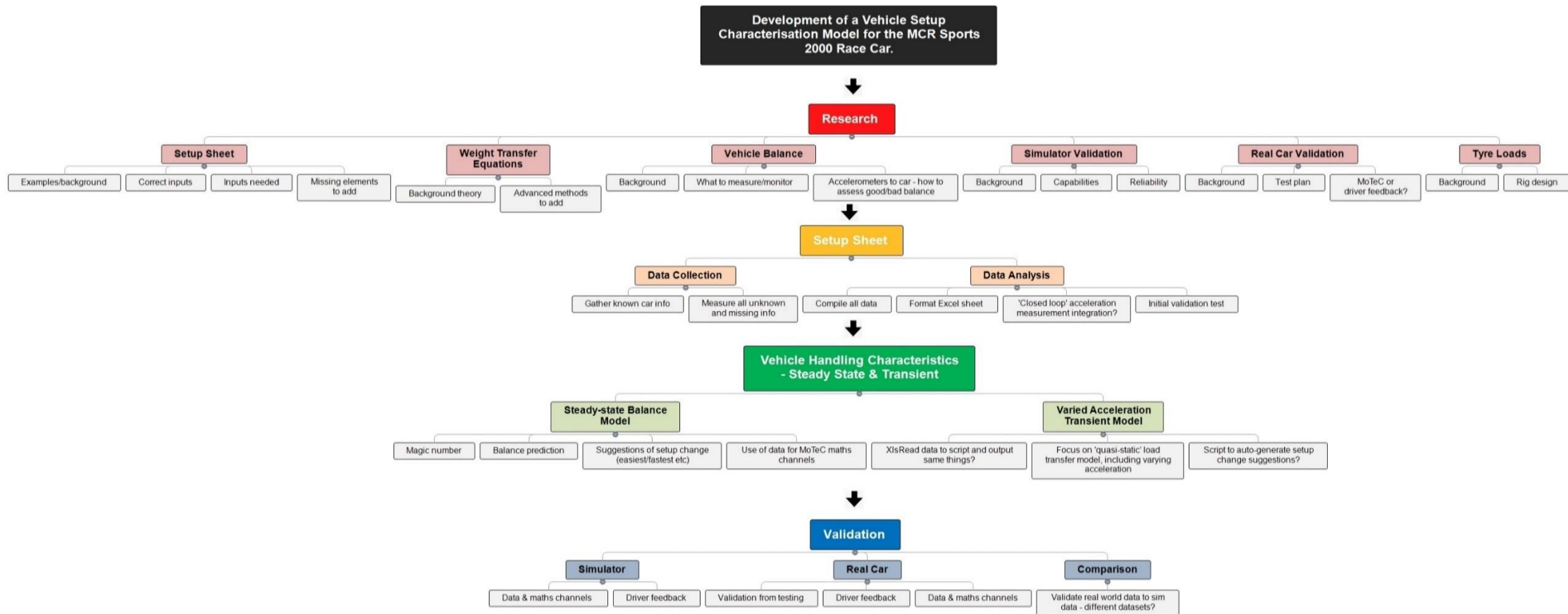


Figure 2.1.1 - Project Plan Flowchart

Figure 2.1.1 shows how the project was clearly and concisely broken down into base components, showing the order in which each task needed to be completed. Research showed that colour coded sections were a key organisation tool; this convention was applied to notes throughout the project. The author referred back to this figure often to check what to work on at the current time, as well as what to research for upcoming sections of the project (Orumwense, 2012).

2.2 Initial Project Gantt Chart

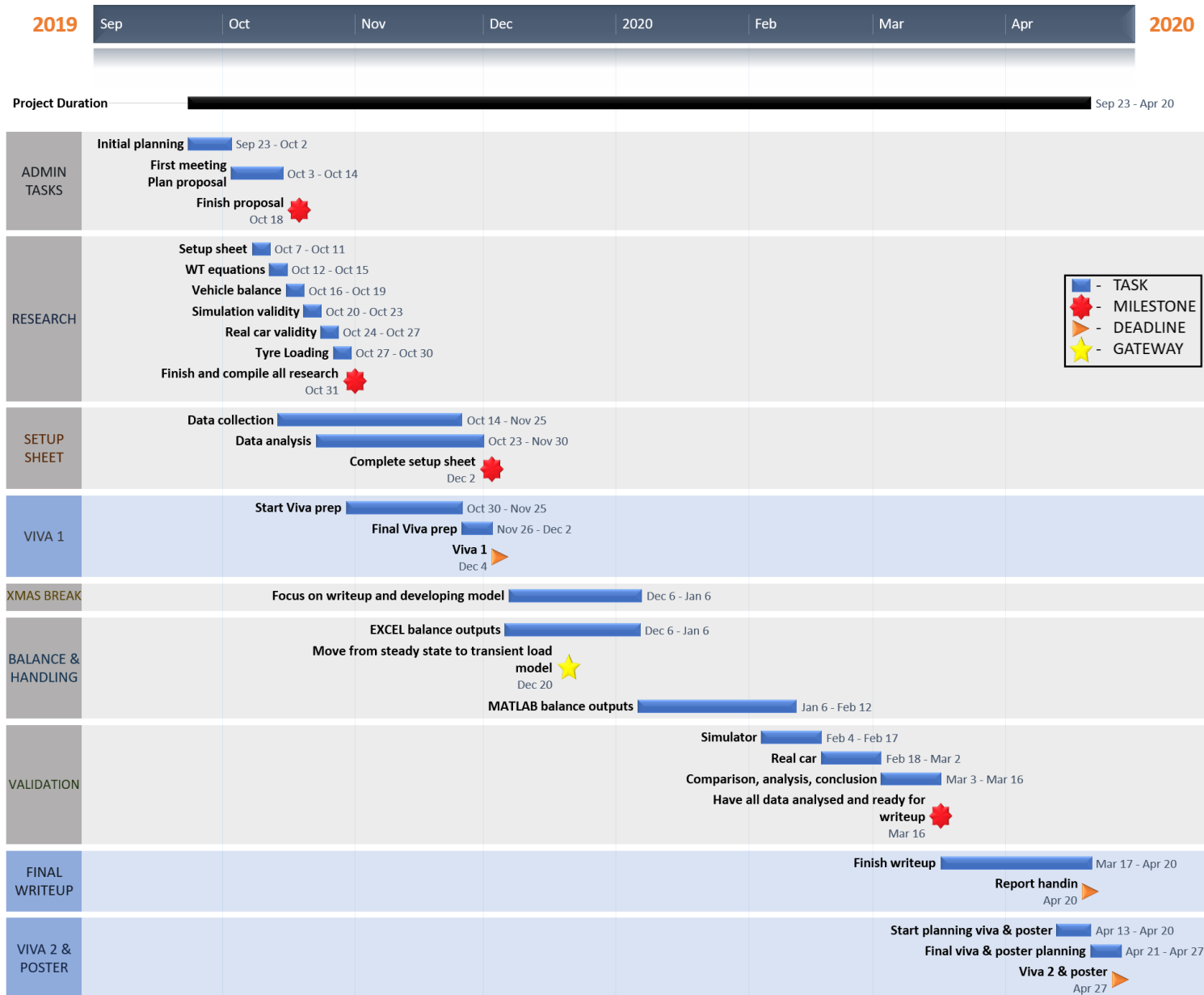


Figure 2.2.1 - Initial Project Time Plan

Figure 2.2.1 shows the initial timing plan. This was created at the very start of the project and formed the basis of time management.

The project was clearly split into sections to help understand the chronology of the tasks to complete. Each section had a milestone, and one project gateway was chosen to help keep track of overall progress and ensure the project ran to schedule (Lester, 1982).

2.3 Revised Project Gantt Chart

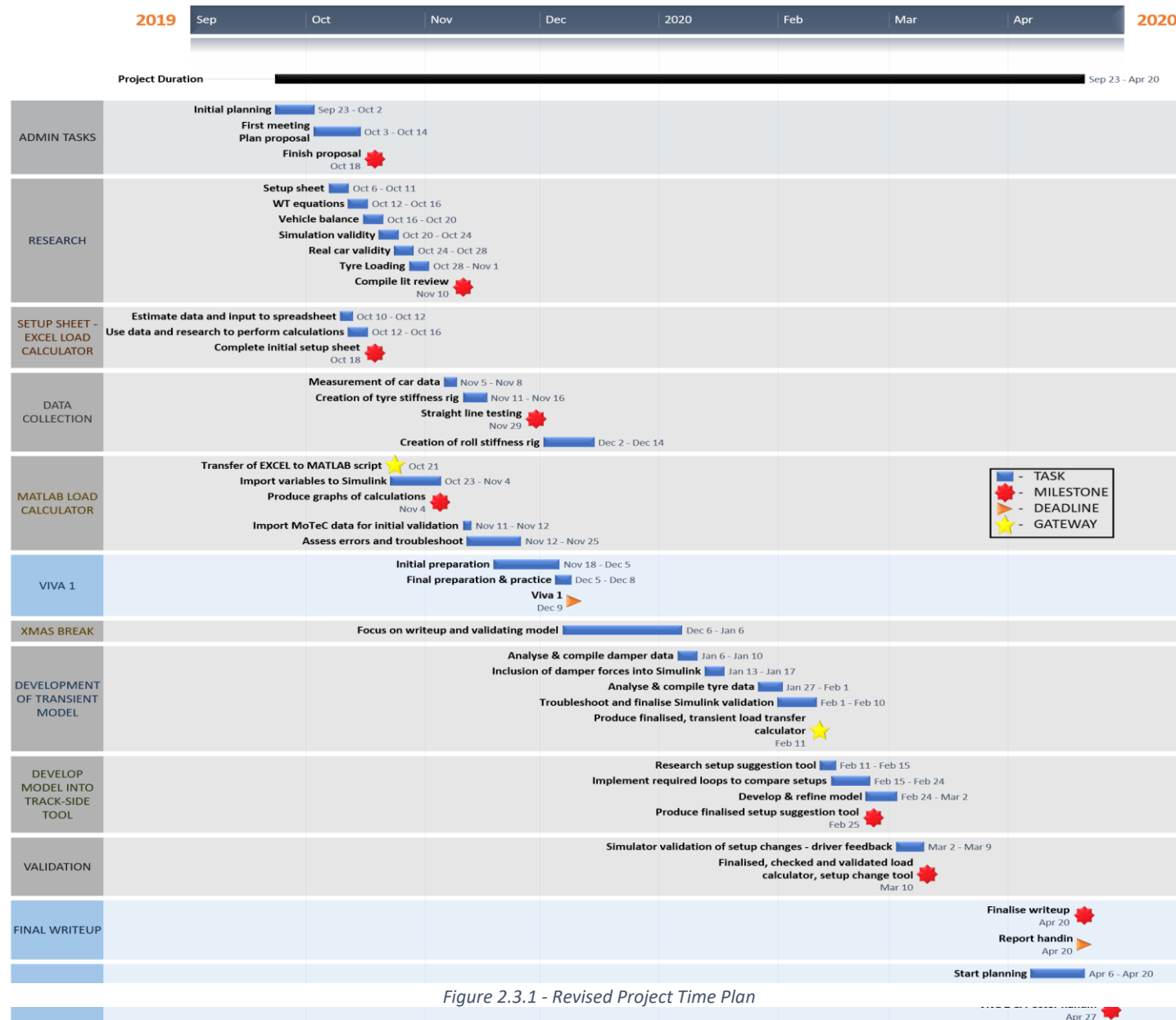


Figure 2.3.1 - Revised Project Time Plan

Figure 2.3.1 shows a revised timing plan. The author broke each task down into more manageable sections, causing more tasks to overlap. The author ensured simultaneously working on tasks was always manageable (Halkjelsvik and Jørgensen, 2018).

A further project gateway was added; both gateways were chosen as these tasks, once completed, provided a good basis for the author to confidently move on to the next phase of the project.

3 Initial Research

Before starting any analysis or modelling for the project, the author conducted thorough background research into various relevant areas and explored new ideas to include.

3.1 Setup Sheets

The author researched different examples of vehicle setup sheets to understand what is required, specifically what data is available and what needs to be measured or further researched; an example can be found in appendix one.

Table 3.1.1 - Setup sheet data (Jones, 2019)

Data available	Data required
• Toe • Camber • Corner masses • Total mass • F/R ride height • Gear ratios • Wing angle	• Wheelbase • F/R track • COG distance from front (weight dist) • COG height (H+h) • Roll axis height @ COG (h) • COG height above roll axis (H) • F/R roll centre height • F/R motion ratio • Sprung/unsprung mass • F/R Rolling radius • Wheel centre height • F/R spring rate • F/R roll bar rate • F/R Roll stiffness

It is clear that a good setup sheet has a simple, user-friendly layout that allows engineers to input data quickly and efficiently when trackside. It is convention to split the sheet into sections, commonly either by data category or location on vehicle, e.g. front left, front right etc.

Extra data such as tyre temperatures and pressures, and temperatures of suspension components could be useful but will not always be essential. Setup sheets often include a notes section in which these values could be recorded (Abe and Manning, 2009; Quick Car, 2014; Longacre Racing, 2016).

3.2 Vehicle Calculations

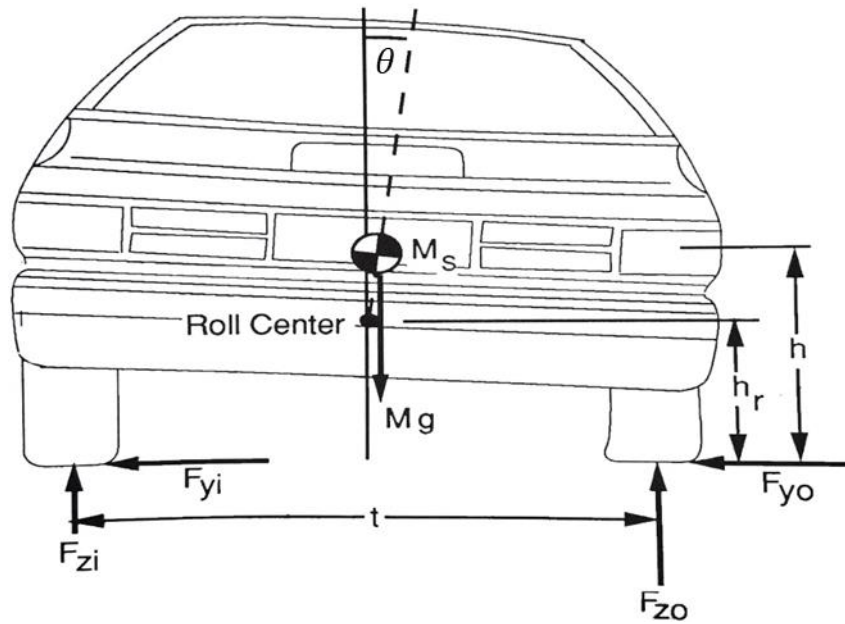


Figure 3.2.1 - Vehicle Experiencing Roll (Bastow et al., 2004)

3.2.1 Roll Angle

The vertical displacement of the body on the outside of the vehicle is noted as δ ; the angle of body rotation is displayed in figure 3.2.1 with respect to the vertical. If the vertical displacement and the track is known, the former can be used as the opposite side of a right angled triangle, and the latter as the adjacent, to calculate the roll angle (Gillespie, 1992; Parczewski, 2017).

$$\tan\theta = \frac{\delta}{\frac{1}{2}t}$$

Equation 3.2.1 - Vehicle Roll Angle

Because of the small angle approximation (Stroud and Booth, 2007), the tan function can be removed, thus:

$$\theta = \frac{\delta}{\frac{1}{2}t}$$

Equation 3.2.2 - Vehicle Roll Angle with Small Angle Approximation

3.2.2 Wheel Rate

The wheel rate characterises the spring rate at the wheel. It also includes the ARB rate (Bastow *et al.*, 2004).

$$K_w = K_s \cdot MR^2 + K_{ARB}$$

Equation 3.2.3 - Wheel Rate

3.2.3 Anti-Roll Moment

The anti-roll moment is the moment when the suspension setup (springs and wishbones) oppose the torque acting through the body roll (Staniforth, 1994).

$$M_\theta = \delta \cdot K_w \cdot t$$

Equation 3.2.4 - Anti-Roll Moment

Despite acting through half-track, in a state of equilibrium clockwise and anticlockwise moments will be equal, so the two instances of half-track can be combined as 't' (Performance Trends, 2007). From equation 3.2.2 it is known that $\delta = \frac{t\theta}{2}$,

$$\therefore M_\theta = \frac{t^2 \cdot \theta \cdot K_w}{2}$$

Equation 3.2.5 - Anti-Roll Moment with Roll Angle Equation

3.2.4 Roll Stiffness

Roll stiffness is the amount of roll moment per degree of roll at front and rear, commonly quantified as the amount of torque required to roll the chassis through 1 radian (Gillespie, 1992).

$$K_\theta = \frac{M_\theta}{\theta} \quad \therefore K_\theta = \frac{t^2 \cdot \theta \cdot K_w}{2\theta} \quad \therefore K_\theta = \frac{t^2 \cdot K_w}{2}$$

Equation 3.2.6 - Roll Stiffness

3.2.5 Overturning Moment

As a vehicle moves through a corner, the sprung mass is rotated about the roll centre as a result of lateral acceleration. This creates another moment, which is resisted by the anti-roll moment. For steady state conditions, these must be equal (Milliken and Milliken, 1995; Bastow *et al.*, 2004).

$$M_{ot} = M_s \cdot A_y \cdot H$$

Equation 3.2.7 - Overturning Moment

$$\therefore \theta = \frac{\text{Overturning Moment}}{\text{Roll Stiffness}}$$

Equation 3.2.8 - Roll Angle in Terms of Overturning Moment

3.3 Weight Transfer Calculations

To understand the basic principles of weight transfer, it is best to first look at a vehicle in steady state:

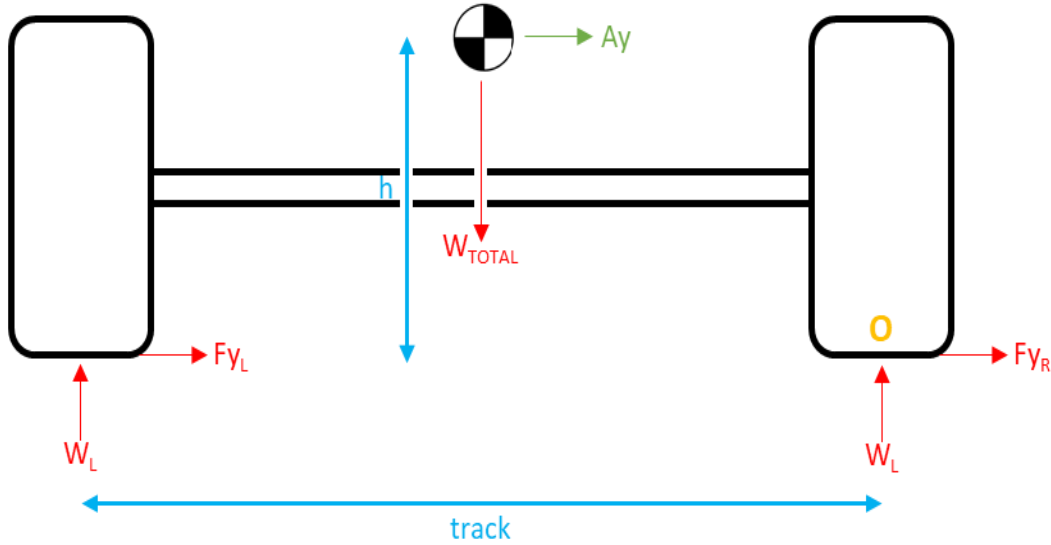


Figure 3.3.1 - Vehicle Experiencing Weight Transfer

To derive the weight transfer equation, take moments about point O in both directions and equate them, as a vehicle in steady state motion will follow the principles of moments (Stroud and Booth, 2007). Components of the following equations correspond to figure 3.3.1.

$$W \cdot Ay \cdot h - WL \cdot t + W \cdot \frac{t}{2} = 0$$

$$\therefore WL \cdot t = W \cdot \frac{t}{2} + W \cdot Ay \cdot h$$

$$\therefore WL = \frac{W}{2} + \frac{W \cdot Ay \cdot h}{t}$$

Equation 3.3.1 - Additional load on Left Tyre

Assuming symmetric L/R weight distribution, with left as outside:

$$WL = \frac{W}{2} + \Delta W$$

$$\therefore \frac{W}{2} + \Delta W = \frac{W}{2} + \frac{W \cdot Ay \cdot h}{t}$$

Equation 3.3.2 - Rearranging Left Tyre Load Equation

The half W's cancel, $\therefore \Delta W = \frac{W \cdot Ay \cdot h}{t}$ (Bastow *et al.*, 2004; Racing Car Dynamics, 2015).

Equation 3.3.3 - Weight Transfer Equation

This accounts for total lateral weight transfer across the vehicle. Unsprung weight transfer is difficult to change as it involves changing either the unsprung mass or centre of gravity height. Because of this, vehicle setup focuses predominantly on changing sprung weight transfer, as roll stiffness distribution can influence this (Racing Car Dynamics, 2015):

$$\Delta W = \frac{Ay \cdot h}{t} (Ms + Mus)$$

$$\therefore \Delta W_s = \frac{Ms \cdot Ay \cdot h}{t} \quad \text{and} \quad \Delta W_{us} = \frac{Mus \cdot Ay \cdot h}{t}$$

Equation 3.3.3 - Sprung and Unsprung Weight Transfer

Following on from this, sprung and unsprung weight transfers can be further broken down:

3.3.1 Elastic Weight Transfer

From Hooke's Law, $F = -k \cdot X$, the force of the elastic weight transfer acting on the sprung mass can be calculated:

$$WT_s = \delta \cdot K_W \therefore WT_s = \frac{K_W \cdot t \cdot \theta}{2}$$

Equation 3.3.4 - Elastic Weight Transfer

3.3.2 Geometric Weight Transfer - Sprung Mass

The geometric weight transfer is that of the mass acting through the roll or wheel centre. Torque through these positions has a moment arm from the centre height, down to the contact patch. This is divided by track as the resistive moment acts upon half-track, again on both sides so combines as t. This is true for both sprung and unsprung mass (Jazar, 2008):

$$WT_{SG} = \frac{M_s \cdot Ay \cdot \text{Roll Centre Height}}{t}$$

$$WT_{USG} = \frac{M_{us} \cdot Ay \cdot \text{Wheel Centre Height}}{t}$$

Equation 3.3.5 - Geometric Weight Transfer

3.3.3 Total Weight Transfer

Combining the above three equations gives the total weight transfer of the vehicle in the corner:

$$WT = WT_s + WT_{SG} + WT_{USG}$$

$$\therefore WT = \frac{K_W \cdot t \cdot \theta}{2} + \frac{M_s \cdot a_y \cdot h}{t} + \frac{M_{us} \cdot a_y \cdot h}{t}$$

Equation 3.3.6 - Total Weight Transfer

3.3.4 Longitudinal Weight Transfer

Longitudinal weight transfer can also be split into these categories, however lateral transfer has much more effect on vehicle handling and balance, for this reason the longitudinal component shall be included in this project as one overall equation, split into just sprung and unsprung components.

$$WT_{LONG} = \frac{M_s \cdot Ax \cdot CoG \text{ Height}}{t} + \frac{M_{US} \cdot Ax \cdot Wheel \text{ Centre Height}}{t}$$

Equation 3.3.7 - Longitudinal Weight Transfer

3.3.5 Weight Transfer Summary

These weight transfer equations can be used to analyse a vehicle's behaviour throughout a corner, to gain a better understanding of loadings on the springs, chassis and body. As the vehicle travels through the corner, the geometric weight transfer happens much sooner. Elastic transfer happens afterwards, as the chassis takes time to roll, loading the springs, dampers and anti-roll bars. This is also heavily influenced by the chassis' inertia forces, which again take time to overcome (Bastow *et al.*, 2004).

3.4 Weight Transfer Equations Limitations

A vehicle corners due to the tyre slip angle creating a side force; the tyre deforms as it moves around the corner, and the tyre's resistance to this deflection creates a lateral loading, which turns the vehicle. These equations do not include tyre deformation; thus the contact patch shape and size is inaccurately taken to remain constant throughout the corner. In reality, the deformation is constantly changing in a non-linear way (Johnson, 1996; Haney, 2003). Moreover, chassis stiffness, dynamic roll centre, changing centre of gravity position and effects of dampers are not included in these equations, reducing accuracy (Milliken and Milliken, 1995).

It is often believed that there are better ways to characterise vehicle balance and handling than with these equations, due to the aforementioned limitations. Studies such as “Mathematical Analysis of Steady State Lateral Load Transfer on a Vehicle” (Kadam and Chauhan, 2018) state that the non-transient nature of this mathematical analysis makes any results too erroneous to deem trustworthy, however, this is contested by the study “Lap Time Simulation: Comparison of Steady State, Quasi- Static and Transient Racing Car Cornering Strategies” (Siegler et al., 2000), which states that a quasi-semi state model that thoroughly analyses transient data can accurately predict vehicle balance. The author wishes to prove this within this project.

3.5 Load Transfer Calculators and Simulation

3.5.1 Steady State Load Transfer Calculator - EXCEL.

For the initial EXCEL Steady State model, the calculations above were implemented using inputted data to analyse a single point within a corner, or an entire corner considered to be perfectly steady state. This formed a good starting point and allowed the author to develop a good understanding of the mathematics involved.

3.5.2 Quasi-Static Transient Load Transfer Calculator - Simulink

To include transient data in the model, the author used a ‘quasi-static’ method. This takes the principle of the EXCEL sheet and applies it to numerous points along the curve (Siegler *et al.*, 2000). This is achieved by replacing the constant values of lateral and longitudinal acceleration and speed with dynamic inputs from MoTeC. These values are taken at different time intervals and strung together to create one varying input. The smaller the time intervals the more accurate the calculations will be. MoTeC can export data at a frequency of up to 1000Hz and this is sufficient (Jacobson *et al.*, 2015).

3.6 Vehicle Balance

Vehicle balance is generally described as the measurement of how much a vehicle understeers or oversteers through a corner; it can be heavily influenced by vehicle setup, speed, conditions and driving style. It is a good way to assess how changing certain elements of a vehicle's setup changes its handling characteristics. The load distribution of a vehicle plays a large role in dictating balance (Wickens, 2003; Riley, 2016).

Weight transfer is a good way of analysing and predicting a vehicle's handling characteristics (Gillespie, 1992; Zhai *et al.*, 2009). Predictions of vehicle handling can be made from the location of the vehicle's loading. As the vehicle moves around a corner, the loading will move to the outside tyre, changing the average lateral force across each axle. Consequently, the tyres have to achieve a greater slip angle to maintain the lateral force necessary to stay on course. If the slip angle is not reached, traction is lost. If this happens to the front tyres, the vehicle will understeer, if this is seen at the rear, the vehicle will oversteer (Gillespie, 1992).

Vehicle balance and load distribution, with regards to suspension setup, is most commonly measured by roll stiffness distribution and weight transfer distribution. A vehicle's roll stiffness distribution, or centre of roll stiffness, shows which end of the vehicle sees a stiffer suspension setup.

$$\text{Centre of Roll Stiffness} = \left(\frac{\text{Front Roll Stiffness}}{\text{Total Roll Stiffness}} \right) \cdot 100$$

Equation 3.6.1 - Centre of Roll Stiffness

Weight transfer distribution, or centre of weight transfer, shows which end of the vehicle is experiencing the highest proportion of overall weight transfer, and thus can predict if understeer or oversteer will be experienced (Milliken and Milliken, 1995).

$$\text{Centre of Weight Transfer} = \left(\frac{\text{Front Weight Transfer}}{\text{Total Weight Transfer}} \right) \cdot 100$$

Equation 3.6.2 - Centre of Weight Transfer

The centres of roll stiffness and weight transfer are closely linked, as the end of the vehicle with the highest suspension stiffness will experience the highest weight transfer, which will have the least grip. A very stiff front suspension setup will result in a centre of roll stiffness above 50%, towards the front of the vehicle. This will coincide with a centre of weight transfer above 50%, towards the rear of the vehicle. Most weight transfer will be seen at the front of the vehicle and thus it will experience understeer.

