

MECHANIO

ADVANCED ENGINE DESIGN.

LECTURE NOTES FOR 17. MARCH.

ENGINE MOUNTING & FLEXIBLE TRANSMISSION THROUGH THE MOUNTS.

A TYPICAL INTERNAL

COMBUSTION ENGINE GENERATES FIRST AND SECOND ORDER INERTIAL FORCES. THE FIRST ORDER ARE USUALLY BRANCHED OUT ON THE CHASSIS, BUT OFTEN SECOND SECOND ORDER FORCES ARE LEFT UNBRANCHED.

IF YOU HAVE A SINGLE

CYLINDER ENGINE MOUNTED VERTICALLY IN A CAR, THE REACTION AND ~~REACTION~~ CYCLIC FORCES LEAD TO

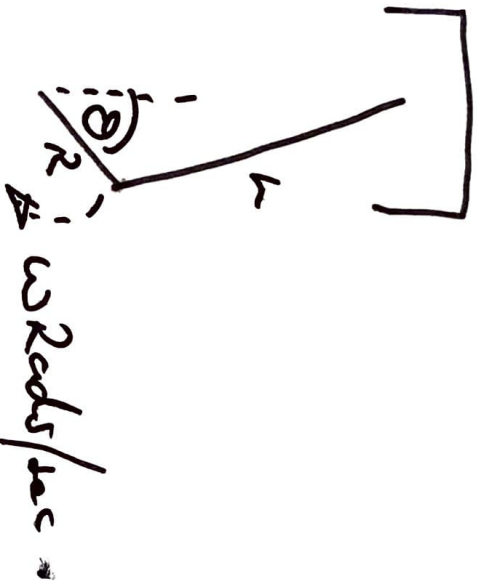
2.

The following vectors:

1. First axle:

$$F = m_{ac} \cdot R \omega^2 \cos \theta.$$

①



2. Second axle:

$$F = R \omega^2 \cos 2\theta.$$

②

In an inline 4 cylinder engine with a single plane crankshaft, the first order forces cancel but the second order forces add up to a total of:

3.

$$F_{\text{Total secondary order}} = 4 \cdot \frac{R}{L} \cdot M_{\text{rec}} R \omega^2 \cos 2\theta$$

————— (3)

$$\left(\begin{array}{l} \text{GIVEN THAT} \\ \cos 2(\theta + 180) \\ = \cos (2\theta + 360) \\ = \cos 2\theta \end{array} \right).$$

IN AN - IN LINE 2 CYLINDER WITH
BOTH CRANKS AT TDC TOGETHER
THE RESULTANT secondary order force
is :

$$F_{\text{Total secondary order}} = \frac{2R}{L} M_{\text{rec}} R \omega^2 \cos 2\theta.$$

————— (4)

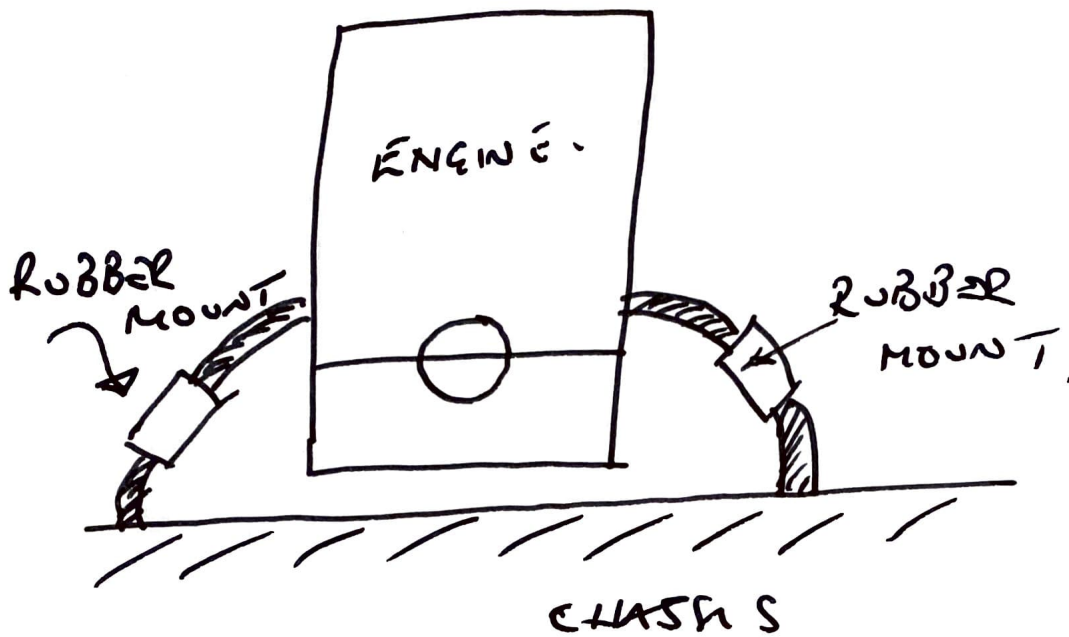
4.

THESE FORCES HAVE SIGNIFICANT MAGNITUDE AND WOULD INTRODUCE A STRONG NOISE AND VIBRATION ELEMENT INTO A CAR WITHOUT THE USE OF RUBBER TYPE ELASTOMERIC ENGINE MOUNTS.

A TYPICAL ENGINE / GEARBOX UNIT CAN USE 3 OR 4 ENGINE MOUNTS TO NOT ONLY PROVIDE STABILITY OVER BUMPS, BRAKING, CORNERING ETC. BUT ALSO TO PROVIDE EFFECTIVE ISOLATION.

THE FIGURE BELOW SHOWS A SCHEMATIC OF THE FRONT VIEW OF AN ENGINE:

5.



SCHEMATIC OF ENGINE:

WE WILL USE AN EXAMPLE
OF 4 ENGINE MOUNTS EQUALLY
LOADED STATICALLY TO DEVELOP THE
PRINCIPLES OF ENGINE MOUNT
ISOLATION.

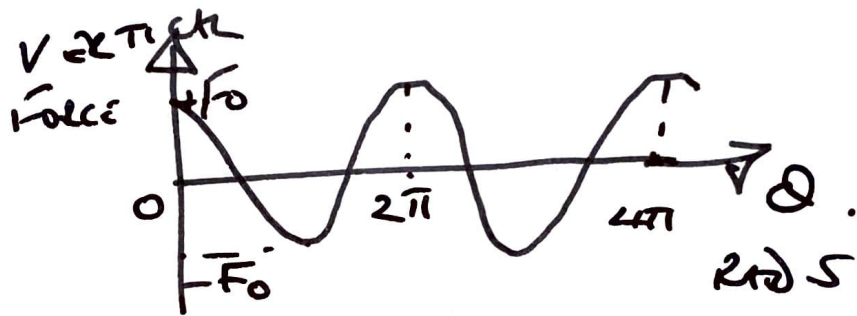
LET THE TOTAL ENGINE / GEARBOX
MASS $= M \text{ kg}$