It is generally considered that the centre of roll stiffness should be roughly central to the vehicle, with a slight bias to the rear. This balances the amount of elastic weight transfer throughout the vehicle (Hathaway, 2009). The optimal front/rear distribution should be applied taking into consideration total vehicle roll stiffness. This affects vehicle balance as well as distribution. If the overall stiffness is too low, body roll will be more pronounced, encouraging oversteer. If overall stiffness is too high, the vehicle will tend to understeer (Ellis, 1994; Milliken and Milliken, 1995).

3.6.1 Characterising Vehicle Balance

Understeer and oversteer levels are the most common way of assessing a vehicle's handling balance. These can be measured simply from driver feedback, or from data from logging systems such as MoTeC. It is possible to calculate how much under or oversteer the vehicle was experiencing through a corner that way.

$$\text{Corner Radius [m]} = \frac{\text{Velocity}^2}{\text{Lateral Acceleration}}$$

$$\text{Theoretical Wheel Steer Angle [rads]} = \frac{\text{Wheelbase}}{\text{Corner Radius}}$$

$$\text{Steering 'X' Constant} = \frac{\text{Theoretical Steer \%}}{\text{Theoretical Steer Angle}} \quad (\text{At low speeds when car is within limit of grip})$$

$$\text{Theoretical Steer \%} = \text{Theoretical Angle} \cdot \text{Steering 'X' Constant}$$

$$\text{Understeer Magnitude} = \text{abs(Steering Wheel \%)} - \text{abs(Theoretical Steer \%)}$$

Equation 3.6.3 - Understeer Analysis Equations

3.7 Simulator Validation

The author assessed the model by comparing calculated data to data gained from the University's Base Performance simulator. This ensured the calculations are reliable before progressing to the next stage of the project.

3.7.1 Simulator Capabilities

The simulator offers the author a constant, reliable source of data acquisition for validation purposes. It has a high level of accuracy, with no fluctuation of results or random error, thus can be trusted and is readily available. Moreover, previous projects have been carried out specifically to match the software as accurately as possible to real-car data (Noble and Bridgeland, 2018). The simulator offers the opportunity to test the car with various different setups easily, as options can be changed from a few clicks on an on-screen menu (de Winter *et al.*, 2009).

The simulator not only removes the physical job of altering vehicle setup, but also means the author does not have to organise real-life testing as often (Tasora, 2008).

3.7.2 Simulator Reliability

The simulator is considered reliable as it runs professional software specially calibrated to ensure close correlation with real world data. However, the simulator showed flaws through coding and programming issues, for example, initial loading geometry and real weather conditions. Contact patch loadings are inaccurate and can be hard to validate/recreate in the real world. Moreover, fuel slosh and tyre deterioration are not accounted for (RFactor, 2005; Purdy, 2006).

3.8 Real Car Validation

The author took the opportunity to take the car to a track to carry out real-world testing, discussed later in the project. This proved a useful opportunity to gain consistent, specific data to put into the load transfer calculator. Moreover, the car was run without ARBs to eliminate ARB rate and see if data matches when ARBs are simulated as zero. This was also a good opportunity to gather data such as aerodynamic constants.

3.8.1 MoTeC or Driver Feedback

Driver feedback can be important to understand the balance of the car and to complement calculations. The model could be validated by gaining results for a particular setup and seeing if driver feedback agrees.

4 MCR Data Collection

Data collection was carried out with two other students who required the same data. It was ensured that all tests were repeated at least three times to minimise error.

4.1 Spring Rates

The author used the University's spring tester to measure spring rates. The MCR springs have a stated maximum force. The spring was compressed to half this value then the gauge was zeroed.

The spring was then decompressed one inch at a time by releasing hydraulic pressure. The force for each increment was recorded and graphed. The gradient of this graph gave the spring rate. This graphical method reduces the possibility of random error, thus increasing accuracy. The rates found were then converted from lbs/inch to N/m.

Table 4.1.1 - MCR Spring Rates

Front Spring Rate	39053.28 N/m
Rear Spring Rate	67598.96 N/m

4.2 Anti-Roll Bar (ARB) Rates

The ARB rate of the vehicle is difficult to measure due to how it is mounted through the drop links. The different materials combined mean that simply removing the ARB and applying a known force and measuring deflected distance would not give an accurate result. A torque needs to be applied over the whole car to investigate how the ARBs work against roll (Simmonds, 2013).

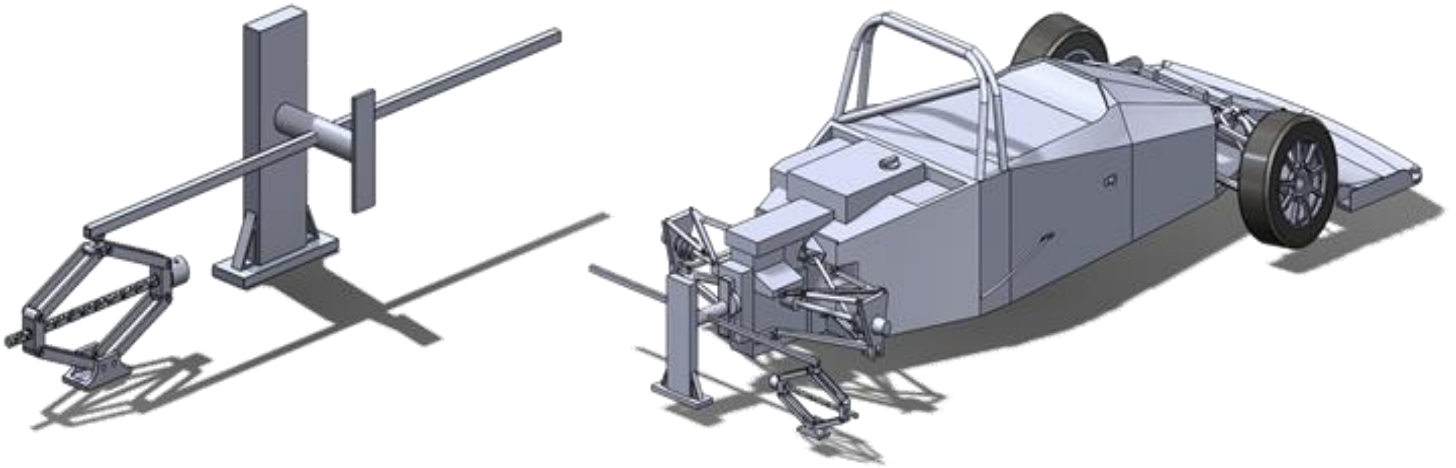


Figure 4.2.1 - Anti Roll Bar Testing Rig

The author developed a plan to measure the total roll stiffness, front and rear, of the vehicle. The rig shown in figure 4.2.1 allows the author to apply a torque to the rear of the car by applying a known force to the bar. This force, multiplied by the perpendicular distance from the pivot, gives a known torque value (Stroud and Booth, 2007). This will rotate the chassis through a known angle, measured by an angle gauge on the chassis. The torque needed to rotate the chassis through one radian can then be found, giving a front roll stiffness (Milliken and Milliken, 1995); this can be repeated for the rear.

The dampers on the end of the car being analysed are to be removed, and dial test indicators (DTIs) put on the wheels to measure tyre deflection, so this can be removed from the calculation. From this, knowing the spring rate and motion ratio, the anti-roll bar rates can be back-calculated using equation 3.2.3. This is the most accurate, time efficient way of achieving an ARB rate value. Unfortunately, due to restrictions placed on working on the car by the current M-SportEng team whilst preparing it for racing, the author could not progress past the development stage of this rig, therefore, data was not collected. For the model, the author used the range of ARB rates on the simulator.

4.3 Motion Ratio

To measure the motion ratio the author set the wheel assembly to its minimum position and increased the height 10mm at a time, recording damper travel. From these values, the total movement of both wheel and damper were recorded and motion ratio was found by $\frac{\text{damper travel}}{\text{wheel travel}}$. These values were plotted for front and rear, the gradient showing the motion ratio. The graphs showed almost perfect linearity within the data, showing that implementing a non-linear motion ratio change into the model would result in negligible difference (Delta Vee Motorsports, 2013).

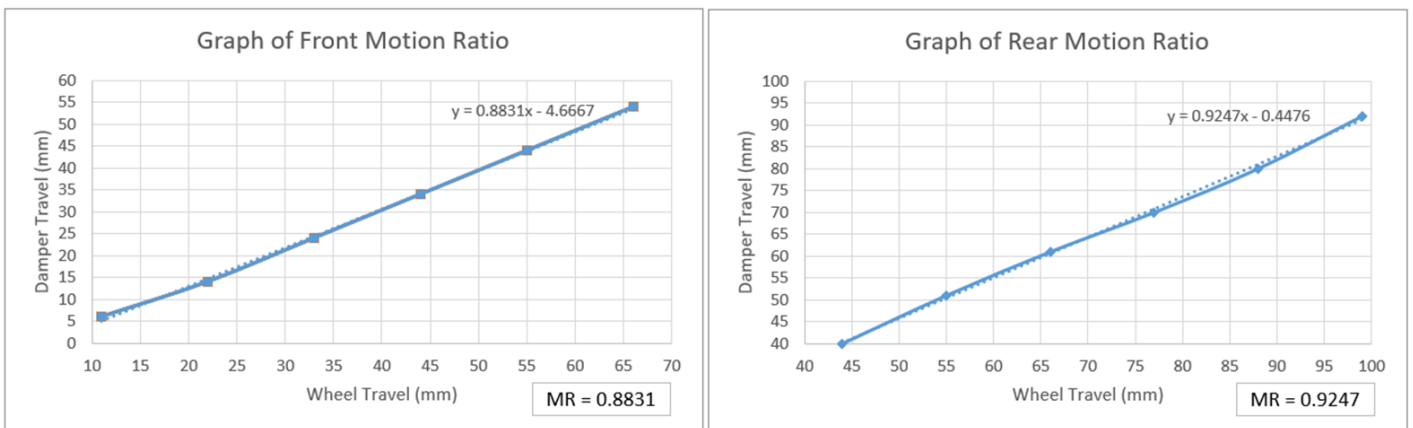


Figure 4.3.1 - Motion Ratio Test Data

4.4 Damping Rates

The damper dynamometer was used to measure vehicle damper rates. Initially, the author took measurements to find damper operating temperature. Heat sensitive stickers were applied to dampers front and rear which recorded their maximum temperature on track.

The author set the dynamometer software to cycle the dampers until they reached operating temperatures. From this, the software was set up to create a force velocity graph. The four main sections of the graph, fast and slow speeds for bump and rebound, were considered to be linear so two points were taken at either end of each section and plotted. The gradient of the line between the two can be found, which is the damping rate (Giaraffa, 2010).

Table 4.4.1 - Damper Test Results

Front at 14°C	Front at cold ($\approx 10^{\circ}\text{C}$)	Rear at 56°C	Rear at ambient ($\approx 14^{\circ}\text{C}$)
– Standard damping	– Standard damping	– Standard damping	– Standard damping
– Rebound 2 clicks in	– Rebound 2 clicks in	– Rebound 2 clicks in	– Rebound 2 clicks in
– Rebound 2 clicks out	– Rebound 2 clicks out	– Rebound 2 clicks out	– Rebound 2 clicks out
– Bump 2 clicks in	– Bump 2 clicks in	– Bump 2 clicks in	– Bump 2 clicks in
– Bump 2 clicks out	– Bump 2 clicks out	– Bump 2 clicks out	– Bump 2 clicks out

The author carried out tests on a limited number of damper settings as a fellow student was completing a more in-depth damper investigation on the MCR. These were compared to the author's measured values, with the maximum difference being just 3.8%. Therefore, using this external data was deemed trustworthy and validated (Farnhill, 2020).

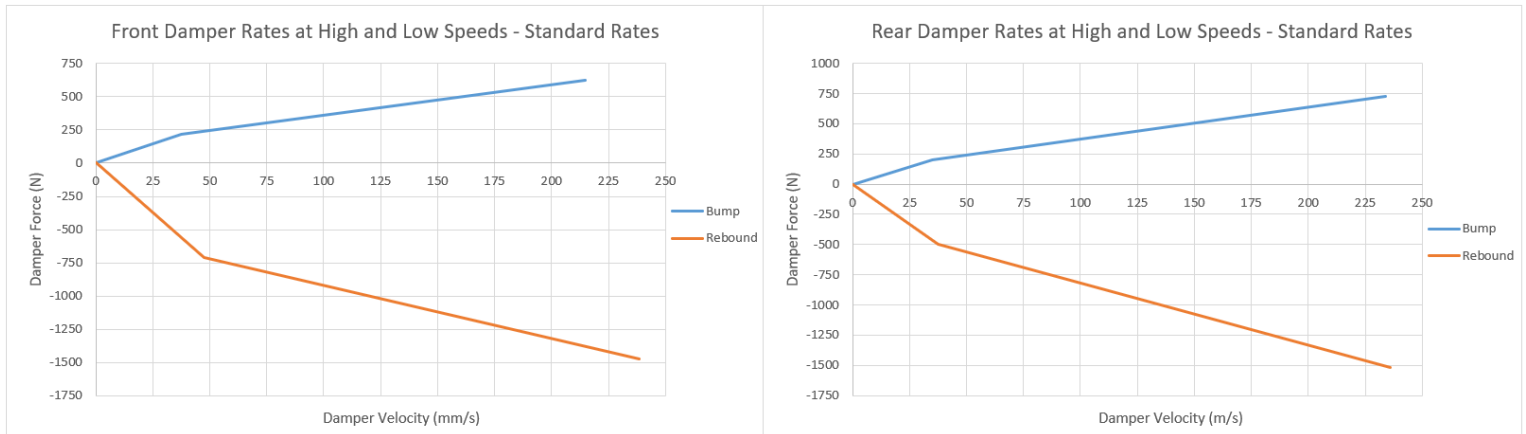


Figure 4.4.1 - Damper Rates Test Data

4.5 Measuring Masses

A thorough way of measuring vehicle mass is to remove and individually measure each component, taking 1/3 component mass values for semi-sprung components.

$$\therefore \text{Corner Mus} = \text{calliper} + \frac{1}{3} \text{push rod} + \frac{1}{3} \text{tie rod} + \text{stub axle} + \\ \text{upright} + \text{rim \& tire} + \frac{1}{3} \text{wishbones} + \frac{1}{3} \text{rocker}$$

Equation 4.5.1 - Unsprung Mass

The total dry mass of the vehicle is known from the corner scales, so this value can be subtracted from the total to find the sprung mass.

4.6 Suspension Geometry - WinGeo Analysis

The author used an existing WinGeo model of the white MCR, created by the 2018/2019 chassis group project team, to find the static and dynamic roll centre heights. The static value was implemented within the steady state models, the dynamic value was used in the transient model. The data and coordinates were confirmed by the author on the yellow car and any changes were made where necessary.

Both sides	Chassis coords	X fore-aft	Y width	Z vertical	Incline angle	Kingpin axis	5.382 degrees
Lower A-arm forward	A	-315.472	175.100	117.320	A to B= -0.525	Scrub radius	105.986
Lower ball joint	B	-11.766	682.230	122.740	C to B= -0.990	Caster	5.549 degrees
Lower A-arm rearward	C	40.418	176.150	113.950	A to C= 0.543	Caster trail	23.690
Upper A-arm forward	D	-84.722	283.750	330.000	D to E= -4.437	Upper A-arm	379.022
Upper ball joint	E	11.296	659.868	360.120	F to E= -4.510	Lower A-arm	506.300
Upper A-arm rearward	F	37.878	281.475	330.200	D to F= -0.093		
Wheelbase, track, tire diameter		2580.005	1599.557	482.600	Rollout1516.133	Track at ground	1599.557
Camber, Toesteer, Toe span		0.188	0.065	711.201	HubTrak 800.570	Track at hub	1601.141
Tire contact patch		0.000	799.779	0.000	B to E= 239.544		

Undo editing

Figure 4.6.1 - WinGeo Front Suspension Coordinates

Knowing that the maximum ride height change is $\pm 20\text{mm}$, the roll centre height was logged in WinGeo for this distance above and below the static ride height, for both front and rear. The effect on roll centre position from roll angle was not measured, as this is not used within the model.

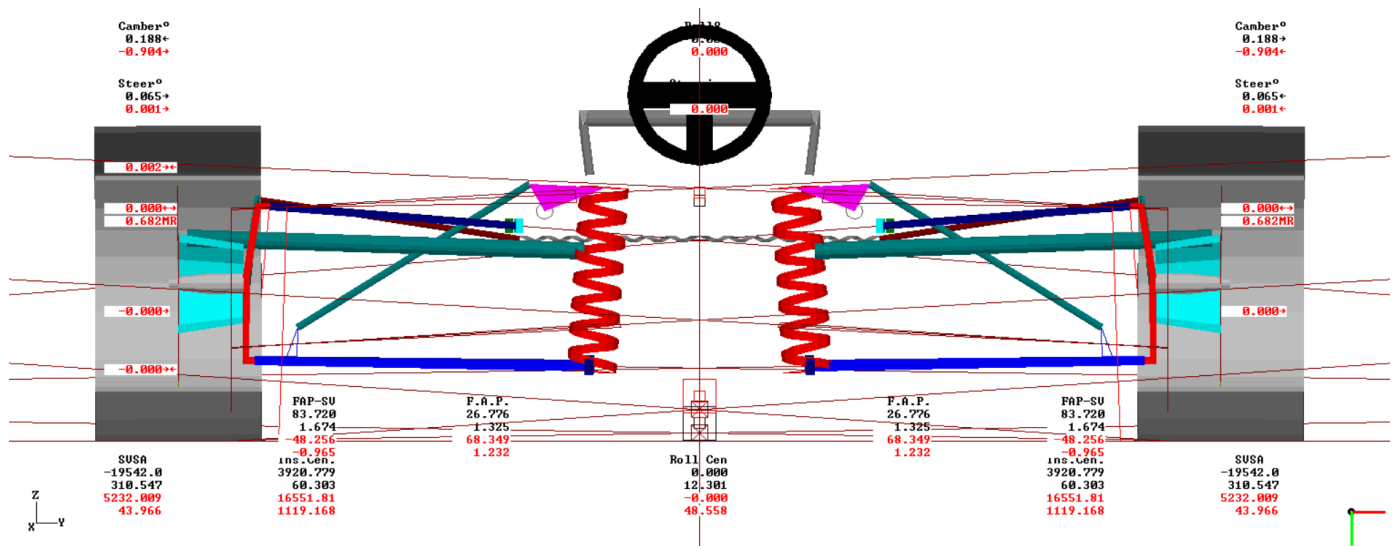


Figure 4.6.2 - WinGeo Rear Suspension Data

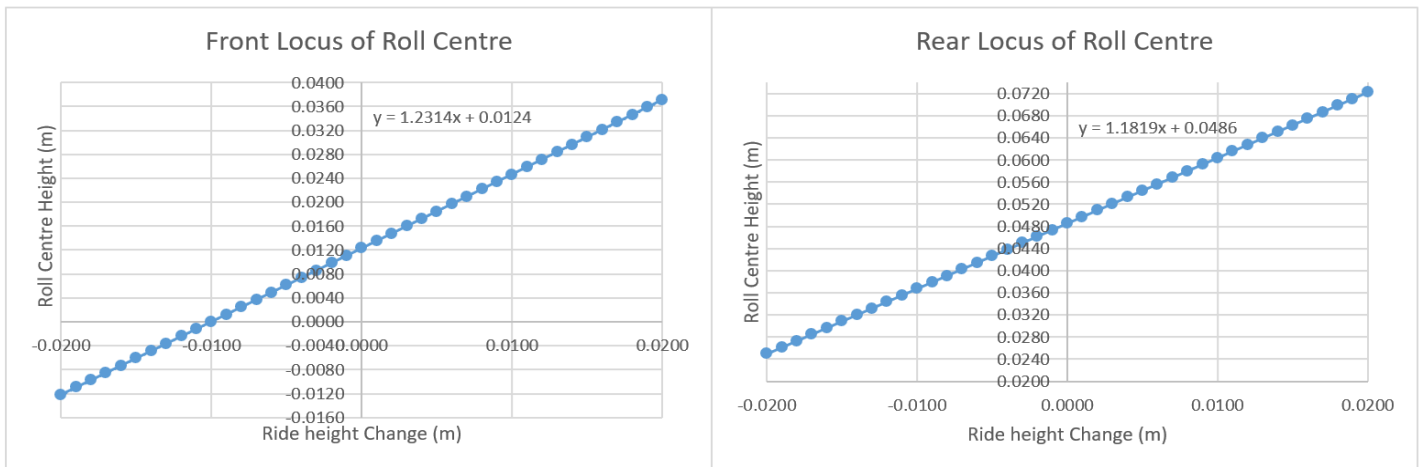


Figure 4.6.3 - Dynamic Ride Height Test Data

Figure 4.6.3 shows the results of increasing the ride height front and rear. It is clear that the correlation is perfectly linear.

4.7 Measurement of Vertical Tyre Stiffness

Elastic weight transfer makes up the highest proportion of total lateral weight transfer and includes wheel rate. This means wheel rate will have a substantial impact on vehicle balance. A key element of the wheel rate is the vertical tyre stiffness; therefore, the author made a rig to measure this for the MCR (Johnson, 1996; Haney, 2003). Any component stiffness is a spring rate, therefore, if a known force is applied and deflection is measured; the result can be found from Hooke's Law (Stroud and Booth, 2007).

4.7.1 Initial Rig Design

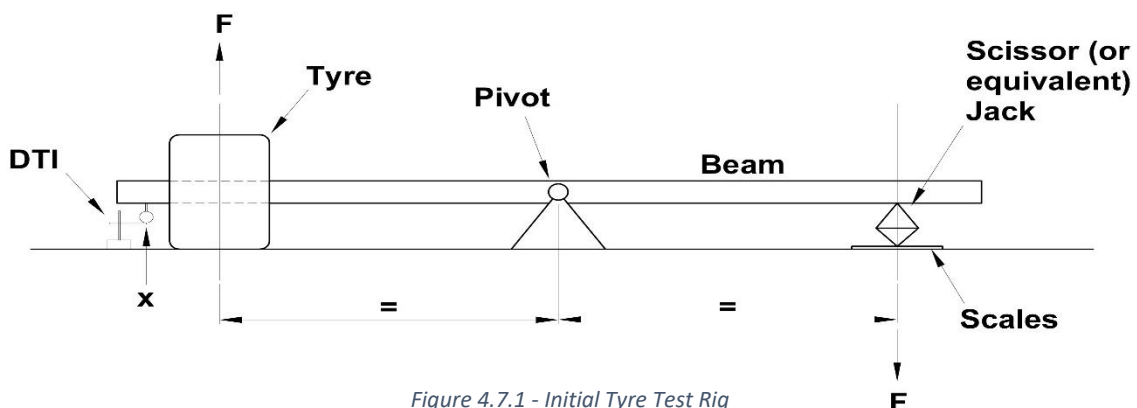


Figure 4.7.1 - Initial Tyre Test Rig

Figure 4.7.1 shows the author's initial design. The rig allows for a known force to be applied vertically to the tyre through a jack and scales, the DTI measures deflection. This initial idea includes a long lever arm, which will deflect and skew the results. Moreover, as the force is applied from one end and pushed through a pivot, the tyre is not loaded perpendicularly, again a source of error (Taylor *et al.*, 2000).

4.7.2 Final Rig Design

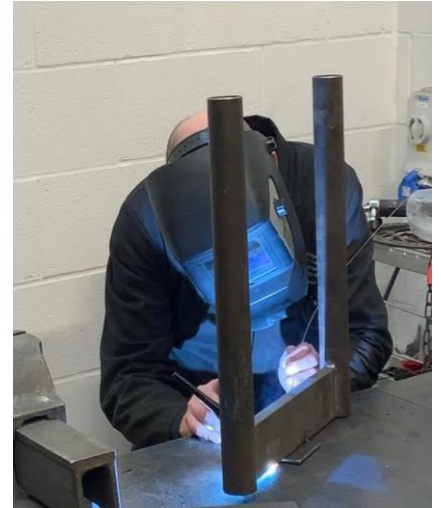
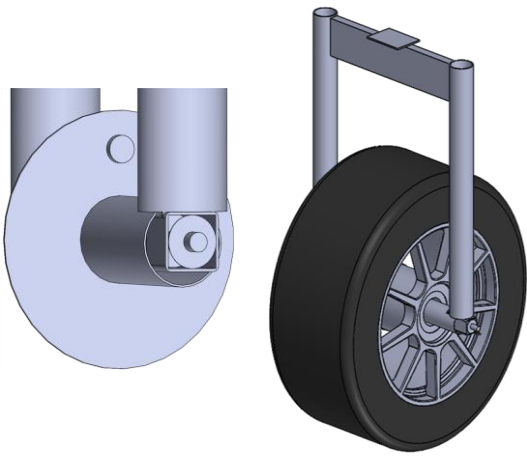


Figure 4.7.2 - Final Tyre Test Rig

To improve this design, the author worked with another student to co-develop the tyre rig. This version enabled the tyre to be placed under the hydraulic press, with two DTIs to measure deflection and check the force is being applied perpendicularly. Moreover, two DTIs were placed on the scales to measure their movement and remove this from the calculations (ISO, 2000).

The author initially started by applying a load equivalent to the vehicle's static corner mass. From here, load was applied in equal increments of 1mm and force, recorded and plotted (Ramshaw, 1985). A separate test was conducted to measure deflection with loads of less than static corner mass, to simulate the loading of the inside tyre throughout a corner.

4.7.3 Results

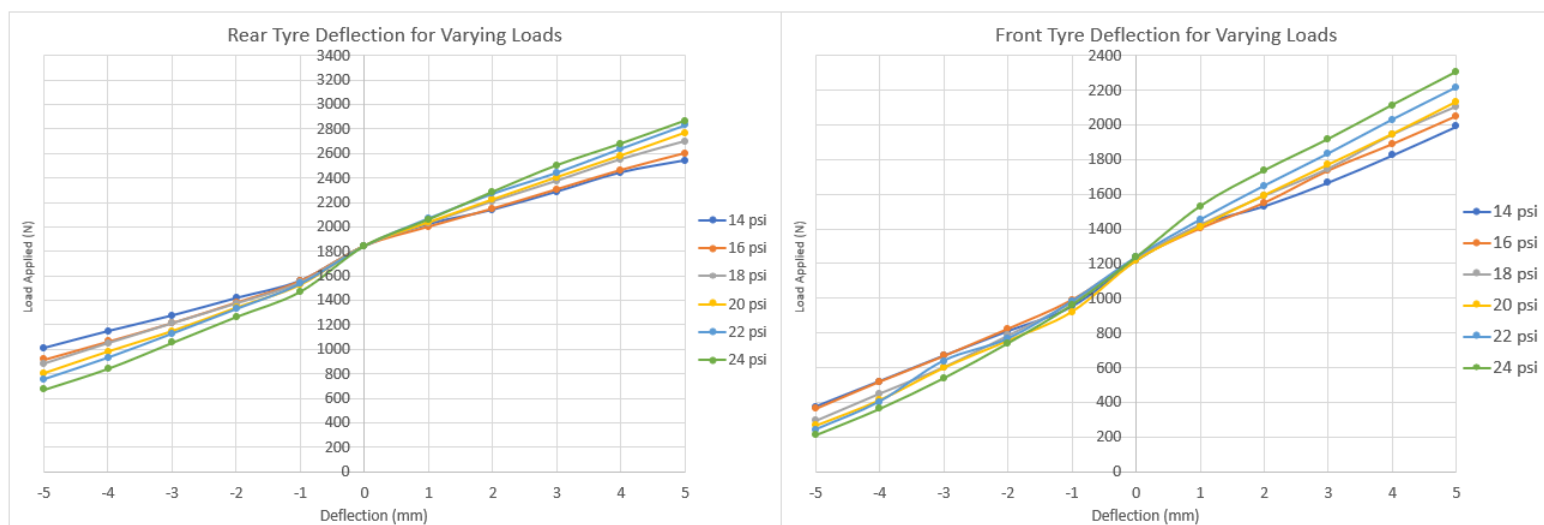


Figure 4.7.3 - Tyre Vertical Stiffness Test Results

Table 4.7.1 - Tyre Vertical Stiffness Test Results

	Tyre Vertical Stiffness (N/m)			
	FRONT +	FRONT -	REAR +	REAR -
24	207970.000	203490.000	204890.000	227310.000
22	195080.000	195500.000	194520.000	213020.000
20	181910.000	184150.000	183590.000	200680.000
18	173780.000	184010.000	171250.000	184430.000
16	164670.000	168170.000	151910.000	179660.000
14	149110.000	161580.000	139860.000	158500.000

Figure 4.7.3 and table 4.7.1 show the results from the tyre stiffness test. To reduce the chance of error, these tests were carried out three times and average values taken. Data from both tests showed a maximum difference of 3.6%, showing validity of results. The initial test produced an erroneous result for the rear positive load test at 18psi, as can be seen in appendix two, so this was discounted and an average of the other measured values was used.

It is clear that the trends for tyre stiffness, under positive and negative loading, are generally linear; the exception to this is the initial change of loading, up to 1mm either way. This trend was expected and could be explained by the very nature of a tyre's non-linearity under loading (Rośkiewicz and Leszczyński, 2016). If greater loads were applied, the results may be much less linear than can be seen above, and certain elements of the tyre, such as contact patch deformation, would behave very differently (Johnson, 1996). The small deviation from linearity here can be considered negligible, and vertical tyre stiffness can be treated as one linearly increasing value.

5 Load Transfer Calculation/Vehicle Set Up Models

5.1 Excel Steady State Load Transfer Calculator

By calculating the individual weight transfers, these could be added to the vehicle static masses to analyse contact patch loadings at each tyre. Throughout this project, the convention was taken that lateral acceleration was acting to the right and longitudinal acceleration was acting forward, as shown in figure 5.1.1. If the opposite of these cases are true at any point, the calculated value is negative; this convention is made clear to the user (Kadam and Chauhan, 2018).

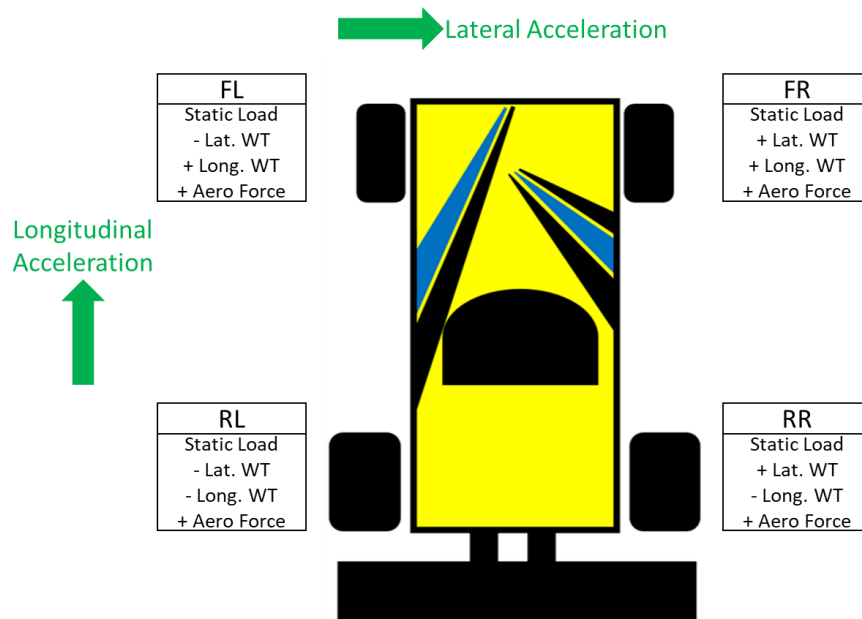


Figure 5.1.1 - Contact Patch Calculations

As the vehicle turns to the right, the left tyres are loaded, hence adding the lateral weight transfer there. As the car brakes, the front tyres are loaded, hence adding the longitudinal transfer there. The mathematics work to accurately calculate the tyre loadings whatever the vehicle's current situation (Wiegand, 2017).

From this, the author developed the EXCEL setup sheet with vehicle data in categorised columns. The values calculated from the data provided were displayed both in a table and through a dynamic and static display of contact patch loads; the dynamic values being from the specific steady state point that the user inputs. This can be seen below in figure 5.1.2 (Velenis *et al.*, 2010).

This model allows the user to take any specific point on the track and gain a better understanding of the vehicle balance. Comparing this to driver feedback is useful, however, a transient model will bring more accurate results and give a better overall view of vehicle balance.

SPORTS 2000 MCR WEIGHT TRANSFER & LOAD DISTRIBUTION CALCULATOR - REAL DATA

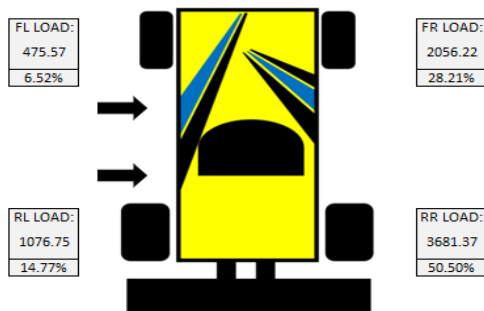
Revised: 11/03/2020

Max Hart

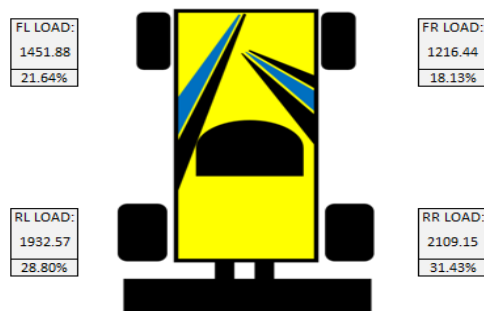
MATLAB variable:	Description:	Value:	Unit:
INPUTS			
INPUTS:			
Ax	Longitudinal Acceleration	0.280	G
Ay	Lateral Acceleration	1.240	G
v	Speed	30.000	m/s
CONSTANTS			
GEOMETRY:			
TrackF	Front Track	1.500	m
TrackR	Rear Track	1.475	m
WheelB	Wheelbase	2.600	m
MASSES:			
FL_M	FL Corner Mass	148.000	kg
FR_M	FR Corner Mass	124.000	kg
RL_M	RL Corner Mass	197.000	kg
RR_M	RR Corner Mass	215.000	kg
MsF	Front Sprung Mass	228.967	kg
MsR	Rear Sprung Mass	358.967	kg
MusF	Front Unsprung Mass	43.033	kg
MusR	Rear Unsprung Mass	53.033	kg
WHEELS:			
RR	Tyre Rolling Radius	0.265	m
Wh	Wheel Centre Height	0.265	m
SUSPENSION DATA:			
hF	Front Roll Centre Height	0.012	m
hR	Rear Roll Centre Height	0.070	m
h_Increase_F	Roll Centre Height Increase Coeff Front	1.231	-
h_Increase_R	Roll Centre Height Increase Coeff Rear	1.182	-
H	CoG Height above Roll Centre	0.350	m
H_plus_h	CoG Height	0.300	m
SPRING RATES:			
KsF	Spring Rate Front	39053.28421	N/m
KsR	Spring Rate Rear	67598.95831	N/m
MR_F	Motion Ratio Front	0.883	-
MR_R	Motion Ratio Rear	0.924	-
KwARBF	Front ARB Rate	63000.000	N/m
KwARBR	Rear ARB Rate	53000.000	N/m
Bump_Knee_Point			
Bump_Knee_Point	Bump Knee Point	37.75	mm/s
Rebound_Knee_Point	Rebound Knee Point	-37.75	mm/s
AERO DATA:			
CiF	Front Coefficient of Lift	0.400	-
CiR	Rear Coefficient of Lift	0.400	-
Area	Frontal Area	1.317	m^2

MATLAB variable:	Description:	Value:	Unit:
OUTPUTS			
WHEEL RATE:			
T_KwF	Total Front Wheel Rate	61735.253	N/m
T_KwR	Total Rear Wheel Rate	70556.140	N/m
ROLL STIFFNESS:			
RollStiffF	Front Roll Stiffness	69452.160	Nm/rad
RollStiffR	Rear Roll Stiffness	76751.851	Nm/rad
T_RollStiff	Total Roll Stiffness	146204.011	Nm/rad
MASS:			
T_Ms	Total Sprung Mass	587.933	kg
T_Mus	Total Unsprung Mass	96.067	kg
T_M	Total Vehicle Mass	684.000	kg
SUSPENSION DATA:			
Mot	Overturning Moment	2503.150	Nm
ROLL ANGLE:			
RollAngle_Deg	Roll Angle (Degrees)	0.981	Degs
RollAngle_Rads	Roll Angle (Radians)	0.017	Rads
LAT. WEIGHT TRANSFER:			
WTEF	Elastic Front WT	792.724	N
WTER	Elastic Rear WT	890.891	N
WTSrcF	Sprung Geometric Front WT	22.839	N
WTSrcR	Sprung Geometric Rear WT	207.229	N
WTusF	Unsprung Geometric Front WT	92.481	N
WTusR	Unsprung Geometric Rear WT	115.903	N
WTRF	TOTAL FRONT WT	908.044	N
WTR	TOTAL REAR WT	1214.022	N
T_WT	TOTAL OVERALL WT	2122.066	N
LONG. WEIGHT TRANSFER:			
WTL	Longitudinal WT	213.234	N
AERODYNAMICS:			
ArF	Front Axle Aero Load	289.933	N
ArR	Rear Axle Aero Load	289.933	N

NOTES:			

DYNAMIC CONTACT PATCH FORCES

LOADINGS	
TOTAL LOAD:	7289.91
INSIDE LOADING:	21.29%
OUTSIDE LOADING:	78.71%
FRONT LOADING:	34.73%
REAR LOADING:	65.27%
MAGIC NUMBER	
WT DISTRIBUTION:	42.79%
IF > 50: OVERSTEER IF < 50: UNDERSTEER	

STATIC CONTACT PATCH FORCES

LOADINGS	
TOTAL LOAD:	6710.04
INSIDE LOADING:	50.44%
OUTSIDE LOADING:	49.56%
FRONT LOADING:	39.77%
REAR LOADING:	60.23%

Figure 0.1 - EXCEL Steady State Load Transfer Calculator

As can be seen, each value has a column with its own variable name beside it to later be read into MATLAB. The spreadsheet also contains a description for the user, as well as a displayed value and units. Because this is intended as a setup sheet as well as steady state calculator, many elements from the author's initial research are included, such as a notes section and parts of the layout. It was important throughout this phase of the project to validate all work as it was carried out. The author validated this model against an existing model, both producing the same results, and also chose a point within real-life MoTeC data to input the speed and accelerations. The weight transfers and contact patch loadings seen on MoTeC correlated to this model within 10%, showing validation. This gave the author confidence to progress to the next version.

5.2 Simulink Steady State Load Transfer Calculator

Once the EXCEL model had been created and fully validated, the author replicated this in Simulink. This formed a starting point to move onto a transient model, allowing the author to learn the Simulink software. Firstly, an initialisation function was created within MATLAB. As the previous model acts as a setup sheet, it was important that the Simulink model read data off of this, as this is where the user will have everything stored. It would be inefficient to request the user to re-input all 31 pieces of required data.

```
%% VEHICLE DATA FROM SETUP SHEET

[num,text,row]=xlsread('STEADY STATE WEIGHT TRANSFER CALCULATOR.xlsx','CALC','B14:D49');
[r,~]=size(row);                                % Outputs size and index locations of all data in (row).
                                                % "num" = all numbers in selection · "text" = all text in selection. · "row" = all data in selection
Count=isnan(num)==0;                            % Creates logical array to find NaN values.
for i=1:r
    if Count(i)==1                              % Finds all data of value 1 - to omit NaN's.
        eval([cell2mat(text(i)) '=num(i)']); % "Eval" stores output from expression in the specified variables.
    end                                         % "Cell2Mat" converts cell array to ordinary array.
end                                             % Therefore, now have a list of required variables in workspace.
```

Figure 5.2.1 - MATLAB Code for Importing Setup Sheet Data

The code above is an advanced use of the 'xlsread' that analyses the vehicle data from the EXCEL spreadsheet, assigning each variable to its given name and omitting any rows which contain no useful data such as section titles (MathWorks, 2013). This code can be assigned as the initialisation function through Simulink model properties, so it automatically runs when the Simulink model is started, further increasing ease of use and ensuring all required data is loaded into the workspace to be used by Simulink.

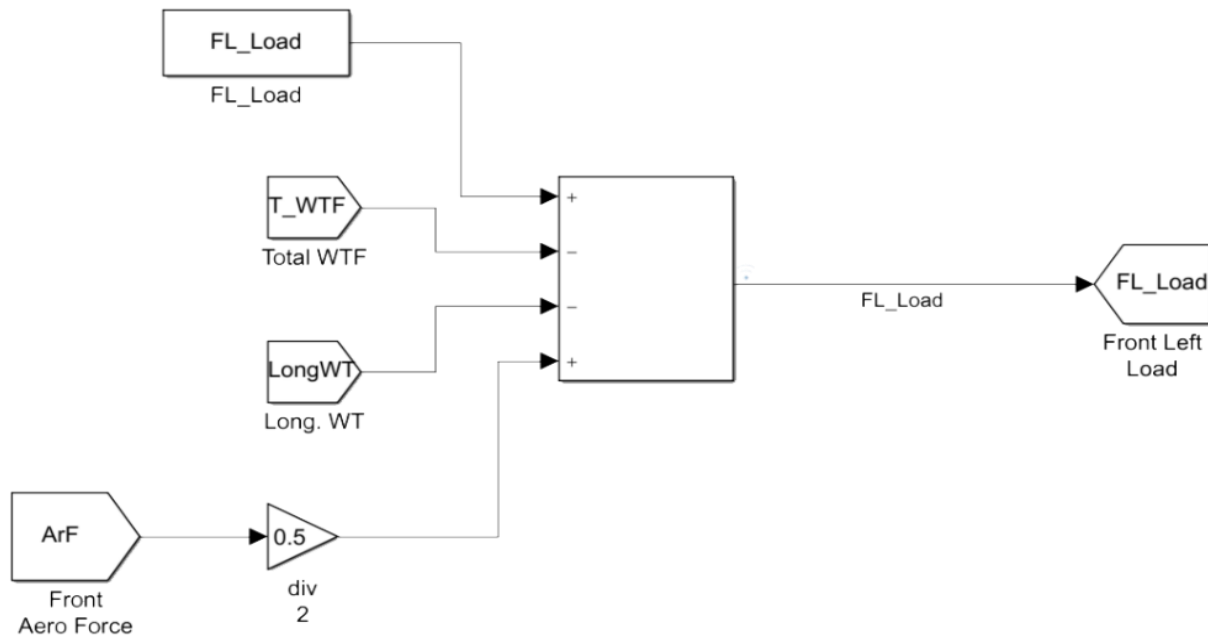
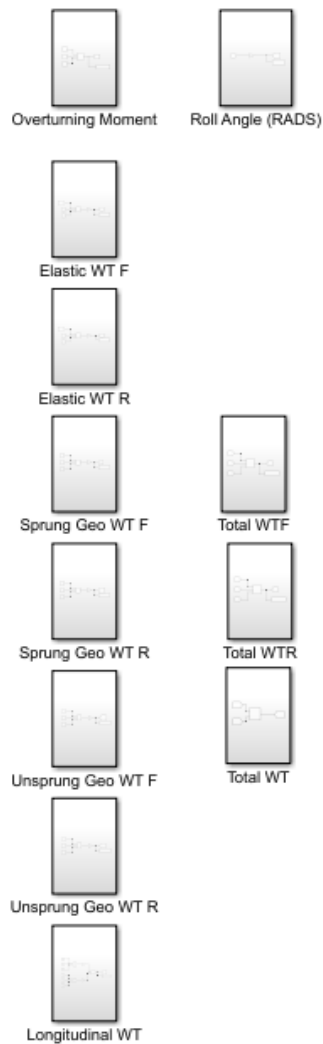


Figure 5.2.2 - Simulink Front Left Contact Patch Load Calculation

Figure 5.2.2 shows the construction of the Simulink system used to calculate the front left contact patch load. The Static load is read in through a 'constant' block, reading the value from the workspace. The other inputs are read in through a 'from' block, feeding in data from another subsystem in the model. Aerodynamic force is multiplied by 0.5 through a 'gain' block. The values are added and subtracted accordingly through an 'add' block, and outputted to a 'goto' block, which will read into a 'from' block in another part of the system.

WORKINGS

WEIGHT TRANSFER VALUES



VEHICLE LOADINGS

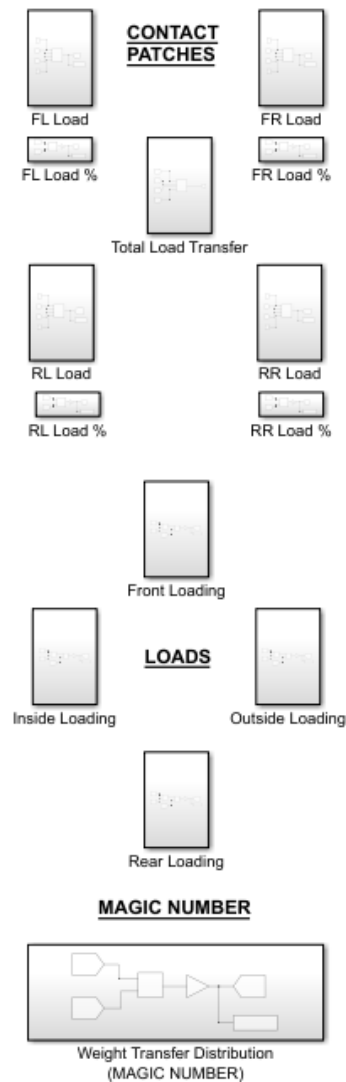


Figure 0.1 - Simulink Steady State Load Transfer Calculator

Figure 5.2.3 shows the steady state Simulink model. This was validated by confirming the displayed values were the same as the previously validated EXCEL model. The subsystems calculate the required values then a 'goto' block sends the data to displays.

5.3 Simulink Transient Load Transfer Calculator

This section of the report will detail the work involved in creating the main part of this project, the Simulink transient load transfer calculator and vehicle setup characterisation model. The author could use the validated Simulink steady state model and adapt this to work with a constantly changing data source; for this application that was initially just lateral and longitudinal acceleration and speed. This data has to be exported from MoTeC by the user into a comma-separated values (csv) file.

```
% Time (s)
Time = xlsread('WT_Calculator_Raw_Data.csv','A18:A55000');

% Lateral Acceleration (converted to m/s^2)
Y_Accel = xlsread('WT_Calculator_Raw_Data.csv','I18:I55000').*(9.80665);
Y_Accel = timeseries(Y_Accel,Time);

% Longitudinal Acceleration (converted to m/s^2)
X_Accel = xlsread('WT_Calculator_Raw_Data.csv','J18:J55000').*(9.80665);
X_Accel = timeseries(X_Accel,Time);

% Speed (converted to m/s)
Speed = xlsread('WT_Calculator_Raw_Data.csv','U18:U55000').*(0.277778);
Speed = timeseries(Speed,Time);
```

Figure 5.3.1 - MATLAB Code for Importing MoTeC Data

The above code, added to the initialisation function, shows how MATLAB reads in the required data from the csv file. The data is read in as a regular double-precision array (double), which is then converted to a timeseries so Simulink can relate each changing value to a specific time throughout the chosen scenario and perform a calculation at each timestep. The accuracy of this will depend on the frequency of the exported MoTeC file. This repetition of calculations is what defines this model as a quasi-semi state model.

Rather than outputting the data in display boxes as per the previous models, the results are now displayed graphically, representing the entire scenario chosen by the user, as a function of time. This is achieved by creating a stop function. This is run automatically when the model finishes the calculations. Data from Simulink is outputted to the workspace via bus creators. This places a variable in the folder of the Simulink model, which can be read into the stop function as a double, and graphed.

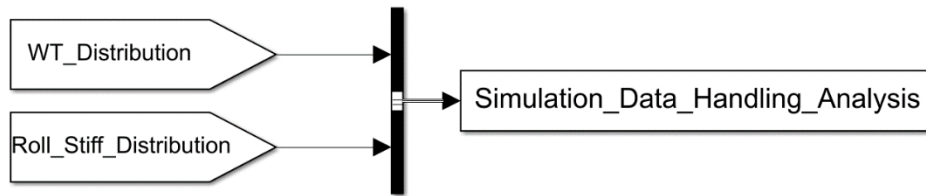


Figure 5.3.2 - Simulink Bus Creator to Output Results

From this, the plot function is used to create separate plots for the following results:

- Handling and balance analysis
- Contact patch loads - full and live plot
- Contact patch load validation
- Damper forces
- Damper rates
- Weight transfers

To validate this model, suspension force data was read in from MoTeC as this is calibrated to represent contact patch loading. These trends could be compared to the Simulink contact patch loads to validate results and ensure the model is calculating accurately. The transient model was frequently checked against the validated steady state models for confirmation of validation.

5.3.1 1-Cos Graph Validation

To validate the mathematics within the model, it was important to initially check that results followed expected trends. To do this, the author first introduced transient analysis by setting a constant speed, then replacing the constant lateral acceleration with data forming a 1-cos graph and removed longitudinal acceleration. This is easy to validate as it will cause the contact patch loadings to take the distinct shape of the graph.

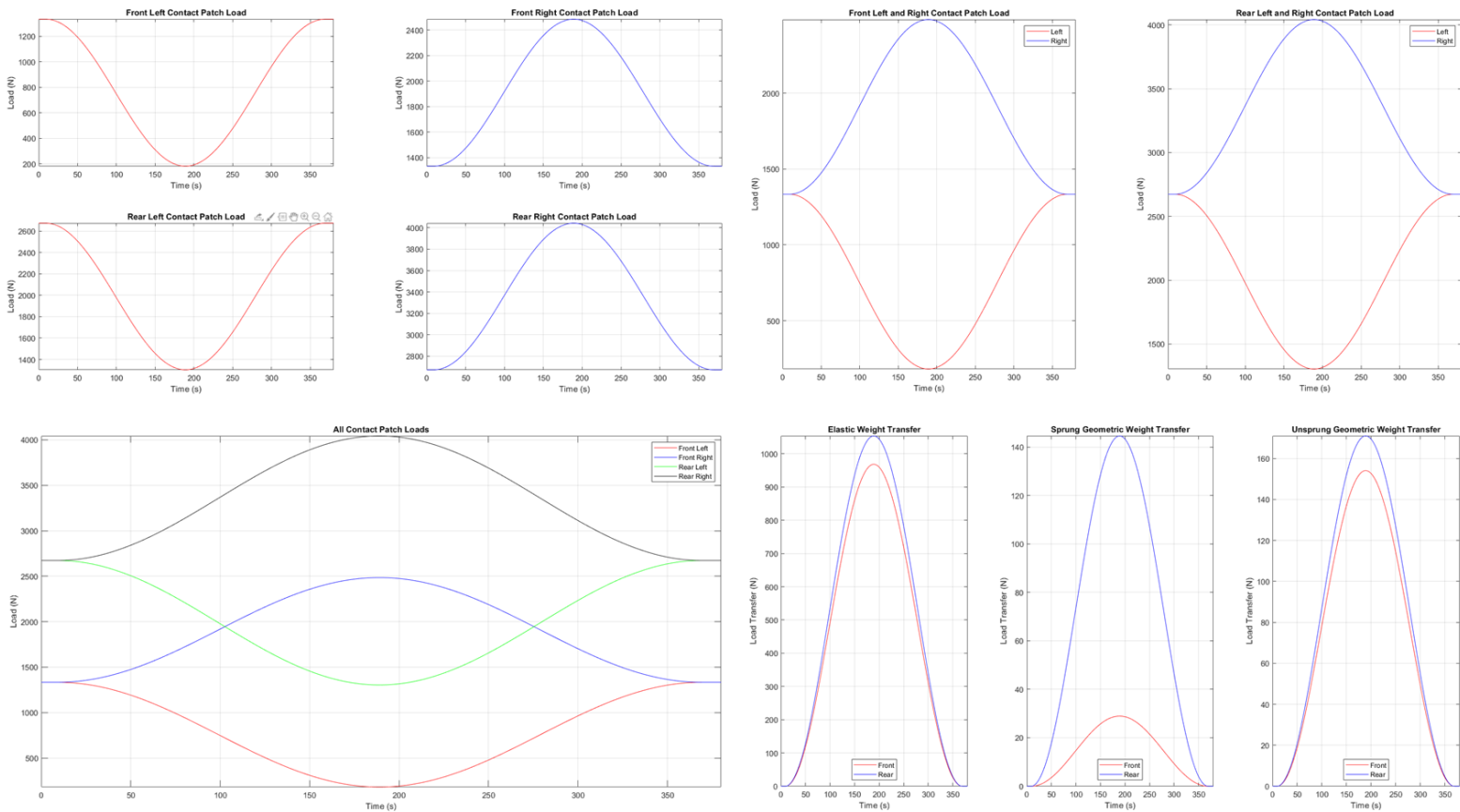


Figure 5.3.3 - 1-Cos Acceleration Results

Figure 5.3.3 shows that the model is working well, as the trends follow expectations:

- All data follows the clear, distinct shape of a 1-cos graph.
- Left and right contact patch loads are equal and opposite.
- When acceleration is zero, the contact patch loads show the static mass of the vehicle.
- The rear contact patch loads are higher than the front, due to vehicle mass distribution.
- Elastic and unsprung geometric weight transfer show rear values higher than front, again due to mass distribution.
- The sprung geometric weight transfer shows a front maximum only 20% as big as the rear.
This calculation includes roll centre height, of which the front is only 20% of the rear.

This shows at the initial stages, the model was working correctly, thus work could carry on.

5.3.2 Initial Silverstone Data

As an initial test of the model, the author inputted the lateral and longitudinal acceleration and speed from the MCR's qualifying lap of Silverstone International from the previous race season. The calculated contact patch loads were plotted with the actual loads from MoTeC to validate.

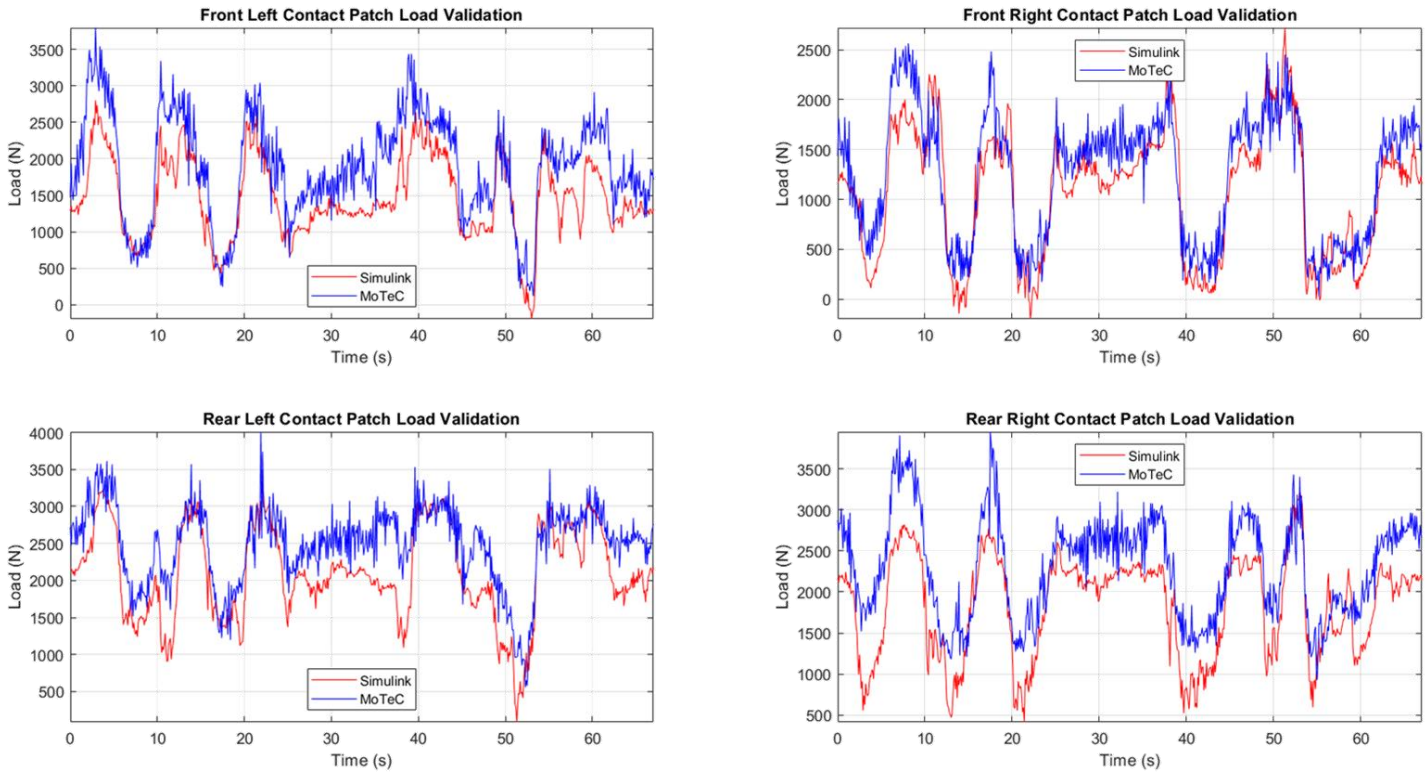


Figure 5.3.4 - Initial Silverstone Data Results

The initial results from the transient load transfer calculator, as in figure 5.3.4, showed a clear correlation in results, validating the mathematics once more. However, it is also clear that there were errors within the model. The main issues were:

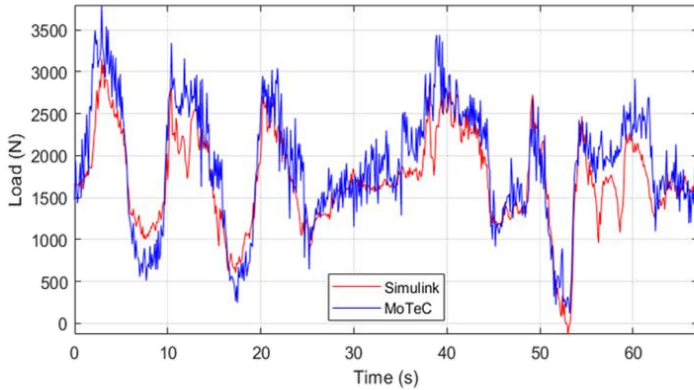
- The start of the graph is on the start finish straight, so minimal longitudinal and no lateral acceleration should be seen. The difference in trends here must be mainly down to static mass errors.
- The peaks at highest steering angles, usually just before the apex are generally lower than the actual values.
- The section between 25 and 38 seconds is the main straight, no acceleration should be acting here so the difference in trends must be due to inaccuracy in aerodynamic forces.

To correct these issues, the author analysed the suspension force channels on the MoTeC data and found a point where the car was stationary. Taking an average of these values, a more accurate static mass for the vehicle on the day the data is from was found. The conversion from g to m/s^2 also contained an error, which was fixed. Moreover, to reduce the aerodynamic force error, the author accessed the Myra wind tunnel data from the white MCR. This showed aerodynamic loading front and rear; from this, using the lift equation, the coefficients of lift front and rear could be back-calculated to find the following data (Horiba Mira, 2015; Hull, 2015).

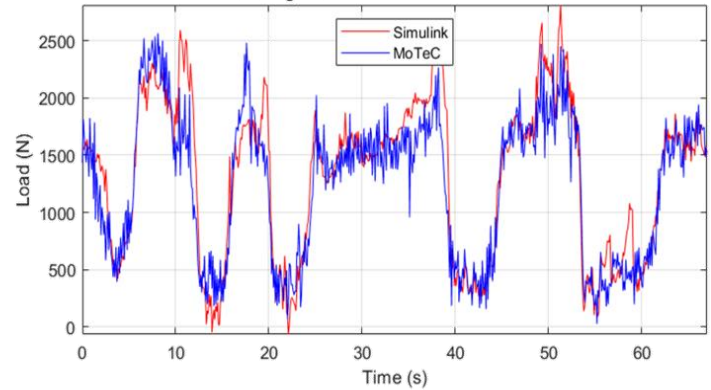
Table 5.3.1 - MIRA Wind Tunnel Aero Data

Front Lift Coefficient	0.400
Rear Lift Coefficient	0.457

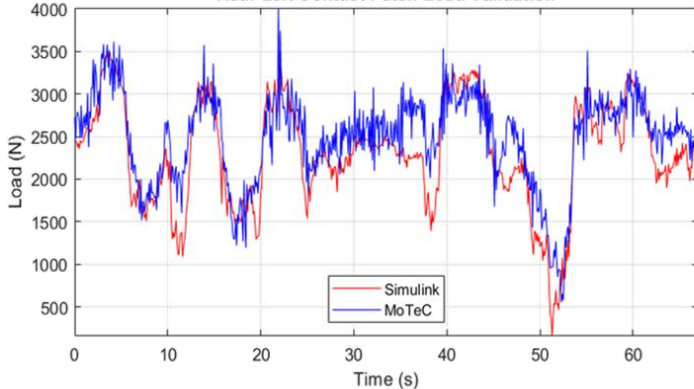
Front Left Contact Patch Load Validation



Front Right Contact Patch Load Validation



Rear Left Contact Patch Load Validation



Rear Right Contact Patch Load Validation

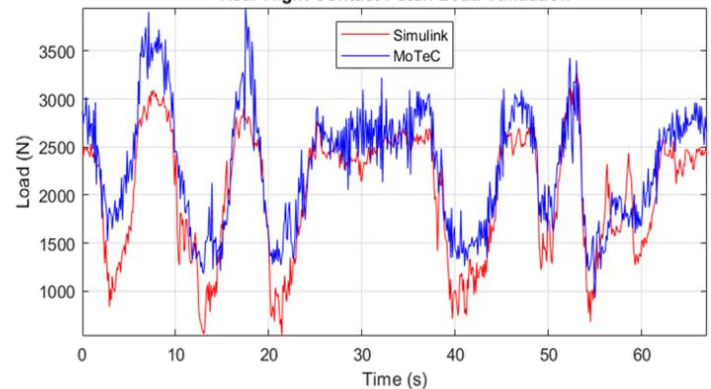


Figure 5.3.5 - Silverstone Data with Revised Static Mass and Aerodynamic Data

Once these errors had been addressed, the model was re-run. A much better correlation of data can be seen, with the trends following much more closely.

At this point, once multiple validation studies had been completed and the model was at an acceptable level of accuracy, the author looked into adding more elements to the model to further increase accuracy, whilst still continually troubleshooting and trying to reduce error within the Silverstone data calculations.

5.4 Adding Effects of Dampers to The Model

As has been discussed, a key area of inaccuracy with the model up to this point is not including the effects of the dampers.

This is particularly important when calculating the roll angle of the vehicle. Dampers are actuated by roll velocity; as the vehicle starts to roll, the dampers resist this motion, reducing the roll velocity and therefore the roll angle as the corner is entered or exited (Bastow *et al.*, 2004; Rosenberger *et al.*, 2015). The model, before including dampers, used the torque applied by the overturning moment divided by the total roll stiffness, the resistance to the torque from the springs and ARBs.

The author first calculated the resistive torque from the dampers as per equation 5.4.1 and brought this into the roll angle calculation, subtracting it from the overturning moment, thus reducing the torque applied to simulate dampers. This was completed in an iterative loop; at every timestep the current torque from the dampers was calculated, subtracted from the overturning moment, then divided by roll stiffness (Bastow *et al.*, 2004; Calvo *et al.*, 2008; Verbaan *et al.*, 2014).

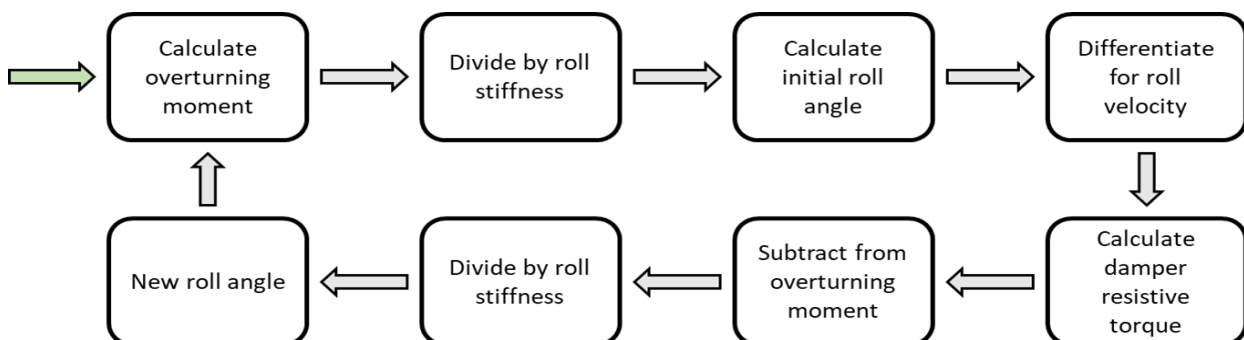


Figure 5.4.1 - Vehicle Dampers Iterative Loop

5.4.1 Calculating Damper Resistive Torque for New Roll Angle

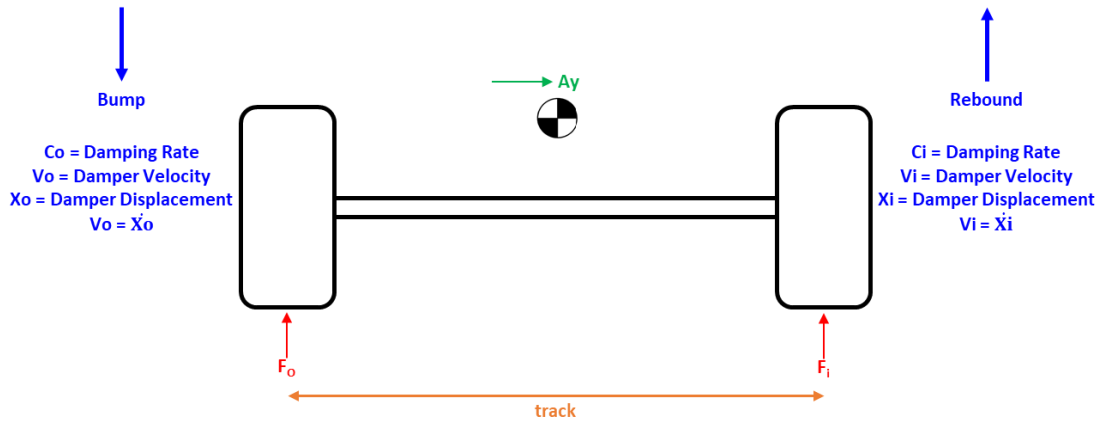


Figure 5.4.2 - Dampers Acting on a Turning Vehicle

Figure 5.4.2 represents a vehicle with dampers travelling around a corner. By taking moments about the inside wheel, the resistive torque created by damping can be calculated for the front and rear. Components of the following equations correspond to figure 5.4.2.

$$F_o = (V_o \cdot C_o) \cdot MR^2 \quad F_i = (V_i \cdot C_i) \cdot MR^2$$

$$\tau_d = F_o \cdot \frac{t}{2} - F_i \cdot \frac{t}{2}$$

$$\therefore \tau_d = \frac{V_o \cdot C_o \cdot MR^2 \cdot t}{2} - \frac{V_i \cdot C_i \cdot MR^2 \cdot t}{2}$$

Equation 5.4.1 - Damper Resistive Torque

This value can then be subtracted from the overturning moment to find the roll angle, as shown by figure 5.4.3. This demonstrates how initially roll will be unaffected by dampers, effectively subtracting zero; the roll angle is only affected when the car starts to roll, thus creating a roll velocity (Calvo *et al.*, 2008; MathWorks, 2017).

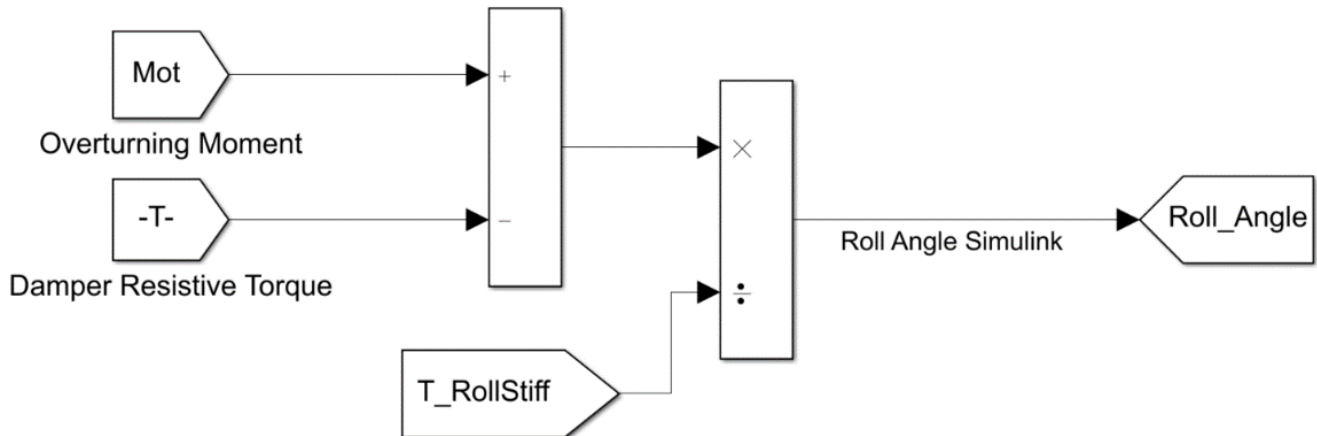


Figure 5.4.3 - Damper Effect on Roll Angle Simulink Calculation

Reducing the roll angle in this way will slightly reduce weight transfer due to the roll angle's effect on elastic weight transfer. Moreover, dampers work by exerting force on the pushrods, this force adds back into the overall weight transfer (Johnston, 2010). This is included by simplistically modelling damper weight transfer:

$$\text{Damper WT} = \frac{(\text{Right Damper Force} - \text{Left Damper Force}) \cdot MR}{2}$$

Equation 5.4.2 - Damper Weight Transfer

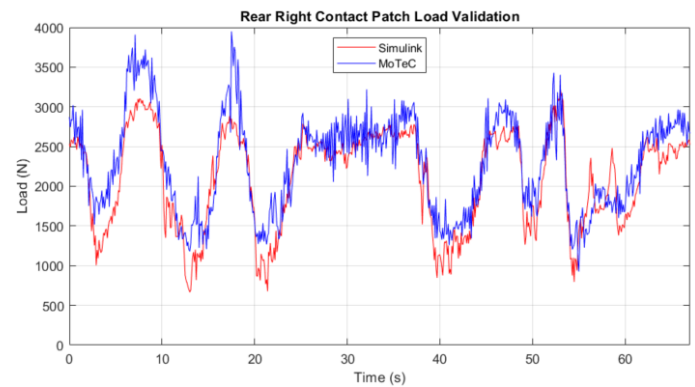
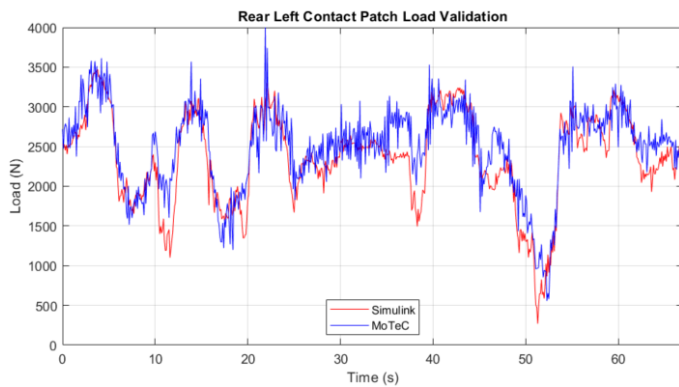
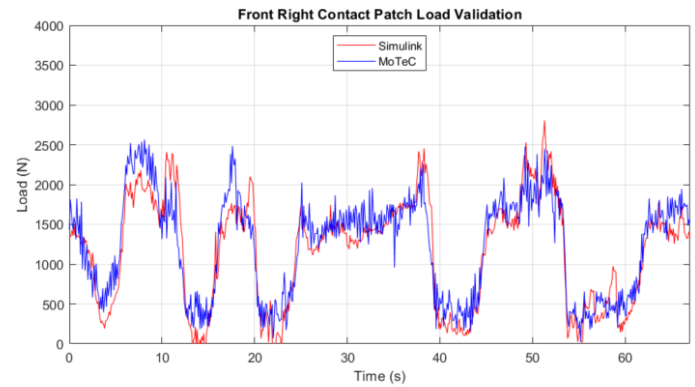
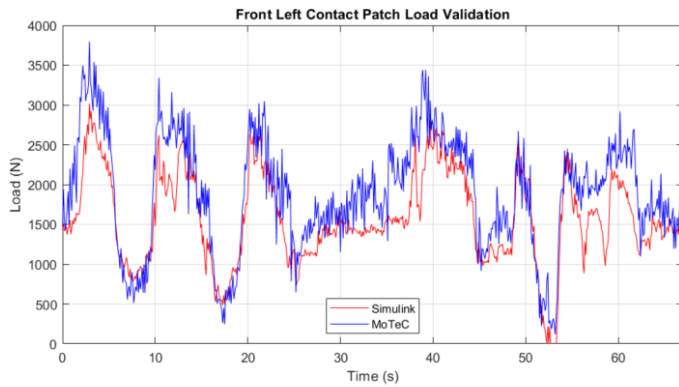


Figure 5.4.4 - Silverstone Data with Effects of Dampers

Figure 5.4.4 shows the Silverstone qualifying contact patch validation after damping effects on roll and weight transfer were added. Compared to figure 5.3.5, there is not a large visual difference, however figure 5.4.5 shows on average a 3.4% decrease in roll angle values, and figure 5.4.6 shows viable results for damper weight transfer, albeit at equally small magnitude. These trends validate the mathematics behind these additions to the model.

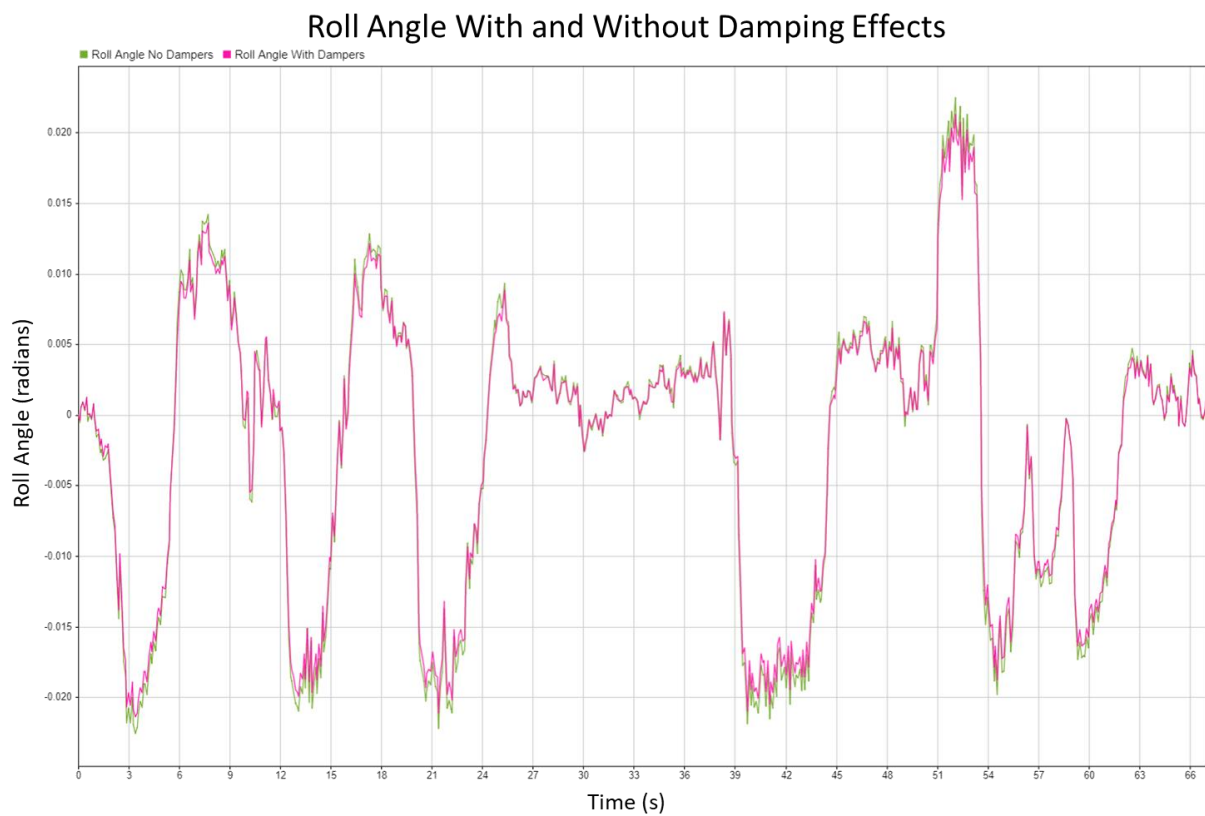


Figure 5.4.5 - Reduction of Roll Angle Due to Dampers

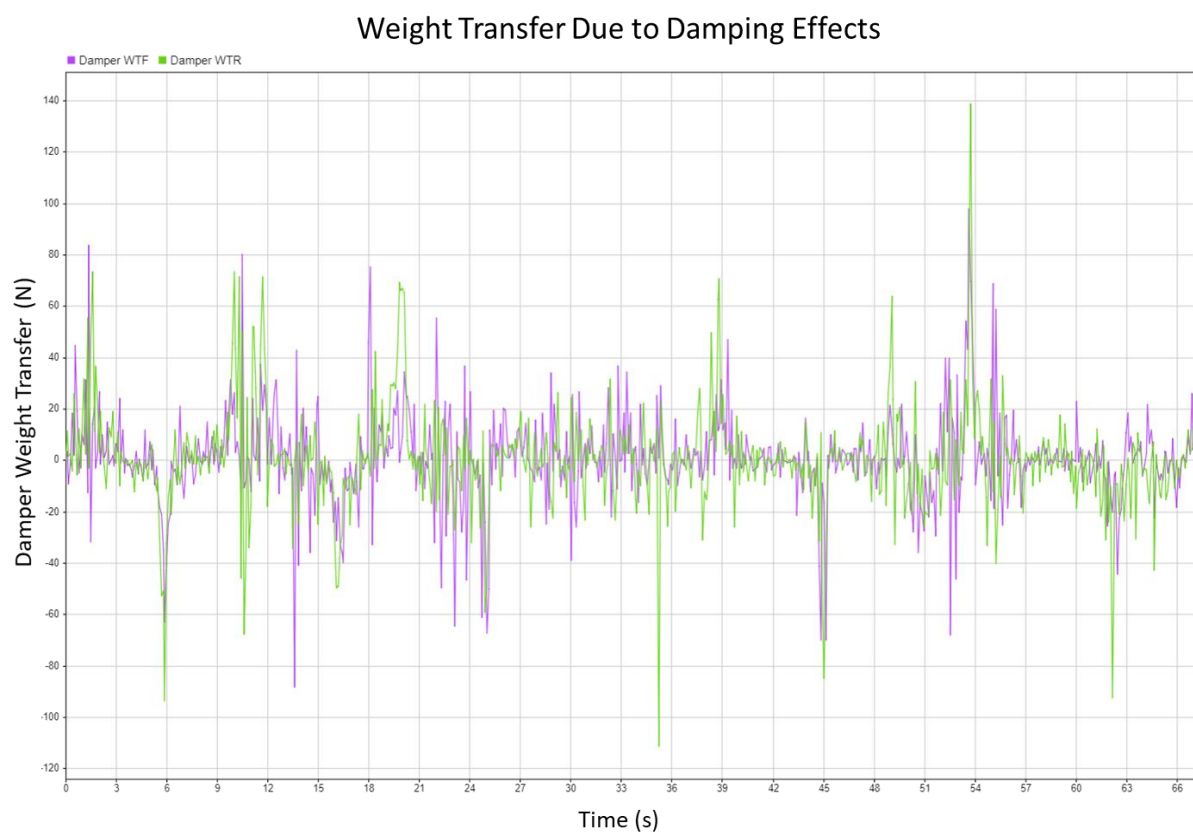


Figure 5.4.6 - Damper Weight Transfer

5.5 Llandow Anti-Roll Bar Rate Testing



Figure 5.5.1 - MCR Testing at Llandow (Hart, 2019)

In November 2019, the car was sent out for three laps with both ARBs detached. This removed the ARB rates, which are unvalidated data, from the calculations to assess how the error in the trends changed.

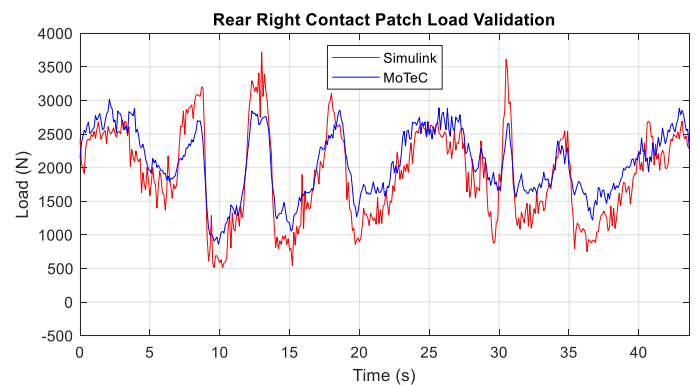
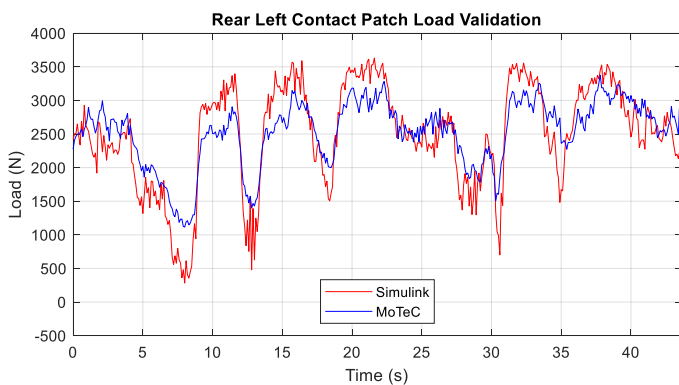
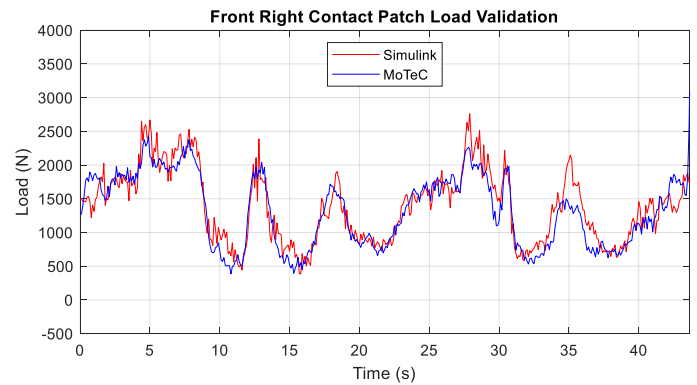
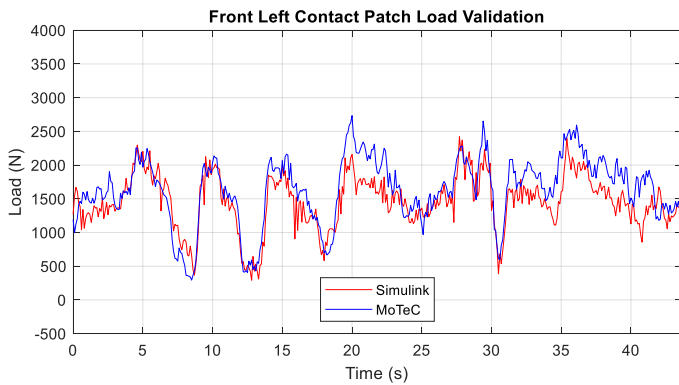


Figure 5.5.2 - Detached ARB Test Data

The author measured and compared the difference in calculated and MoTeC values between figures 5.4.4 and 5.5.2 and found the difference to be exactly the same. This proves that the standard ARB rates acquired from the simulator were accurate and were not the main contributing factors to the inaccuracy of the model. This process allowed the author to validate ARB rates within the model without creating a testing rig.

5.6 Analysing Pushrod Calibration Errors

The main source of validating this model is through comparison of contact patch loads. Although this is a good way to analyse similarities in the trends, there are inaccuracies when correlating this to real data. Contact patch loads are calculated through push rod sensors. This does not always give an accurate representation of the forces seen at the contact patch. The calibration process, completed most recently in November 2019 is as follows (Hart *et al.*, 2018):

- Consult the wiring diagram and find the pushrod sensor input locations.
- Plug all sensors into the correct inputs on the car.
- Find MoTeC math channels and output values.
- Apply a load by hand to check the signal and check if it is plugged into the right place.
- Use the Longacre coil spring tester to set constant pressure in pounds.
- Convert this to Newtons and input pressure exerted into MoTeC software.
- Repeat this for each 10 Newton increment between 0 and 100 Newtons first of all.
- Check inputted data on MoTeC; a linear graph should be seen
- Repeat for remaining sensors, only 3 readings need to be taken as linearity confirmed.
- View output channels on MoTeC - applied load to all by hand to check live output.
- Mark each with the correct position on the vehicle.

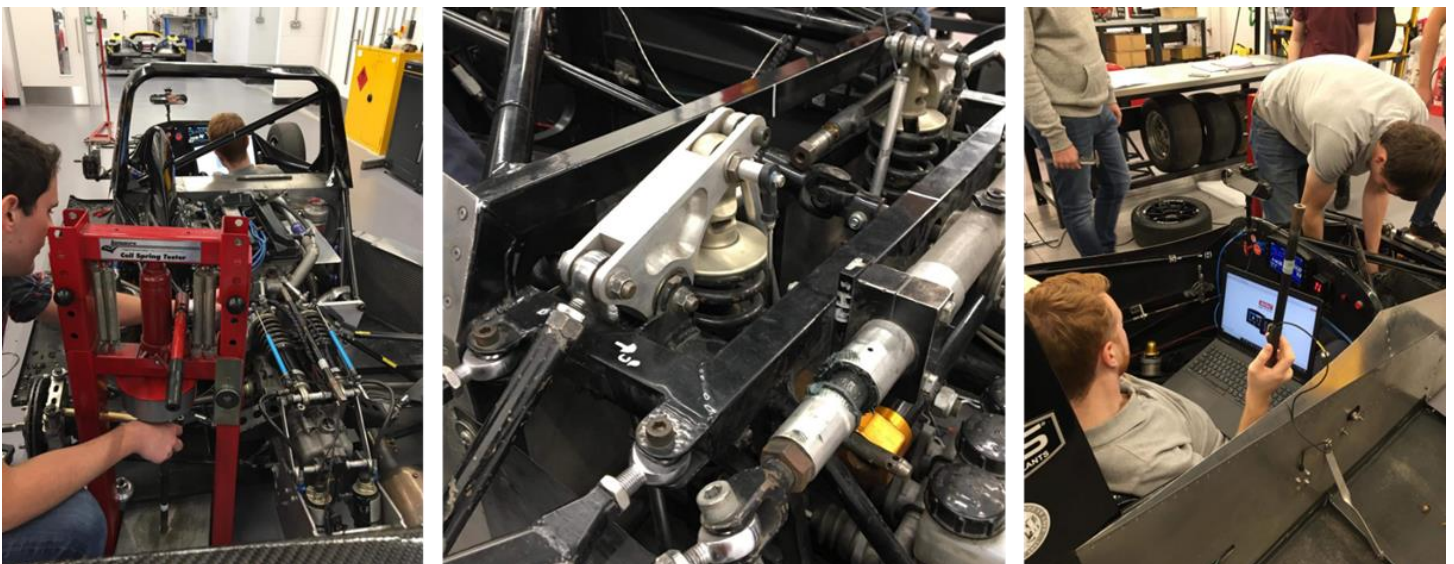


Figure 5.6.1 - Pushrod Calibration Process (Hart, 2019)

The inaccuracy from this calibration occurs when the pushrods are put under high load. The non-linearity of the loading can cause erroneous results as they are not tested within this region in the workshop and the calibration does not account for it.

Moreover, the position of the pushrod within the vehicle means it will experience different forces to that at the contact patch. Despite this, the trends are clearly close and are still a valid method of checking results.

To account for this inaccuracy, the author altered the static masses of the vehicle until the trends visually matched more closely. This approach worked well and allowed a more representative view of how closely the trends matched.

5.7 Effect of Movement of Centre of Gravity on Weight Transfer

The height of the centre of gravity has a noticeable effect on weight transfer, and therefore the vehicle balance. Another factor to consider is the lateral movement of the centre of gravity as the vehicle corners, due to body roll. This effect is usually considered negligible; however, to further increase the model's accuracy the author included this mathematically (Racing Car Dynamics, 2015).

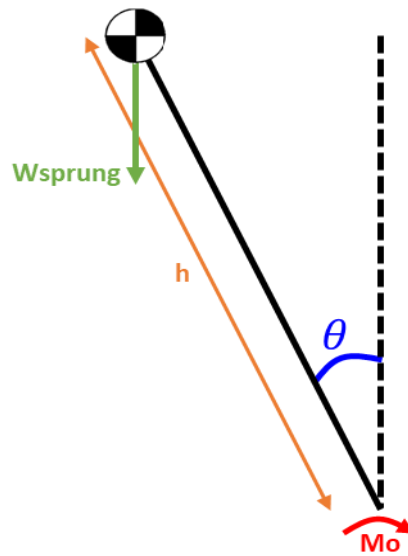


Figure 5.7.1 - Changing Position of CoG as Vehicle Corners

Figure 5.7.1 represents the centre of gravity being moved to the left through a fast corner through roll angle θ . It is now at a height of h , which can be worked out trigonometrically.

The inertia force acting on the centre of gravity generates the moment M_o about the roll axis. Assuming symmetrical left/right weight distribution and the small angle theory, take moments about the roll axis:

$$\begin{aligned}
 M_o - M_s \cdot h_{COG} \cdot \sin\theta - M_s \cdot Ay \cdot h_{COG} \cdot \cos\theta &= 0 \\
 \therefore M_o - M_s \cdot h_{COG} \cdot \theta - M_s \cdot Ay \cdot h_{COG} \cdot 1 &= 0 \quad (\text{Small angle theory}) \\
 \therefore M_o &= M_s \cdot h_{COG} \cdot (\theta + Ay)
 \end{aligned}$$

Equation 5.7.1 - Taking Moments about the Roll Axis

Here, M_o is the roll resistance moment from the springs and ARBs, and $W_s \cdot h_{COG} \cdot \sin\theta$ is the vertical, gravitational component (Racing Car Dynamics, 2015; Sebastian Siyad *et al.*, 2016). At any timestep, the total roll resistance moment will equal the roll moment. This means $M_o = K_o$ where K_o = the resistive roll moment from the springs.

$$\begin{aligned}\therefore M_s \cdot h_{COG} \cdot (A_y + \theta) &= (K\theta F + K\theta R) \cdot \theta \quad \text{where } K\theta = \text{roll stiffness} \\ \therefore M_s \cdot h_{COG} \cdot A_y &= (K\theta F + K\theta R - M_s \cdot h_{COG})\theta \\ \therefore \theta &= \frac{M_s \cdot h_{COG} \cdot A_y}{(K\theta F + K\theta R - M_s \cdot h_{COG})} \quad \therefore \frac{\theta}{A_y} = \frac{M_s \cdot h_{COG}}{(K\theta F + K\theta R - M_s \cdot h_{COG})}\end{aligned}$$

Equation 5.7.2 - Amount of Body Roll per Uni Lateral Acceleration

Equation 5.7.2 shows $\frac{\theta}{A_y}$, the amount of body roll per unit lateral acceleration. By looking at the roll angle, the moment from the roll resistance moment and side shift of centre of gravity can be found. Here, the roll resistance moment (M_o) will be the roll stiffness * roll moment, and 'A' shows the value for the axle currently being analysed.

$$M_o = \frac{K\theta A \cdot M_s A \cdot h_{COG} - A_y}{K\theta F + K\theta R - M_s \cdot h_{COG}}$$

Equation 5.7.3 - Roll Resistance Moment

For a single axle, the moment must only take into consideration that axle's sprung mass. Therefore, the Moment for axle A is:

$$M_{COG} = M_s A \cdot h_{COG} \cdot \theta$$

Equation 5.7.4 - Moment about the Centre of Gravity

Substituting this expression into equation 5.7.2:

$$M_{COG} = M_s A \cdot h_{COG} \cdot \frac{M_s \cdot h_{COG} \cdot A_y}{(K\theta F + K\theta R - M_s \cdot h_{COG})}$$

Equation 5.7.5 - Moment about the CoG

Therefore the total moment from the roll angle on a single axle will be:

$$M_o = M_s A \cdot A_y \cdot h_{COG} \cdot \frac{K\theta A \cdot M_s A \cdot h_{COG}}{(K\theta F + K\theta R - M_s \cdot h_{COG})}$$

Equation 5.7.6 - Total Moment about CoG

Therefore the sprung geometric weight transfer including centre of gravity movement is:

$$\Delta WTs = \frac{M_s A \cdot A_y \cdot h_{COG}}{t} \cdot \frac{K\theta A \cdot M_s A \cdot h_{COG}}{(K\theta F + K\theta R - M_s \cdot h_{COG})}$$

Equation 5.7.7 - Revised Sprung Geometric Weight Transfer Equation

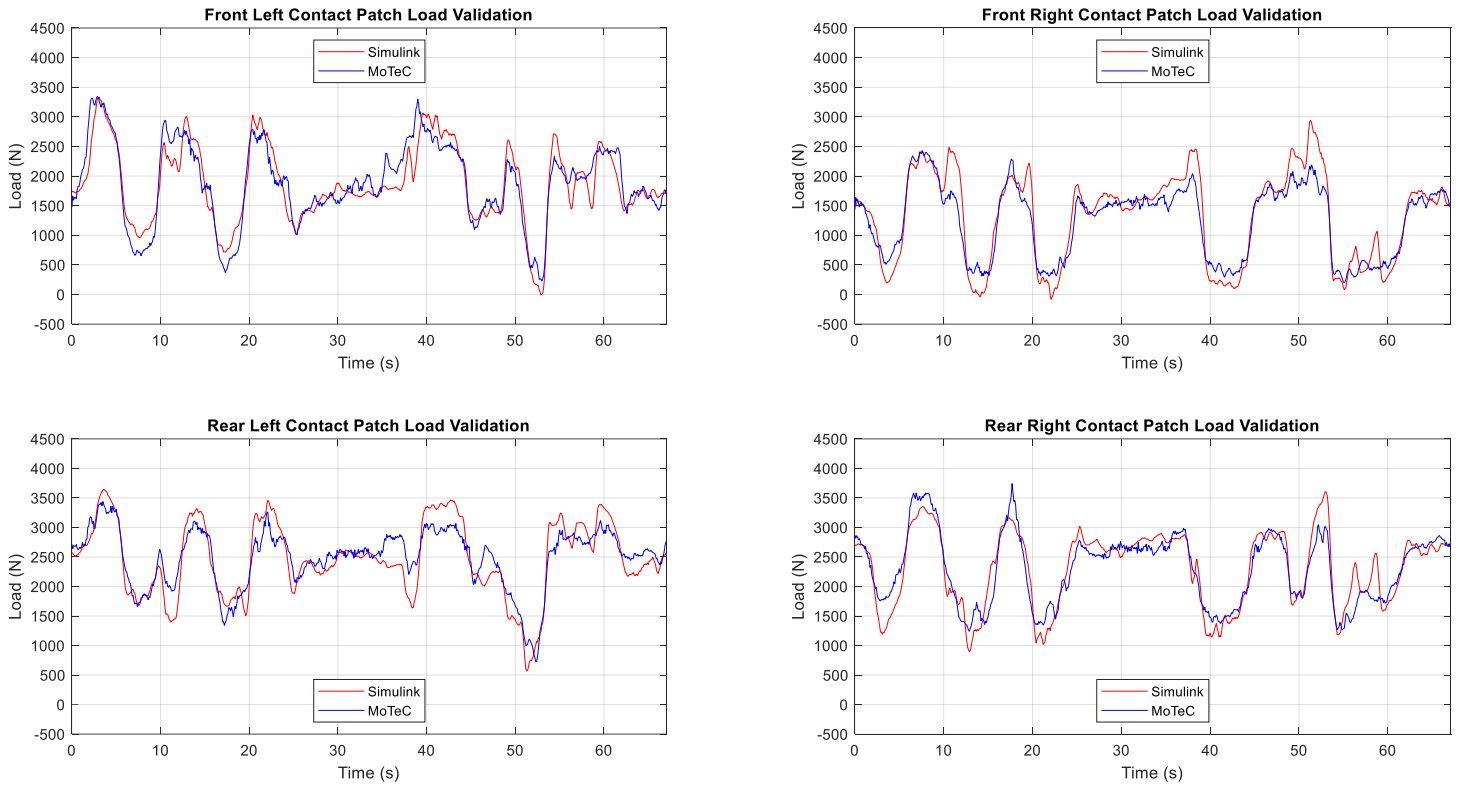


Figure 5.7.2 - Silverstone Data with Revised Elastic Weight Transfer and Static Masses

Figure 5.7.2 includes data filtered by the MATLAB smooth function. It is clear to see that the altered static masses and including elastic weight transfer with effects of lateral centre of gravity movement produce a much closer correlation, validating the mathematics in this section.

5.8 Simulator Data Validation

To eliminate errors associated with the real data, the author devised a test on the simulator, where contact patch loads are generated by the software with much greater accuracy. The test consisted of the following components (ISO, 2012):

- Vehicle stationary for five seconds to register accurate static corner masses.
- Harsh accelerations in two clear steps to analyse trends in longitudinal weight transfer.
- Vehicle continues straight at constant speed for five seconds to analyse aerodynamic forces.
- Series of high speed slaloms to analyse trends in lateral weight transfer.
- Constant braking down to standstill to again analyse trends in longitudinal weight transfer.

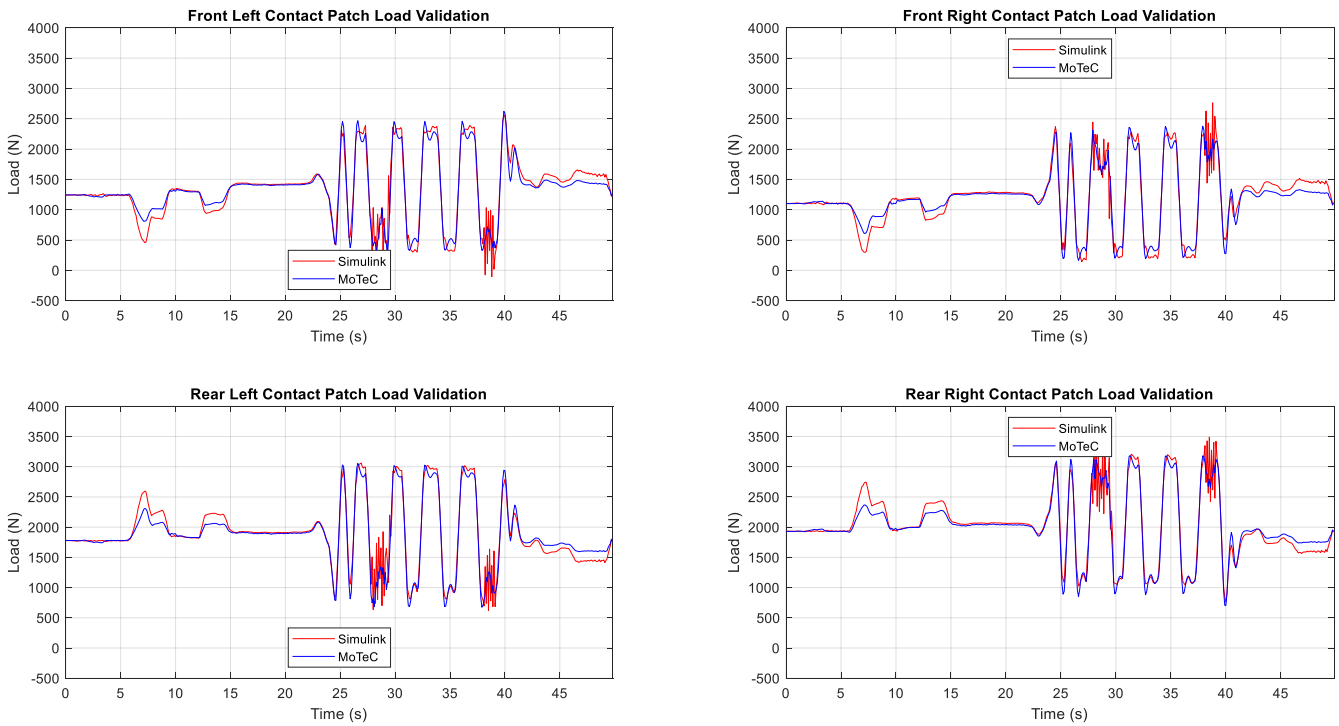


Figure 5.8.1 - Initial Simulator Data Comparison

Figure 5.8.1 shows the initial correlation of trends between calculated contact patches, and those from the simulator. It is clear the trends are much closer now the inaccuracy within the pushrods has been significantly reduced. The initial accelerations and final deceleration show poor correlation, showing an error within the longitudinal weight transfer. Moreover, there is clear noise within the Simulink data even after filtering, and the trends do not match at the maximum points of lateral acceleration.

Upon investigation, it was discovered that the simulator logs damper velocity whereas the real data logs damper displacement. Because of this, the Simulink model was differentiating displacement twice, causing an error. To overcome this, the author implemented a user selection within the initialisation code to select either simulator or real life data. A switch will select if the damper data is differentiated or not depending on what the user selects. Figure 5.8.2 Shows a clear improvement in correlation once this issue was fixed.

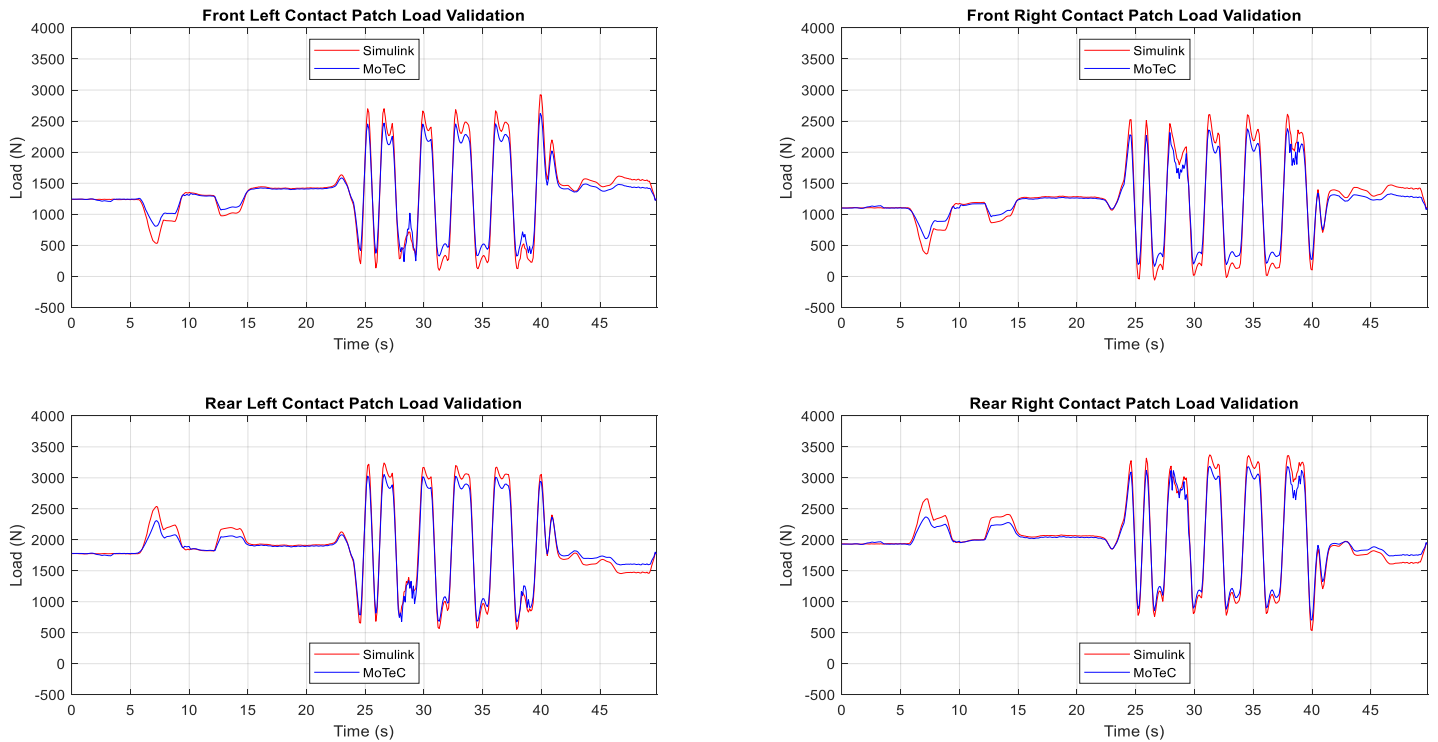


Figure 5.8.2 - Simulator Data with Revised Damper Data

5.9 Effect of Changing Roll Centre Height

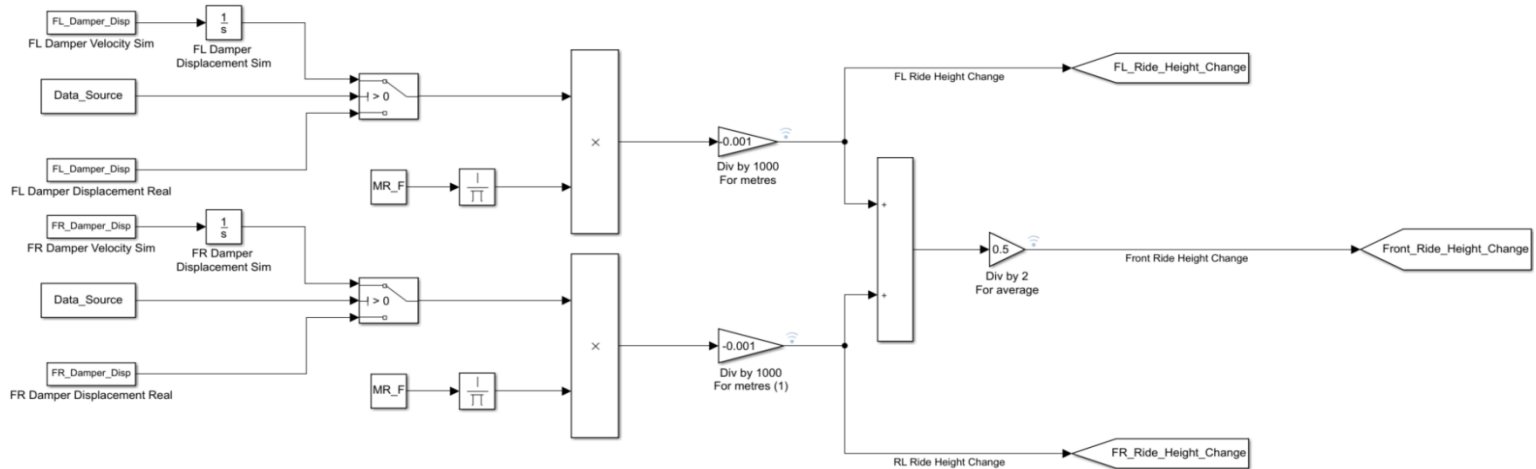


Figure 5.9.1 - Simulink Ride Height Change Calculation

Figure 5.9.1 shows the Simulink subsystem that calculates how much the ride height changes as the vehicle travels around the circuit. This is calculated for front and rear. The switches, as mentioned in section 5.8, dictate whether the MoTeC data is damper displacement or velocity, depending on if the simulator or real car has been used. From here, the damper position is multiplied by the inverse of the motion ratio, giving the wheel position relative to the initial, static ride height. This value is calculated for the left and right wheels, and an average is taken. This assumes the roll centre stays central to the vehicle (Sakai and Satoh, 1994; Bastow *et al.*, 2004).

From here, the Simulink model multiplies the change in ride height by the gradient of the graph in figure 4.6.3, calculating the roll centre height. There was no visual change in validation graphs from this addition, however the author multiplied the ride height by 10 and saw a substantial increase in lateral weight transfer, validating the mathematics behind this part of the model.

5.10 Effects of Including Vertical Tyre Stiffness

The Simulink model calculates change in ride height by measuring wheel movement in relation to the chassis. For direct comparison, the rig was made to measure wheel movement in the same way in relation to the hydraulic press, therefore, the calculation of ride height change could be used to calculate the deflection of the tyre. As the vehicle corners one tyre will increase loading, the other will decrease; there is no way of implementing this into the weight transfer calculations, therefore, the model finds the tyre stiffness rate for each tyre and takes an average of the two. This will be a source of inaccuracy within the results but gives more realistic results than not adding at all, as shown in figure 5.10.1

The tyre stiffness is included in the model through wheel rate. The tyre's spring rate acts in series with the spring and ARB rates:

$$Total\ Wheel\ Rate = \left(\frac{1}{Wheel\ Rate} + \frac{1}{Tyre\ Stiffness} \right)^{-1}$$

$$\therefore Total\ Wheel\ Rate = \left(\frac{1}{Spring\ Rate \cdot MR^2 + ARB\ Rate} + \frac{1}{Tyre\ Stiffness} \right)^{-1}$$

Equation 5.10.1 - Wheel Rate Including Tyre Stiffness

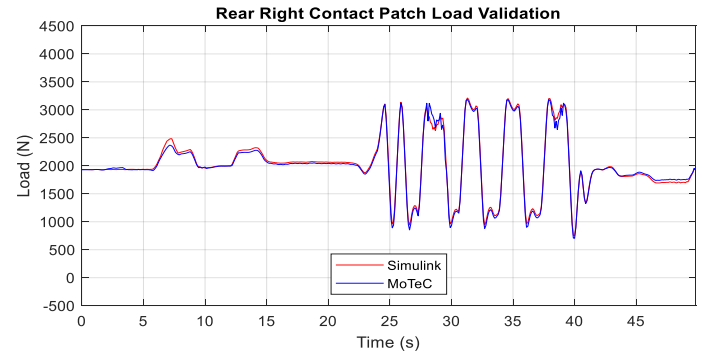
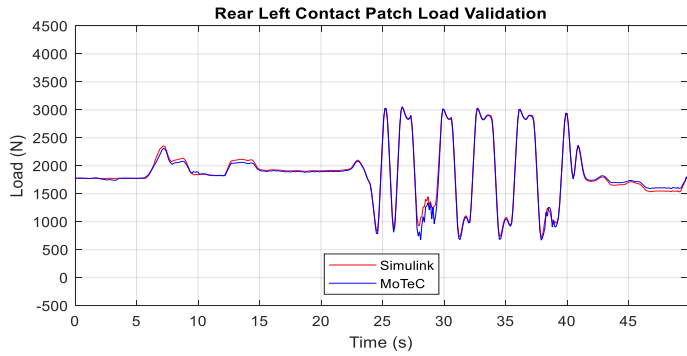
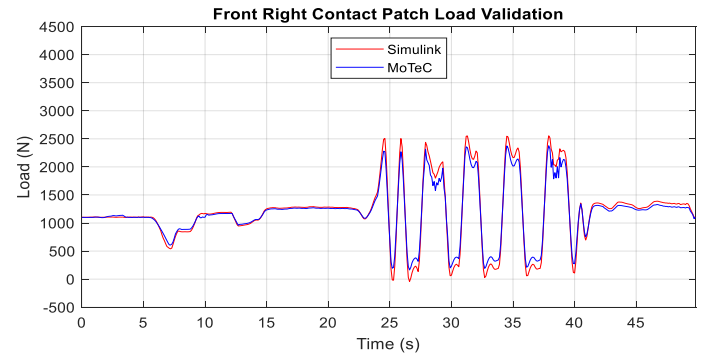
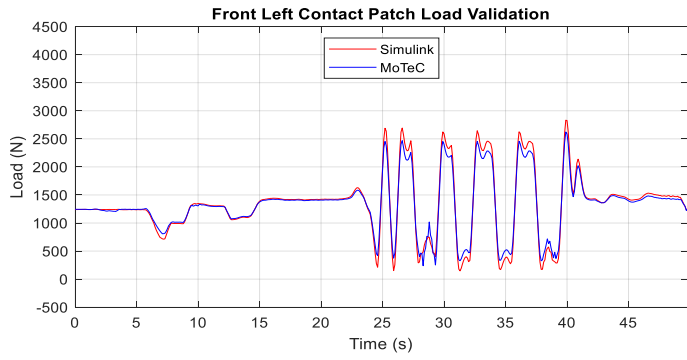


Figure 5.10.1 - Simulator Data Including Vertical Tyre Stiffness

It is clear that the trends throughout the slalom match much more closely compared to figure 5.8.2, proving that this addition to the model increases accuracy significantly. Moreover, the author noticed by analysing the suspension coordinate file (PM file) for the MCR on the simulator, the wheelbase is modelled incorrectly at 3.225m. Once this was changed on the setup sheet, the error in the longitudinal weight transfer was significantly reduced.

5.11 Final Simulink Model Layout

SPORTS 2000 MCR WEIGHT TRANSFER & LOAD DISTRIBUTION CALCULATOR

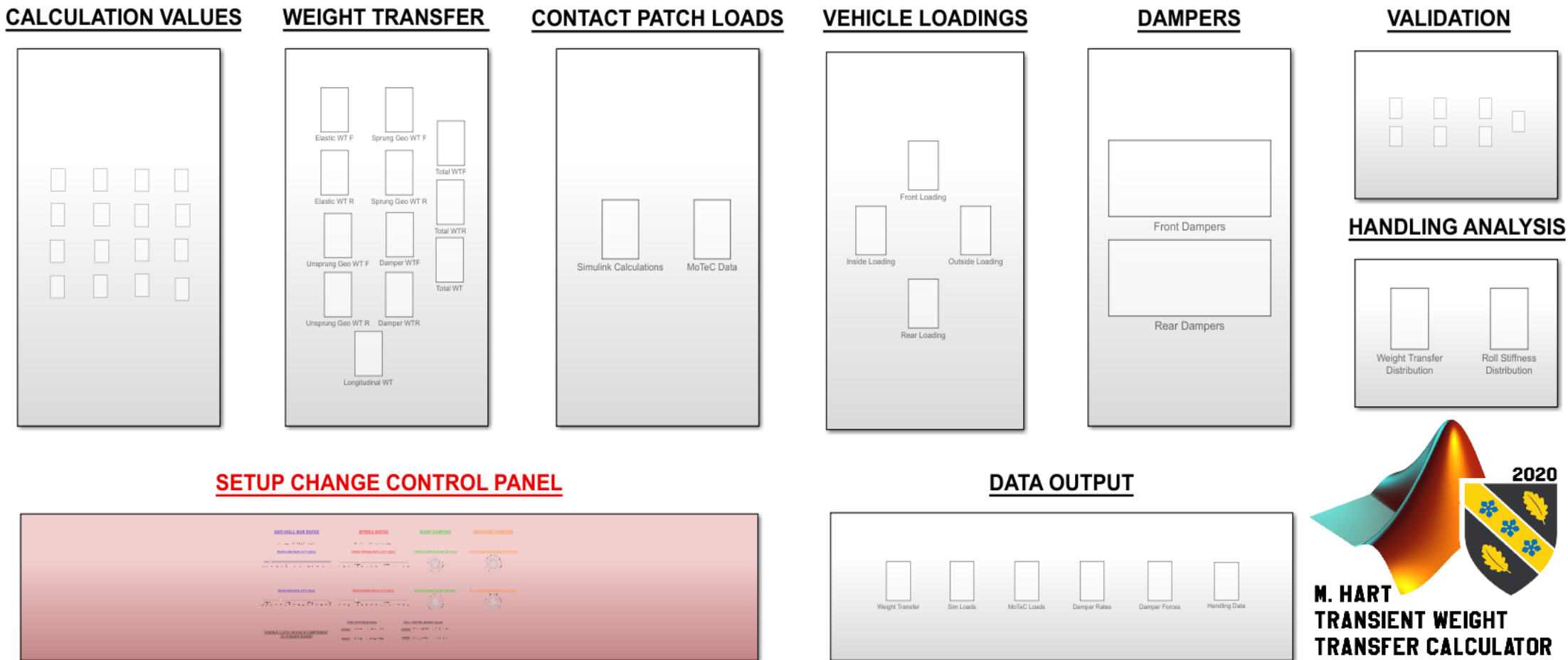


Figure 5.11.1 - Layout of Final Transient Simulink Model

Figure 5.11.1 shows the layout of the Simulink model. Each section of mathematics is grouped in subsystems and labelled accordingly, following accepted modelling and naming conventions (MathWorks, 2020).

5.12 Diagram of Simulink Model Mathematics

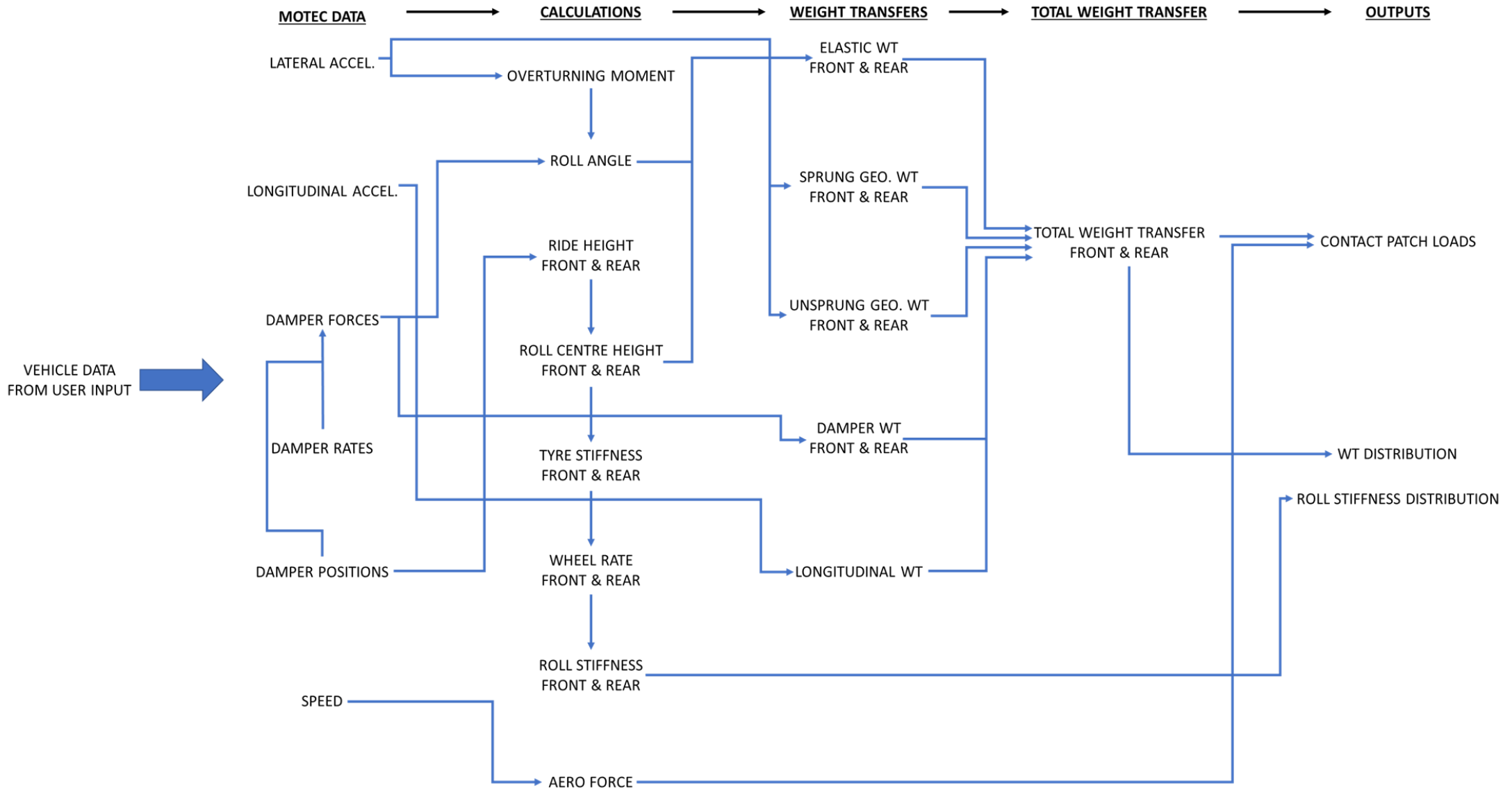


Figure 5.12.1 - Transient Simulink Model Mathematics

Figure 5.12.1 shows how the subsystems within the Simulink model are connected by goto and from blocks.

6 Integration of User-Friendly Features

In the final version of the model, the author included slider and knob components for an easy and intuitive user experience. The value selected by the user is outputted to a constant block to define the required input for the model. A colour coding system makes the interface more simplistic.

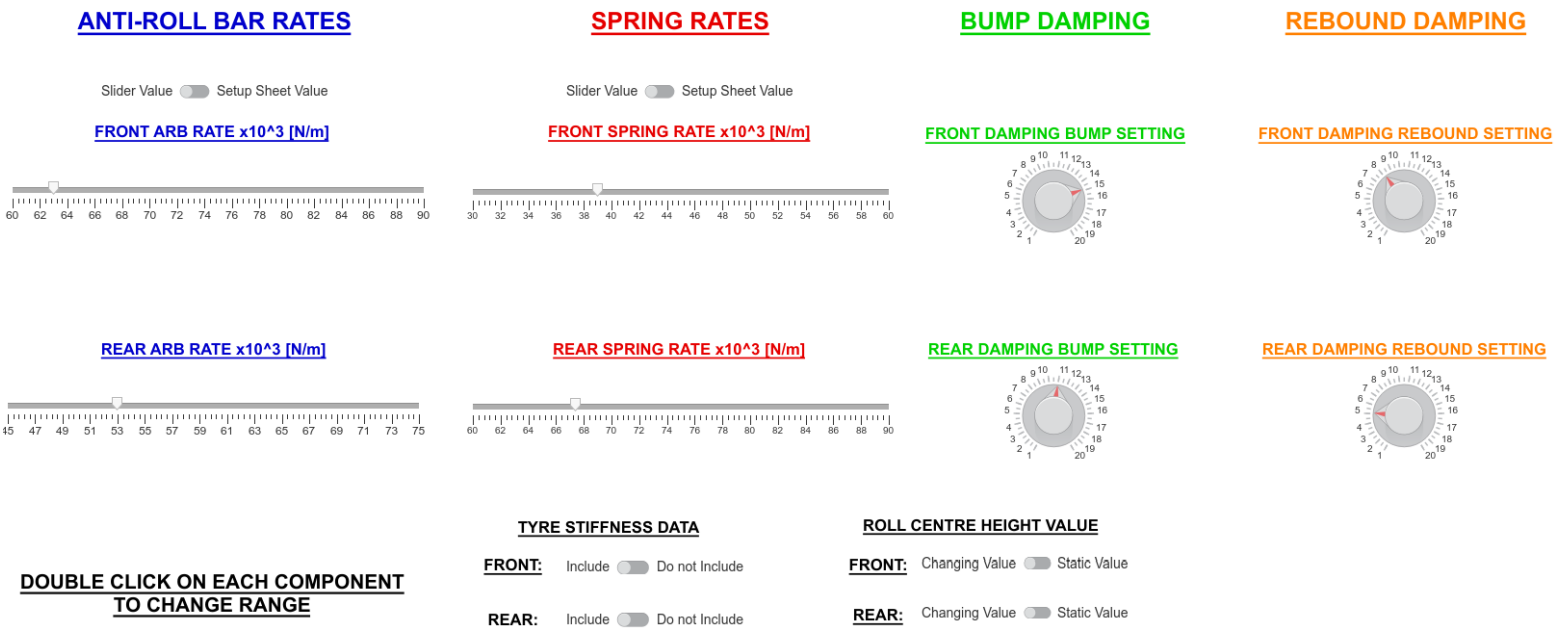


Figure 6.1 - Simulink Model Setup Change Control Panel

As shown in figure 6.1, through GUI components, the user can change ARB and spring rates, and damper settings. The ranges for ARB and spring settings were taken from the simulator. The user can easily change these ranges to suit any vehicle.

To make the model accessible for a variety of applications, the author included switches to give the option of removing various features. These allow the user to turn off the sliders and use ARB and spring rates on the EXCEL setup sheet, or to remove tyre stiffness and changing roll centre height values from the model, as this data may not be available to the user.

The initialisation function generates a reminder to define the data source, as shown in figure 6.2. Moreover, the model also reminds the user to define the calculation stop time.

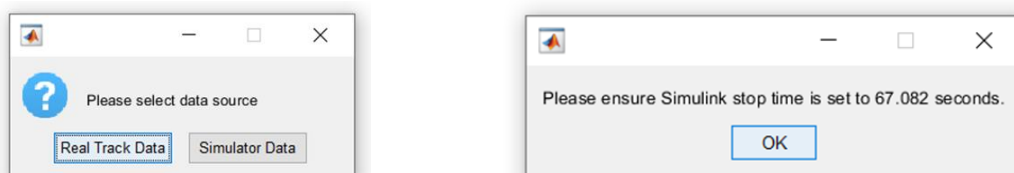


Figure 6.2 - Simulink Model Message Boxes

7 Validation of Results

7.1 Lateral Acceleration Check

Within the real-world data, it is clear that the Simulink calculations are too high at the points of maximum lateral acceleration. To check the lateral acceleration data from the real car, the author re-created the Silverstone qualifying lap as similarly as possible in the simulator to compare the trends. Although a different driver was used, the lap times were within 3% of each other.

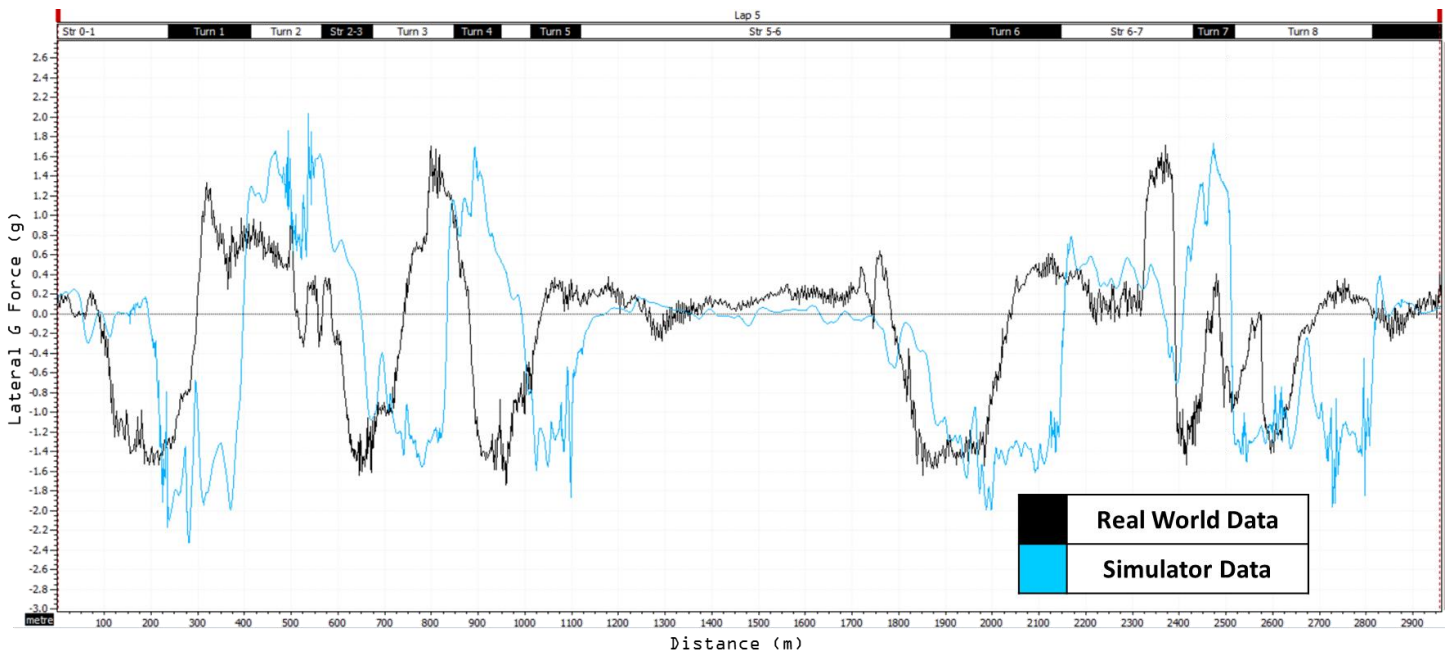


Figure 7.1.1 - Lateral Acceleration Check

Figure 7.1.1 shows a clear difference in driving style between drivers, with much later corner entry on the simulator. However, crucially it shows the magnitude of lateral acceleration to be very similar, validating the data from the real car. This proves that lateral acceleration is not the source of inaccuracy and the error lies elsewhere in the model; this will be investigated in more detail later in the report.

7.2 Silverstone Real-World Data Validation

As mentioned, throughout the project the author has used the real-world data from the Silverstone qualifying session to keep track of data correlation. Figure 7.2.1 shows the trends of the actual MoTeC data, initial Simulink data from the first version of the transient model, and the final Simulink model data. It is clear that throughout the course of the project, the model accuracy improved significantly.

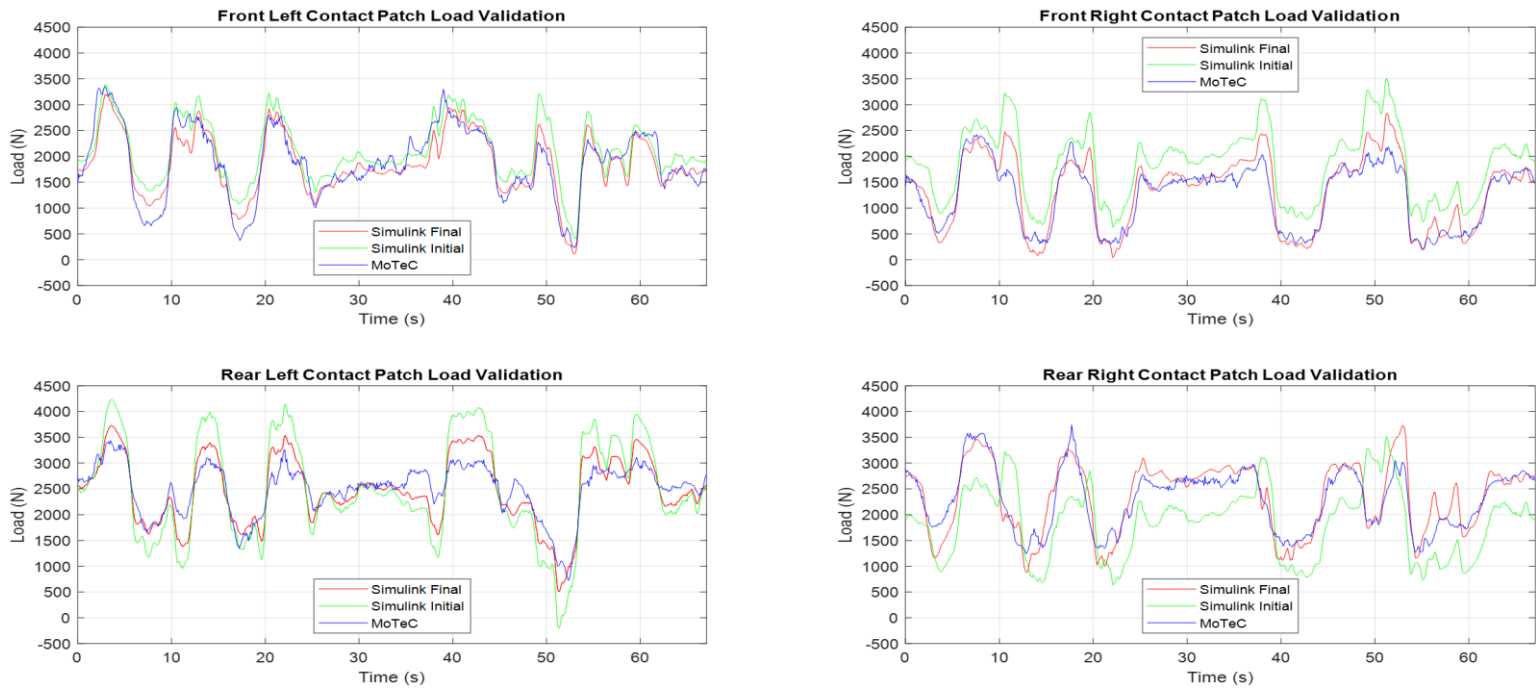


Figure 7.2.1 - Comparison of Initial and Final Model Results

To fully analyse the correlation in trends, the author plotted MoTeC against Simulink contact patch load results. A perfect correlation would produce a straight, 45 degree line; any deviation from this highlights inaccuracy within the model. The graphs are labelled with a $\pm 10\%$ error region; keeping all points within this was the author's aim (Taylor, 2013). Figure 7.2.2 shows this was not achieved with the real-world Silverstone data, however a clear correlation is visible. The values see closest correlation around the area of the static load, this is expected. As the weight transfer increases around the circuit, the correlation becomes worse. In the real life data, this is most likely due to the aforementioned inaccuracy in the pushrod sensors.

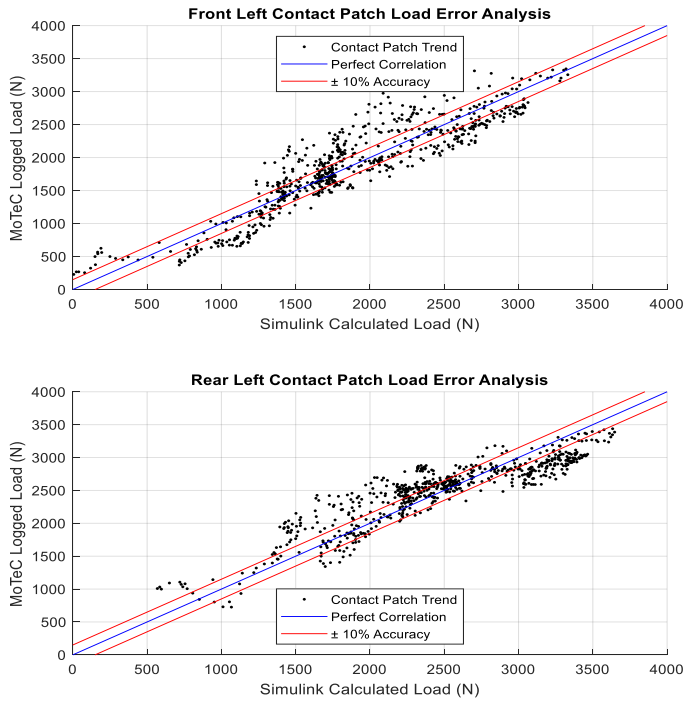


Figure 7.2.2 - Silverstone Data Validation

7.3 Simulator Test Track Data Validation

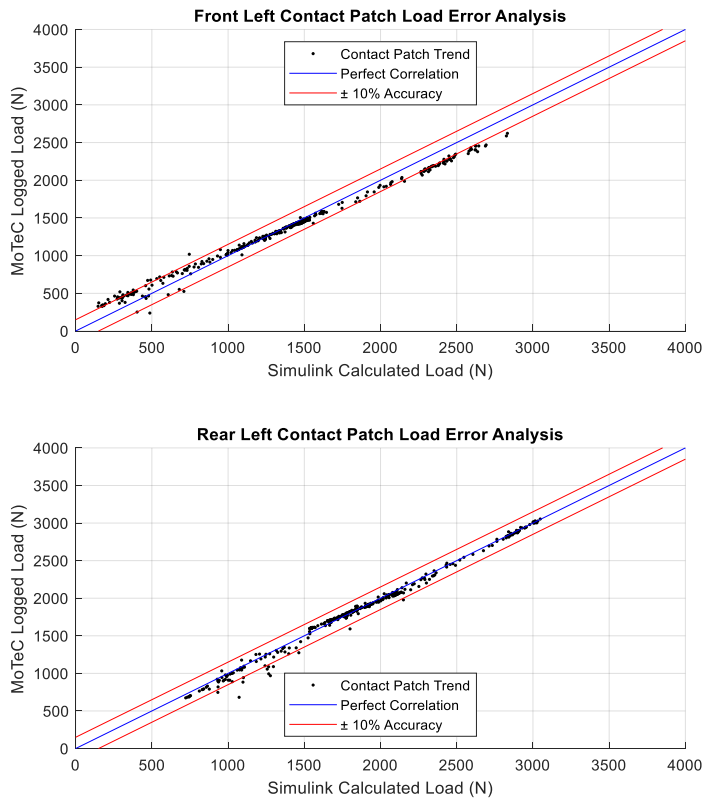


Figure 7.3.1 - Simulator Data Validation

Figure 7.3.1 shows the correlation of data from the simulator test. Without the errors from the pushrod calibration, it is clear that the data correlates much better, and almost all of the data is within the target $\pm 10\%$ error region.

Once more, the points that land furthest from perfect correlation are at areas of highest lateral acceleration, as they correlate to the maximum tyre loads experienced (Dhasarathy *et al.*, 2010). This could be error from elastic weight transfer; wheel rate is used to calculate this. The ARB rates were not fully validated, meaning this could be a source of error within the wheel rate; a wheel rate value that is too large will over-predict lateral weight transfer, thus increasing contact patch loading.

Moreover, roll angle is the other likely source of error. When comparing the calculated roll angle to the MoTeC values, as in figure 7.3.2, it is clear there is inaccuracy. At the start the vehicle is traveling perfectly straight, however, MoTeC is registering a roll angle of around 0.002 radians. The maximum roll angle seen is only 0.02 radians, thus this is an inaccuracy of around 10%, which could explain some of the points that lie outside of the error region, especially at maximum lateral acceleration when the highest roll angle would be seen. This is likely to be due to the asymmetrical lateral weight distribution of the vehicle. The driver, or an offset fuel tank could cause more mass on one side, possibly causing the vehicle to roll slightly. The model's calculations do not take this into account, as the roll angle at zero lateral acceleration should be taken as zero.

The Simulink value could be multiplied by a factor of the error seen; however, this would not be transferable to other vehicles as the error may not be the same. For this reason, and because the error overall makes little difference to the calculated value, this was not implemented.

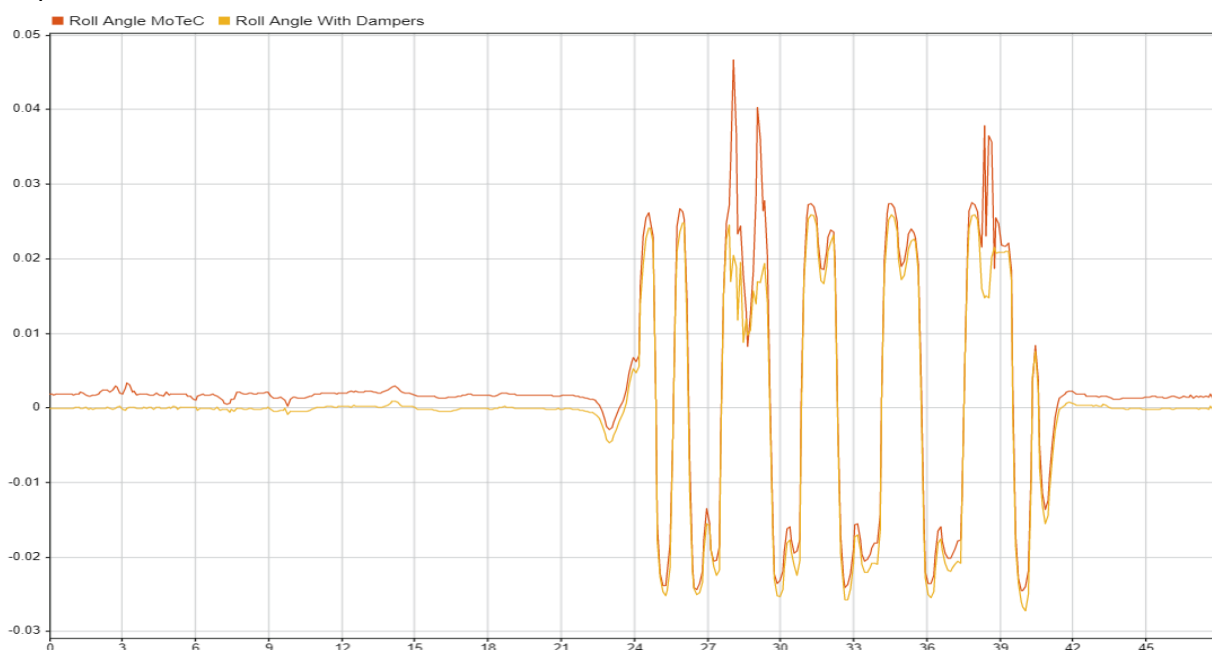


Figure 7.3.2 - Roll Angle Validation

7.4 Further Data Validation

To confirm the model was working as the Silverstone data suggested, the author ran a series of other tests. MoTeC and vehicle data from other sources were inputted into the model. The tests were as follows:

- Brands Hatch Indy qualifying real-world data - Sports 2000 MCR.
- Donington Park simulator data - Sports 2000 MCR.
- Donington Park Simulator data - Aston Martin DBR9 GT1.

The results of the first two tests showed identical trends overall to the Silverstone data, validating findings up until this point. Graphs of this data can be found in appendix 3.

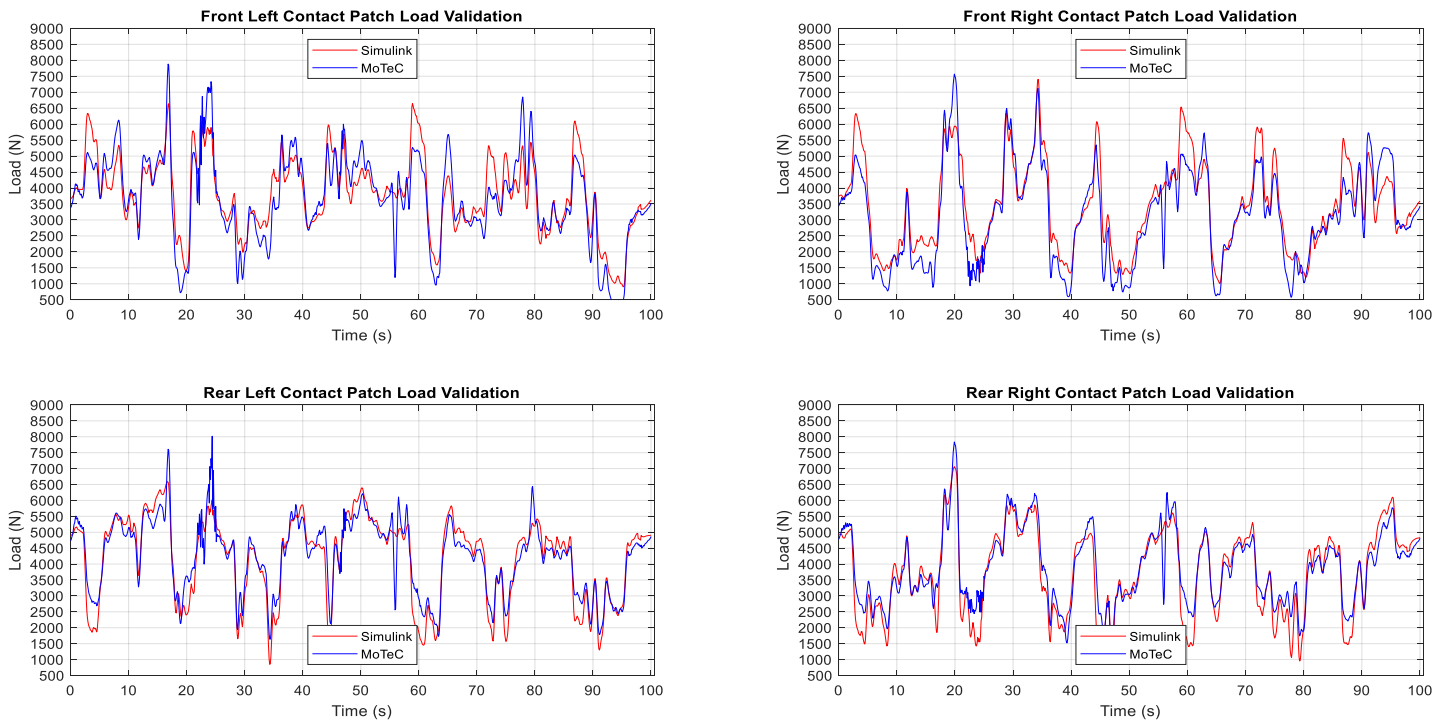


Figure 7.4.1 - Donington Park Aston Martin Data

Figure 7.4.1 shows the contact patch trends for the simulator lap of Donington Park in an Aston Martin DBR1. This validates the model further and shows that the model can be used successfully with a different vehicle. The author did not have changing roll height or tyre stiffness data for this vehicle, but could use the switches to turn these features off, moreover, an existing setup sheet for the car could be used by selecting the model to read data straight from EXCEL, not the GUI sliders.

8 Results Analysis

To simulate analysing various setup changes in a real-world situation, the author ran a series of tests and analysed the vehicle handling characteristics predicted by the model.

8.1 Changing Springs and Anti-Roll Bar Rates

Table 8.1.1 - Changing Roll Stiffness Distribution Results

Test	Front Springs	Rear Springs	Front Arbs	Rear Arbs	Roll Stiffness Distribution (%)	Weight Transfer Distribution (%)
1	Standard	Standard	Standard	Standard	49.48	47.04
2	Max	Min	Standard	Standard	53.34	50.54
3	Min	Max	Standard	Standard	45.44	43.39
4	Standard	Standard	Max	Min	55.21	52.25
5	Standard	Standard	Min	Max	45.98	43.87
6	Max	Min	Max	Min	58.37	55.15
7	Min	Max	Min	Max	42.46	40.71
8	Standard	Standard	Standard	Standard	49.46	47.03

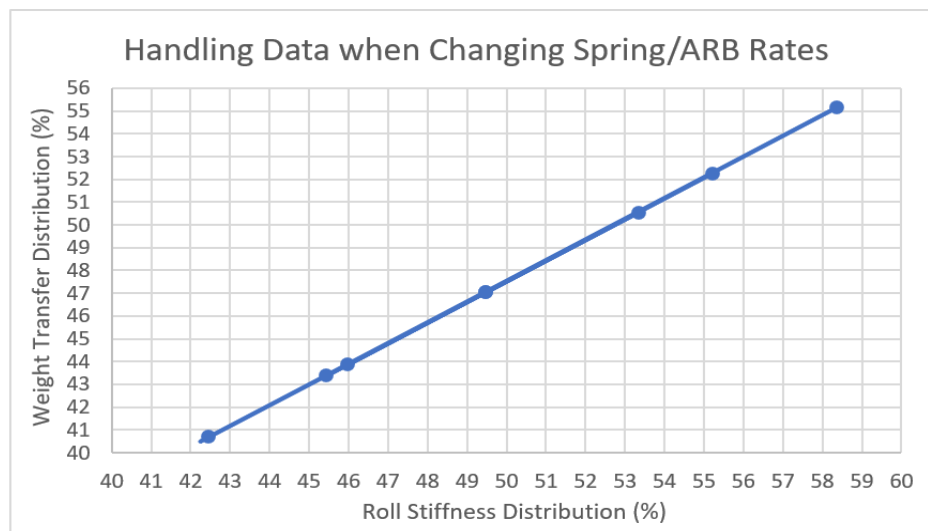


Figure 8.1.1 - Changing Roll Stiffness Distribution Results

Figure 8.1.1 and table 8.1.1 show the trends expected from changing spring and ARB rates. There is a linear, positive correlation between roll stiffness and weight transfer distributions, showing that a stiffer front will result in more front weight transfer, causing more understeer. (Hathaway, 2009; Jazar, 2017). These results are based solely on the model's calculations, validating the mathematics behind it.

8.2 Changing Damper Settings

Table 8.2.1 - Changing Damper Settings Results

<u>Test</u>	<u>Front Bump</u>	<u>Front Rebound</u>	<u>Rear Bump</u>	<u>Rear Rebound</u>	<u>Roll Stiffness Distribution (%)</u>	<u>Weight Transfer Distribution (%)</u>
<u>Standard Damper Rates</u>						
1	Standard	Standard	Standard	Standard	49.47	47.02
2	Max	Max	Min	Min	49.47	47.03
3	Min	Min	Max	Max	49.47	47.02
4	Max	Min	Max	Min	49.47	47.02
5	Min	Max	Min	Max	49.47	47.01
<u>Enlarged Damper Rates</u>						
6	Max x100	Max x100	Min	Min	49.48	48.80
7	Min	Min	Max x100	Max x100	49.48	44.30

Table 8.2.1 shows how dampers do not have a large effect on weight transfer distribution. Damper weight transfer makes up only around 10% of the total weight transfer, thus this does not change the value significantly. Moreover, the effect dampers have on roll angle is not great enough to numerically quantify its effect on weight transfer distribution; dampers do not necessarily affect the magnitude of roll angle, just the rate at which it increases (Kasprzak, 2007; Alexander, 2015). This means the overall weight transfer analysis is not largely affected.

To confirm the mathematics behind the damper's effects on weight transfer within the model, the author increased the damping rates by a factor of 100, thus increasing weight transfer by this much also. As can be seen by tests six and seven, this worked as expected.

8.3 Silverstone Qualifying Data Input

Throughout the weekend at Silverstone last season, various setup changes were implemented following data analysis and driver feedback:

Table 8.3.1 - Silverstone Qualifying Setup Changes

<u>Session</u>	<u>ARB Rate Change</u>	<u>Damper Rate Change</u>
Test 1:	Middle front and rear	Standard all round
Test 2:	Full soft front, full hard rear	Full hard all round
Test 3:	No change	Full hard bump rear, Full soft rebound front
Qualifying:	No Change	Full hard bump and rebound rear. Front bump to baseline (15), increase rebound by 4 to 18. Stiffer damping at rear required.
Quali. pit in:	No Change	Soften front bump by 2, soften rear rebound by 2.

Table 8.3.1 details setup changes that were made in a real-world situation through instinct, validated only by trial and error. This process could be improved by this model as numerical results could back up the team's decisions before the car is sent out.

Table 8.3.2 - Silverstone Qualifying Setup Changes Results

<u>Session</u>	<u>Roll Stiffness Distribution (%)</u>	<u>Weight Transfer Distribution (%)</u>
Test 1:	50.56	48.09
Test 2:	45.96	43.85
Test 3:	45.96	43.85
Qualifying:	45.96	43.90
Quali. pit in:	45.96	43.90

Although damper changes have little to no effect on weight transfer distribution, trends are visible from table 8.3.2. To understand how dampers are affecting vehicle balance further, the user would have to analyse this data through one corner entry or exit, depending on where the car is unbalanced.

9 Limitations and Further Work

This model has been proven to be a useful tool in vehicle setup, however, more elements could be added to increase accuracy, for example caster, toe and camber. The calculations within this model are not greatly affected by these factors, as they relate primarily to tyre dynamics and how this affects contact patch loading. This could be analysed by testing the tyre stiffness rate at various camber and toe angles but may not influence the results hugely in this application.

9.1 Including Chassis Stiffness

A potential source of error within the model is the fact that the vehicle is assumed to have infinite chassis stiffness. This, in reality, is not the case and will bring inaccuracy to the results.

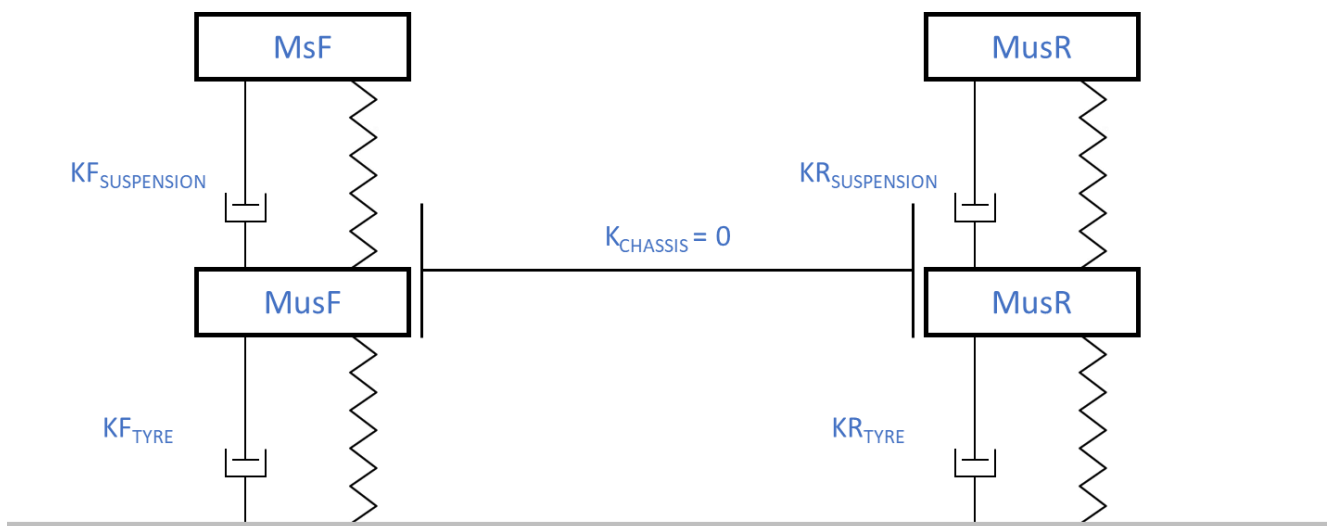


Figure 9.1.1 - Vehicle Spring Rates with Infinite Chassis Stiffness

Figure 9.1.1 shows how the model currently views the car, with an assumed infinite chassis stiffness.

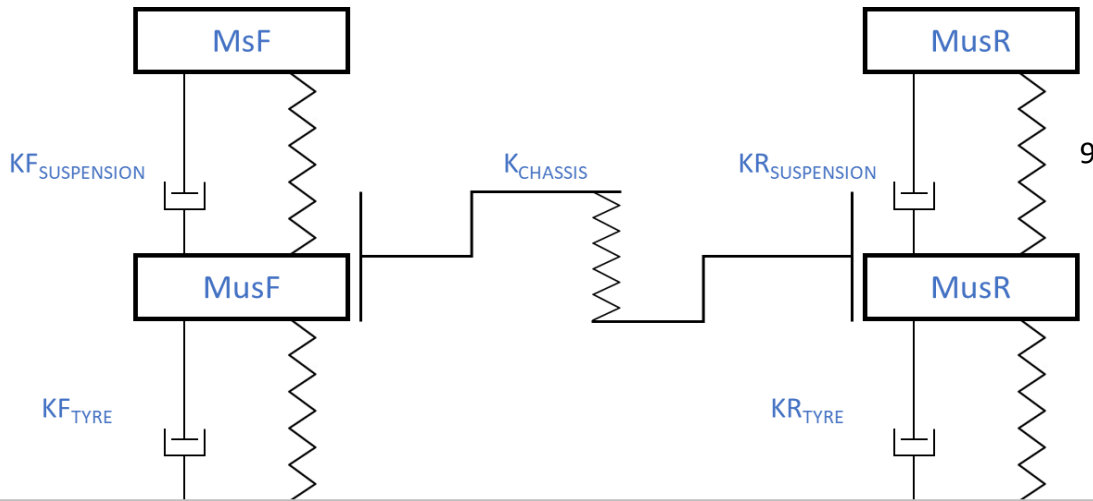


Figure 9.1.2 - Vehicle Spring Rates with Finite Chassis Stiffness (Skovajsa and Sedlacek, 2017)

Figure 9.1.2 shows how chassis stiffness should be included to produce more accurate results. To include this value numerically, equation 5.7.7 (characterising weight transfer in terms of roll stiffness) can be developed. The roll and chassis stiffness values can be seen as springs in series. These equations were sourced from researched reports by Skovajsa and Tripathi. The forces acting through these springs within these equations relate to various data throughout the vehicle:

Table 9.1.1 - Chassis Stiffness Equation Nomenclature

K_F	Front roll stiffness
K_R	Rear roll stiffness
K_T	Total roll stiffness
Z_f	Front roll centre height
Z_r	Rear roll centre height
h_f	Front roll centre height @ CoG
h_r	Rear roll centre height @ CoG
F_w	Front weight distribution
R_w	Rear weight distribution

From this, the weight transfer with finite chassis stiffness can be quantified (Skovajsa and Sedlacek, 2017; Tripathi *et al.*, 2017):

$$WTF = \frac{m}{t} \left(\frac{K_F}{K_T} \cdot R_w \cdot h_f + \frac{K_F}{K_T} \cdot F_w \cdot h_r + R_w \cdot Z_f \right) \cdot a_y \rightarrow \text{Eqn. 1}$$

$$WTR = \frac{m}{t} \left(\frac{K_R}{K_T} \cdot R_w \cdot h_f + \frac{K_R}{K_T} \cdot F_w \cdot h_r + R_w \cdot Z_r \right) \cdot a_y \rightarrow \text{Eqn. 2}$$

Equation 9.1.1 - Weight Transfer Characterised in Terms of Roll Stiffness with Infinite Chassis Stiffness

The h_f and h_r values are the height of roll centres in line with the front and rear centres of gravity. These locations are unknown for the MCR; therefore, the author rearranged the above equations to calculate them. This rearranging can be found in appendix 4.

$$h_f = 0.2724 \quad h_r = 0.4164$$

Knowing these values, the author could calculate the theoretical chassis stiffness of the MCR by inputting MoTeC weight transfer values into the following equation, and back-calculating (Skovajsa and Sedlacek, 2017; Tripathi *et al.*, 2017):

$$WTF_{KC} = \frac{m}{t} \left(\left(\frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_R + K_C}} \right) (Rw)(hf) + \left(\frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}} \right) (Fw)(hr) + (Rw)(Zf) \right) \cdot ay$$

$$WTR_{KC} = \frac{m}{t} \left(\left(\frac{\frac{K_R \cdot K_C}{K_R + K_C}}{K_F + \frac{K_R \cdot K_C}{K_R + K_C}} \right) (Rw)(hf) + \left(\frac{K_R}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}} \right) (Fw)(hr) + (Rw)(Zr) \right) \cdot ay$$

Equation 9.1.2 - Weight Transfer Characterised in Terms of Roll Stiffness with Finite Chassis Stiffness

The chassis stiffness value for the white MCR on a rig was measured as 1850Nm/rad in a recent test (Dowden, 2020). This value is thought to be below the actual value, as the rig deflects, reducing overall stiffness and rigid engine mounts were not in place. Moreover, the yellow MCR has aluminium honeycomb panels bonded to the chassis, increasing stiffness (Kawade and Student, 2015; Chambers *et al.*, 2016). For these reasons, the accepted value for the chassis stiffness is around 2000Nm/rad.

A MATLAB script was used to write out these equations in smaller sections; the chassis stiffness value was originally inputted as 2000Nm/rad and trial and error was used to get the weight transfer values to match MoTeC.

For both front and rear weight transfer, the author calculated the chassis stiffness to be 2287.815Nm/rad. This is only a 13.4% difference, which is small enough to validate the mathematics behind this research; however, for ease of use of the model, and due to the various unvalidated data, the author did not include chassis stiffness.

However, this could easily be included as a separate subsystem that can be turned off if not required, using the following equations to quantify the weight transfer effects solely from chassis stiffness, the derivations of which can be found in appendix 5:

$$\Delta WTF = \left(\frac{m}{t} \cdot ay\right) \cdot \left[(Fw \cdot hf) \left(\frac{K_F}{K_F + K_R} - \frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_R + K_C}} \right) + (Rw \cdot hr) \left(\frac{K_F}{K_F + K_R} - \frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}} \right) \right]$$

$$\Delta WTR = \left(\frac{m}{t} \cdot ay\right) \cdot \left[(Fw \cdot hf) \left(\frac{K_R}{K_F + K_R} - \frac{\frac{K_R \cdot K_C}{K_R + K_C}}{K_F + \frac{K_R \cdot K_C}{K_R + K_C}} \right) + (Rw \cdot hr) \left(\frac{K_F}{K_F + K_R} - \frac{K_R}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}} \right) \right]$$

Equation 9.1.3 - Weight Transfer Solely Due to Chassis Stiffness

9.2 Including Non-Dynamic Steer Induced Weight Transfer

When the front wheels of a vehicle steer about an inclined steer axis, a diagonal weight transfer occurs (Tudor and Tudor, 2016). As the vehicle is steered, the contact patches move vertically, relative to the chassis, asymmetrically causing the front inside wheel to be loaded, resulting in a roll torque across the steering axle. As the vehicle corners, the outside wheels are loaded. These two events cause the front inside wheel to be loaded, whilst the rear outside wheel is simultaneously loaded; thus, a diagonal weight transfer (Milliken and Milliken, 1995; Tudor and Tudor, 2016).

This phenomenon works in the opposite direction to weight transfer at the front, and with it at the rear. Results show the model over-calculates at the front contact patches and under-calculates at the rear, at the points where the highest steer angle is seen. Including steer-induced weight transfer could help to reduce this error and improve the accuracy of the model.

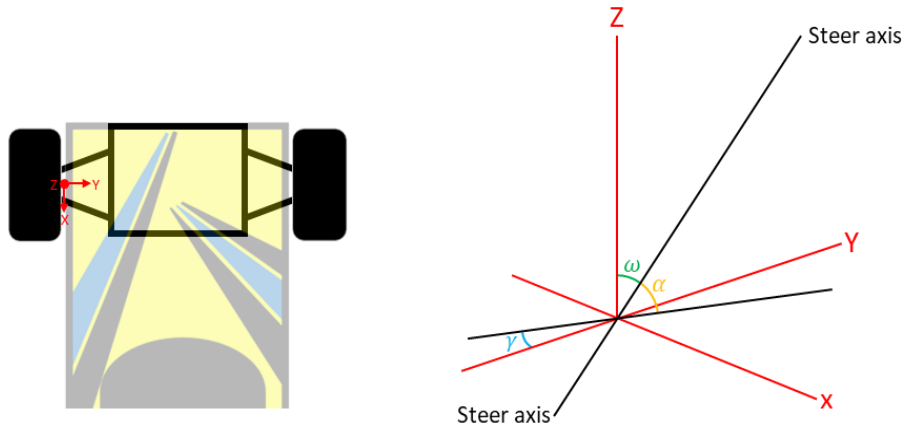


Figure 9.2.1 - Steering Calculations Coordinate System (Tudor and Tudor, 2016).

To calculate steer-induced weight transfer, a cartesian coordinate system with origin at the bottom of the steer axis can be created as shown in figure 9.2.1 for the outside wheel, and similarly for the inside. These coordinates are linked by ω (angle of steering axis to vertical) and α , the complementary angle. From datapoints along these coordinates and knowing that $\theta = 90^\circ$ - caster angle, and $\phi = 90^\circ$ - king pin inclination, the following calculations can be made. Components of the following equations correspond to figure 9.2.1.

$$\sin \alpha = \frac{1}{\sqrt{\frac{x^2}{2} + \frac{y^2}{2} + 1}} \quad \therefore \alpha = \sin^{-1} \frac{1}{\sqrt{\frac{x^2}{2} + \frac{y^2}{2} + 1}}$$

$$\omega = 90 - \sin^{-1} \frac{1}{\sqrt{\frac{1}{\tan^2 \theta} + \frac{1}{\tan^2 \phi} + 1}}$$

$$\gamma = \tan^{-1} \frac{\tan \phi}{\tan \theta}$$

Equation 9.2.1 - Steering Geometry Equations

From here, front and rear chassis roll with respect to suspension planes can be found:

$$\theta_F = \alpha - \frac{\alpha}{\frac{K_R}{K_F} + 1} \quad \theta_R = \frac{\alpha}{\frac{K_R}{K_F} + 1}$$

Equation 9.2.2 - Chassis Roll Equations

From this, steer-induced weight transfer is quantified as:

$$WTF = \frac{KW_F \cdot \tan \theta_F \cdot T_F}{2} \quad WTR = \frac{KW_R \cdot \tan \theta_R \cdot T_R}{2}$$

Equation 9.2.3 - Steer-induced Weight Transfer

9.3 General Improvements

To improve the model, the author would add into future versions the possibility of the user inputting a desired vehicle balance at a given section of track, and receiving a suggested setup change to reflect this, considering what is most simple to change in real-time, in a real-world application. The model could also sweep every available setup and plot the results from each, allowing the user to visually compare what works better for any given scenario. Both of these additions would be simple to add, using code within the initialisation and stop functions to analyse data within the Simulink model, log results, change the setup accordingly, and log a new set of results.

To improve the usability of the model further, the author could research the possibility of exporting through C++ code to create a standalone application. This, however, was not researched in detail due to the wide use of MATLAB and Simulink in industry.

10 Conclusion

Developing a vehicle setup characterisation model offered the chance to explore and develop a vast array of skills and fully consolidate understanding of the basic principles of vehicle dynamics. The work completed provides a good platform to move onto a more specialised masters project, within which the author can use experience gained to model more advanced elements of vehicle dynamics.

Throughout this project, the author took the proven, basic equations of weight transfer and applied them in a quasi-steady state fashion, which made for good correlation with simulator and real-world data from MoTeC. Combining the model's results with the author's knowledge and further research of the principles of centres of weight transfer and roll stiffness, a reliable and accurate vehicle setup characterisation model was produced. The inclusion of transient data analysis such as tyre stiffness and effects of dampers has further improved the accuracy of results. The author gained a thorough insight into damper behaviour and function through numerical analysis. The iterative loop defining damper effect on roll very clearly demonstrates, in real time, what dampers do within the vehicle and the effect this has.

The main limitation faced throughout the project was the inaccurate correlation of data from pushrod sensors. This source of error frequently suggested that the model's calculations were incorrect. However, this was rectified by comparing results to simulator data, where the trends were much closer. This, and a few other more minor inaccuracies, did not reduce the validity of the model produced. It is still fully trustworthy; results from this model were continually validated throughout the project and proven to show a clear correlation with actual data, the trends becoming much closer as the project progressed. It has been proven that the model produced is a useful tool not only for developing understanding, but in a practical, trackside application also

11 Pandemic Impact

Due to the COVID-19 global pandemic, the university was closed to students from 16/03/2020.

This mainly impacted the project by rendering the author unable to carry out coordinate measurement for the MCR's steering geometry, as detailed in section 9.2. Due to this, the effect of weight transfer from steer angle could not be implemented within the model.

Also, the author was unable to fully validate and further test chassis stiffness of the MCR. Due to this, results could not be validated and thus were also not included in the model.

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13 Appendices

13.1 Appendix One - Setup Sheet Research

Chassis Setup Sheet

DATE	TRACK	DRIVER	CAR	WEATHER	AIR TEMP	AIR DENSITY %
SWAY BAR	SWAY PRELOAD	DIAMETER	TOE	F. SPOILER HEIGHT	R. SPOILER ANGLE	

Weight			
Spindle			
Spring Rate			
Shock	C	R	P
Length			
Static Load			
Measurement (Body to Adj Nut)			
Pressure			
Bump Stop			
Load			
Shims			
Spring Rubbers			
Ride Height Before Bar			
Ride Height After Bar			
Caster			
Camber			
Bump Steer			
Ackerman			
A-Arm Length			



Weight			
Spindle			
Spring Rate			
Shock	C	R	P
Length			
Static Load			
Measurement (Body to Adj Nut)			
Pressure			
Bump Stop			
Load			
Shims			
Spring Rubbers			
Ride Height Before Bar			
Ride Height After Bar			
Caster			
Camber			
Bump Steer			
Ackerman			
A-Arm Length			

Weight			
Spring Rate			
Shock	C	R	P
Length			
Static Load			
Measurement (Body to Adj Nut)			
Pressure			
Bump Stop			
Load			
Shims			
Spring Rubbers			
Ride Height Before Bar			
Ride Height After Bar			
Camber			
Trailing Arm			
Frame to Rotor			
LR to LF			



Weight			
Spring Rate			
Shock	C	R	P
Length			
Static Load			
Measurement (Body to Adj Nut)			
Pressure			
Bump Stop			
Load			
Shims			
Spring Rubbers			
Ride Height Before Bar			
Ride Height After Bar			
Camber			
Trailing Arm			
Frame to Rotor			
RR to RF			

Figure 13.1.1 - Longacre Racing Setup Sheet (Longacre Racing, 2016)

Chassis Setup Sheet

TIRE - COMP.	PRESSURE	SIZE	FRONT STAGGER	SIZE	PRESSURE	TIRE - COMP.
	Cold Hot	Cold Hot	Cold Hot	Cold Hot	Cold Hot	
	Cold Hot	Cold Hot	Cold Hot	Cold Hot	Cold Hot	
	Cold Hot	Cold Hot	Cold Hot	Cold Hot	Cold Hot	

TRACK BAR	REAR END	GEARS	FUEL
Rear End Side "	Pinion Angle	Gear Ratio	Laps
Frame Side "	3rd Link Angle	Gear Set #	Gal. Used
Split "	F. Height of 3rd Link	RPM	MPG
Length "	R. Height of 3rd Link		LPG

NOTES/TIMES

Figure 13.1.1 - Longacre Racing Setup Sheet (Longacre Racing, 2016)

Figure 13.1.1 shows an example of a researched setup sheet. As well as proving useful to check data, the idea of splitting data into corners of the car, as well as a large notes section was implemented by the author.

13.2 Appendix Two - Initial Tyre Vertical Stiffness Data

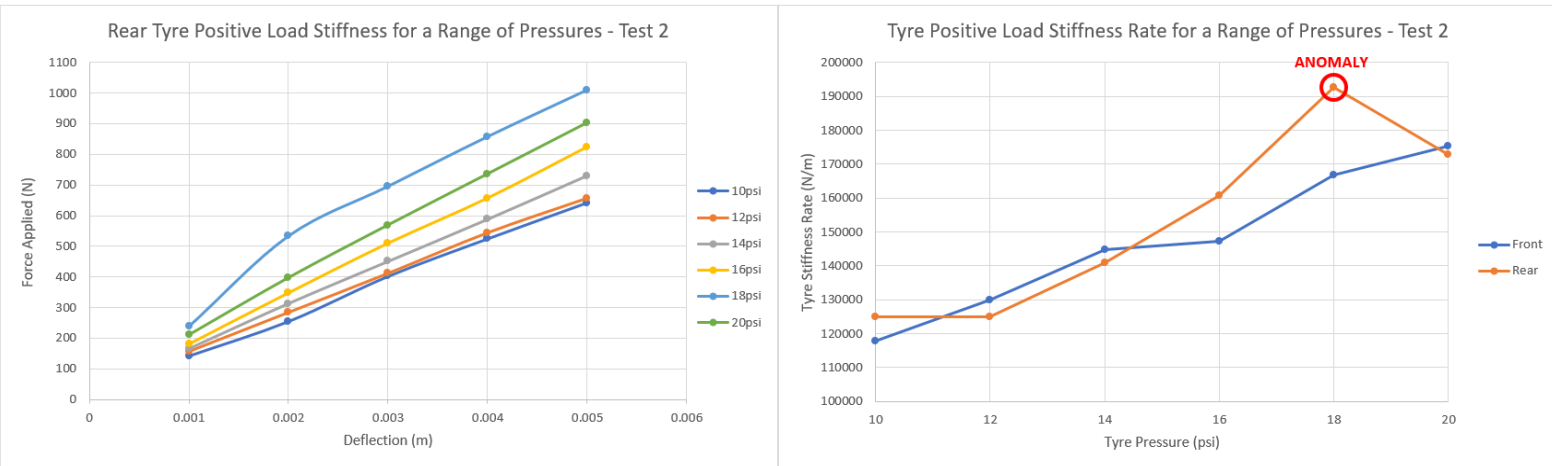


Figure 13.2.1 - Initial Vertical Tyre Stiffness Test Data

Figure 13.2.1 shows a clear anomaly at 18psi. Due to this, the tests were re-run for validation. The data across all three tests correlated well, enabling the author to trust the accuracy of these results.

13.3 Appendix Three - Extra Validation Tests

13.3.1 Brands Hatch Qualifying Real-World Contact Patch Load Comparison

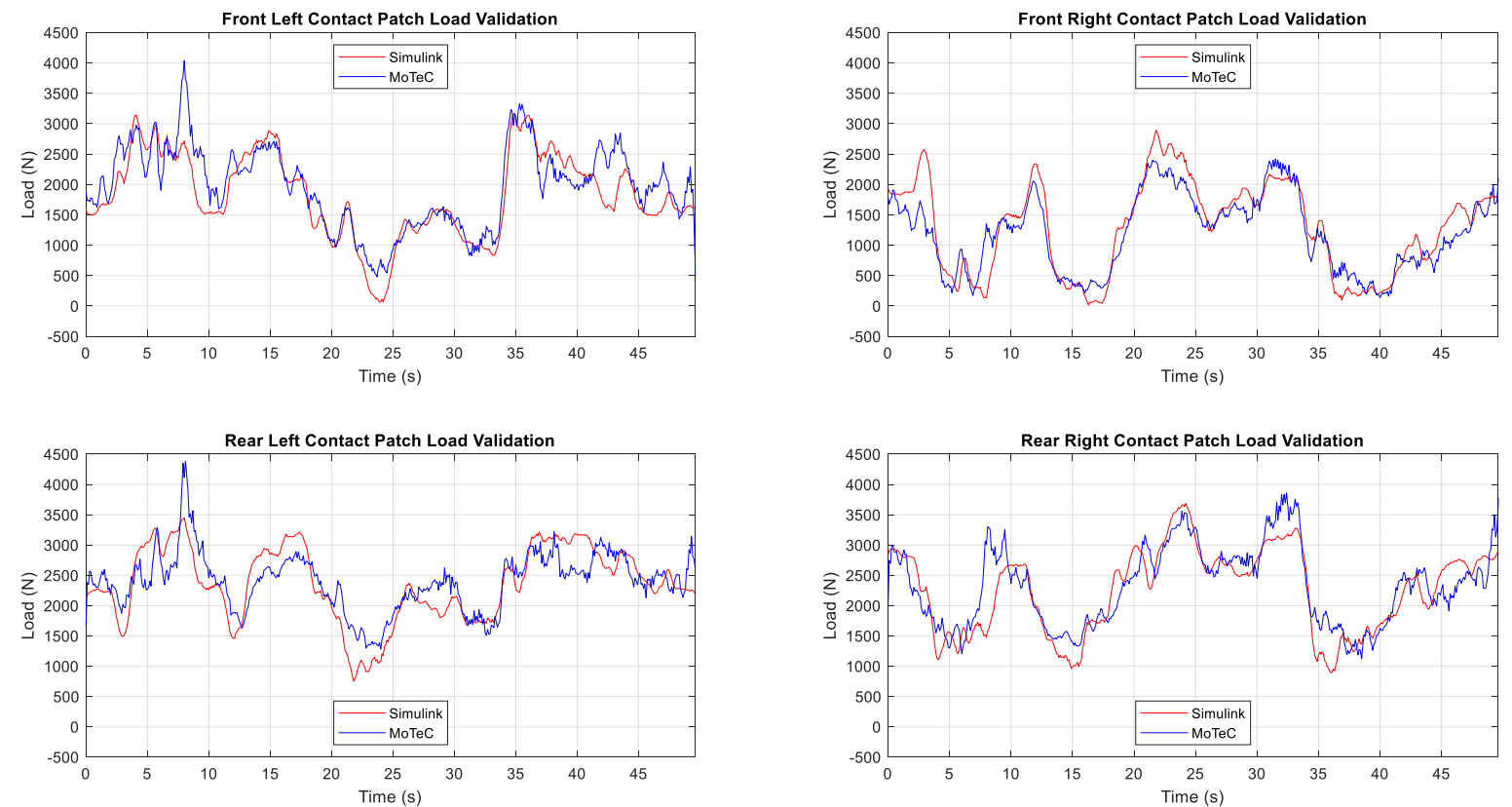


Figure 13.3.1 - Brands Hatch Qualifying Real-World Contact Patch Load Comparison

13.3.2 Brands Hatch Qualifying Real-World Contact Patch Load Validation

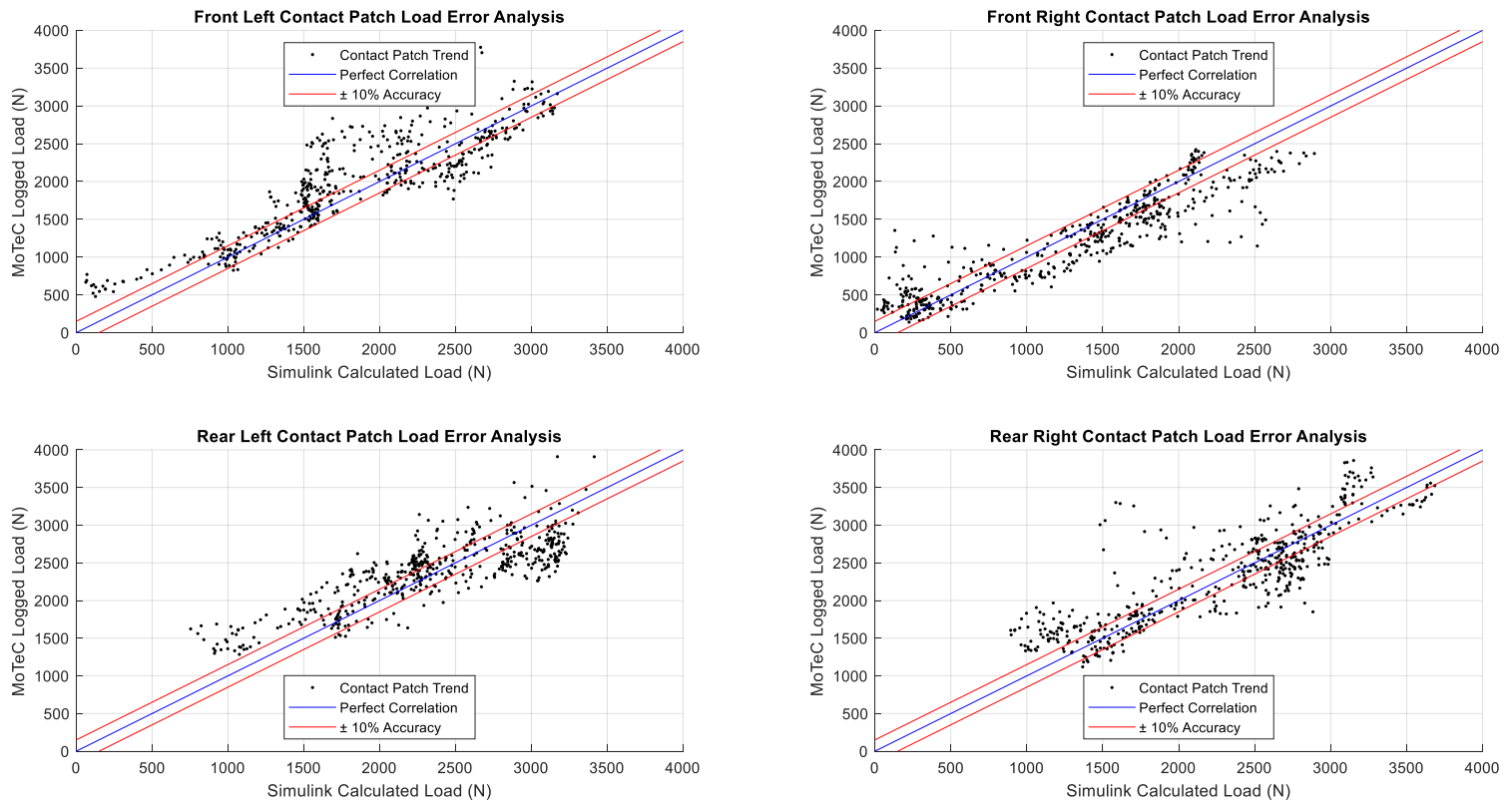


Figure 13.3.2 - Brands Hatch Qualifying Real-World Contact Patch Load Validation

Figures 13.3.1 and 13.3.2 show very similar correlation between Simulink and MoTeC results, compared to the Silverstone data. This proves that the areas where the trends follow accurately are not a one-off and are due to correctly calculated data. This data also shows the same errors in the same areas as the Silverstone data, again suggesting that the pushrod sensor calibration may be a significant source of error.

13.3.3 Donington Park Simulator Real-World Contact Patch Load Comparison

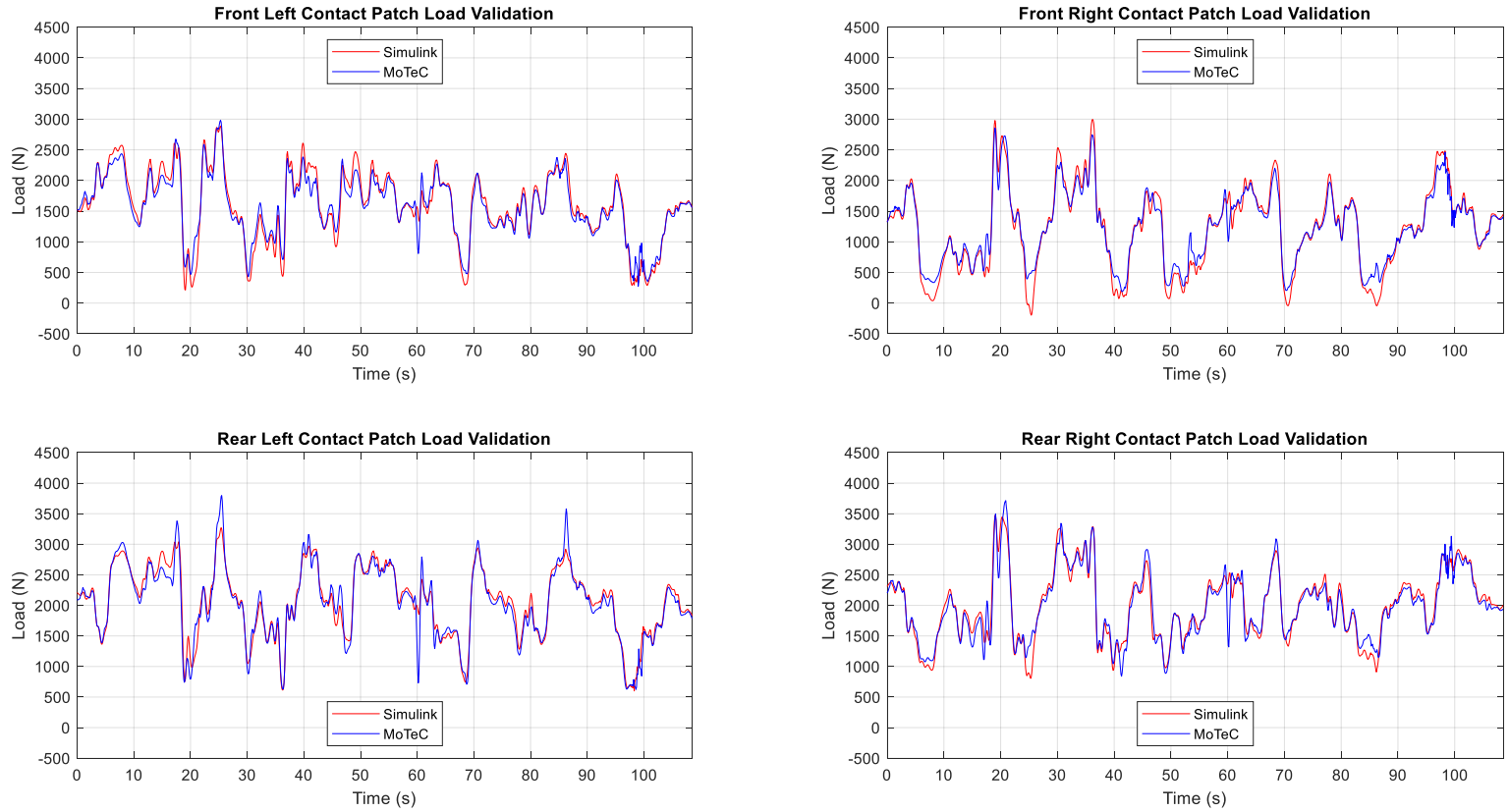


Figure 13.3.3 - Donington Park Simulator Real-World Contact Patch Load Comparison

13.3.4 Donington Park Simulator Real-World Contact Patch Load Validation

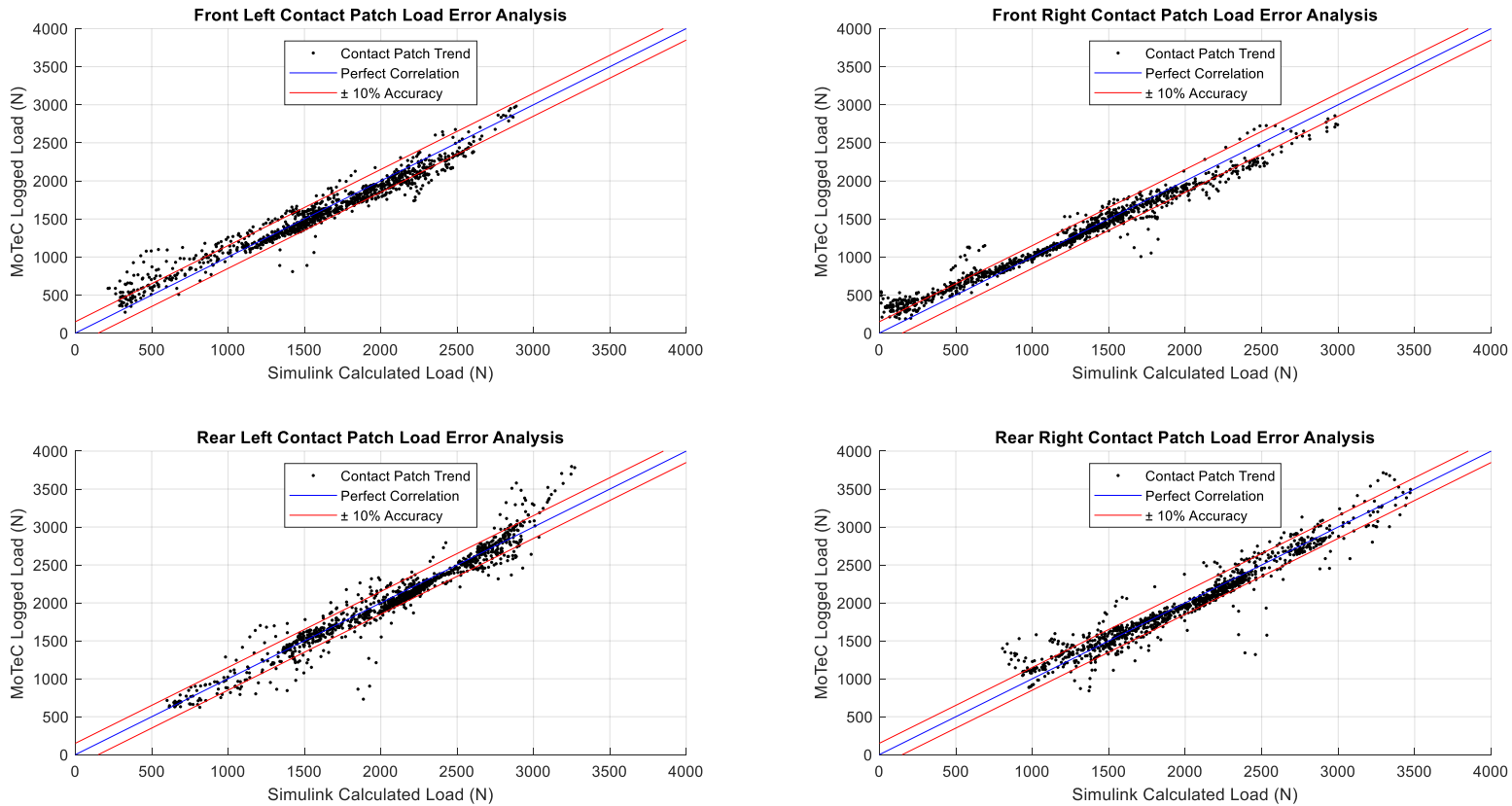


Figure 13.3.4 - Donington Park Simulator Real-World Contact Patch Load Validation

Figures 13.3.3 and 13.3.4 show trends that match the correlation of the other simulator data. The trends are much closer, again suggesting the inaccuracy within the real-world data is down to the error in pushrod sensor calibration. Generally, the data lies within the author's targeted 10% margin of error, showing validity within the mathematics used.

These figures prove that the model can work with other vehicles to the same degree of accuracy as it has on the MCR, even when dynamic roll centre height and tyre stiffness data is unavailable. This was important to clarify, as it was the author's intention from the start to ensure this model is useful in a variety of applications.

13.4 Appendix Four - Derivation of hr and hf Values

Rearrange equation 1 for hf:

$$\begin{aligned}
 WTF &= \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf + \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot F_w \cdot hr + \frac{m}{t} \cdot R_w \cdot Z_f \cdot ay \\
 \therefore \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf &= WTF - \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot F_w \cdot hr - \frac{m}{t} \cdot R_w \cdot Z_f \cdot ay \\
 \therefore hf &= \frac{WTF - \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot F_w \cdot hr - \frac{m}{t} \cdot R_w \cdot Z_f \cdot ay}{\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w} \quad \therefore hf = \frac{WTF - \frac{m}{t} \left(\frac{K_F}{K_T} \cdot F_w \cdot hr - R_w \cdot Z_f \cdot ay \right)}{\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w} \rightarrow \text{Eqn. 3}
 \end{aligned}$$

Follow same steps to rearrange equation 2 for hr:

$$hr = \frac{WTR - \frac{m}{t} \left(\frac{K_R}{K_T} \cdot R_w \cdot hf - F_w \cdot Z_r \cdot ay \right)}{\frac{m}{t} \cdot \frac{K_R}{K_T} \cdot F_w} \rightarrow \text{Eqn. 4}$$

Substitute equation 4 into equation 3 and solve for hf

$$\begin{aligned}
 hf &= \frac{WTF - \frac{m}{t} \left(\frac{K_F}{K_T} \cdot F_w \cdot \frac{WTR - \frac{m}{t} \left(\frac{K_R}{K_T} \cdot R_w \cdot hf - F_w \cdot Z_r \cdot ay \right)}{\frac{m}{t} \cdot \frac{K_R}{K_T} \cdot F_w} - R_w \cdot Z_f \cdot ay \right)}{\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w} \\
 \therefore \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf &= WTF - \frac{m}{t} \left(\frac{K_F}{K_T} \cdot F_w \cdot \frac{WTR - \frac{m}{t} \left(\frac{K_R}{K_T} \cdot R_w \cdot hf - F_w \cdot Z_r \cdot ay \right)}{\frac{m}{t} \cdot \frac{K_R}{K_T} \cdot F_w} - R_w \cdot Z_f \cdot ay \right) \\
 \therefore \frac{m^2}{t} \left(\frac{K_F}{K_T} \cdot R_w \cdot hf \cdot \frac{K_R}{K_T} \cdot F_w \right) &= WTF - \frac{m}{t} \cdot \left(\frac{K_F}{K_T} \cdot F_w \cdot WTR - \frac{m}{t} \cdot \frac{K_R}{K_T} \cdot R_w \cdot hf - F_w \cdot Z_r \cdot ay \right) - R_w \cdot Z_f \cdot ay \\
 \therefore \frac{m^2}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf \cdot \frac{K_R}{K_T} \cdot F_w + \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot F_w \cdot WTR - \frac{m^2}{t} \cdot \frac{K_R}{K_T} \cdot R_w \cdot hf - \frac{m^2}{t} \cdot F_w \cdot Z_r \cdot ay &= WTF - R_w \cdot Z_f \cdot ay \\
 \therefore \frac{m}{t} \left(\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf \cdot \frac{K_R}{K_T} \cdot F_w + \frac{K_F}{K_T} \cdot F_w \cdot WTR - \frac{m}{t} \cdot \frac{K_R}{K_T} \cdot R_w \cdot hf - \frac{m}{t} \cdot F_w \cdot Z_r \cdot ay \right) &= WTF - R_w \cdot Z_f \cdot ay \\
 \therefore \frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot hf \cdot \frac{K_R}{K_T} - \frac{m}{t} \cdot \frac{K_R}{K_T} \cdot R_w \cdot hf &= \frac{WTF - R_w \cdot Z_f \cdot ay}{\frac{m}{t}} - \frac{K_F}{K_T} \cdot F_w \cdot WTR + \frac{m}{t} \cdot F_w \cdot Z_r \cdot ay \\
 \therefore hf \left(\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot \frac{K_R}{K_T} - \frac{m}{t} \cdot \frac{K_R}{K_T} \cdot R_w \right) &= \frac{WTF - R_w \cdot Z_f \cdot ay}{\frac{m}{t}} - \frac{K_F}{K_T} \cdot F_w \cdot WTR + \frac{m}{t} \cdot F_w \cdot Z_r \cdot ay \\
 \therefore hf &= \frac{\frac{WTF - R_w \cdot Z_f \cdot ay}{\frac{m}{t}} - \frac{K_F}{K_T} \cdot F_w \cdot WTR + \frac{m}{t} \cdot F_w \cdot Z_r \cdot ay}{\left(\frac{m}{t} \cdot \frac{K_F}{K_T} \cdot R_w \cdot \frac{K_R}{K_T} - \frac{m}{t} \cdot \frac{K_R}{K_T} \cdot R_w \right)} \quad \therefore hf = 0.2724
 \end{aligned}$$

Substitute this value into equation 4:

$$hr = 0.4164$$

13.5 Appendix Five - Equation for Weight Transfer Due to Chassis Stiffness

$WTF = \text{Front WT for infinite chassis stiffness}$

$WTF_{KC} = \text{Front WT for finite chassis stiffness}$

$$\therefore \Delta WTF = WTF - WTF_{KC}$$

$$\therefore \Delta WTF = \left(\frac{m}{t} \cdot ay\right) \cdot \left[\left(\frac{K_F}{K_F + K_R}\right)(Fw)(hf) + \left(\frac{K_F}{K_F + K_R}\right)(Rw)(hr) + (Fw)(Zf)\right] - \left(\frac{m}{t} \cdot ay\right) \cdot \left[\left(\frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_F + K_C}}\right)(Fw)(hf) + \left(\frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}}\right)(Rw)(hr) + (Fw)(Zf)\right]$$

$$\therefore \Delta WTF = \left(\frac{m}{t} \cdot ay\right) \cdot \left[\left(\left(\frac{K_F}{K_F + K_R}\right)(Fw)(hf) + \left(\frac{K_F}{K_F + K_R}\right)(Rw)(hr) + (Fw)(Zf)\right) - \left(\left(\frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_F + K_C}}\right)(Fw)(hf) + \left(\frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}}\right)(Rw)(hr) + (Fw)(Zf)\right)\right]$$

$$\therefore \Delta WTF = \left(\frac{m}{t} \cdot ay\right) \cdot \left[\left(\left(\frac{K_F}{K_F + K_R}\right)(Fw)(hf) + \left(\frac{K_F}{K_F + K_R}\right)(Rw)(hr)\right) - \left(\left(\frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_F + K_C}}\right)(Fw)(hf) + \left(\frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}}\right)(Rw)(hr)\right)\right]$$

$$\therefore \Delta WTF = \left(\frac{m}{t} \cdot ay\right) \cdot \left[(Fw \cdot hf) \left(\frac{K_F}{K_F + K_R} - \frac{K_F}{K_F + \frac{K_R \cdot K_C}{K_F + K_C}}\right) + (Rw \cdot hr) \left(\frac{K_F}{K_F + K_R} - \frac{\frac{K_F \cdot K_C}{K_F + K_C}}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}}\right)\right]$$

Using the same derivation as above for WTR:

$$\therefore \Delta WTR = \left(\frac{m}{t} \cdot ay\right) \cdot \left[(Fw \cdot hf) \left(\frac{K_R}{K_F + K_R} - \frac{\frac{K_R \cdot K_C}{K_F + K_C}}{K_F + \frac{K_R \cdot K_C}{K_F + K_C}}\right) + (Rw \cdot hr) \left(\frac{K_F}{K_F + K_R} - \frac{K_R}{K_R + \frac{K_F \cdot K_C}{K_F + K_C}}\right)\right]$$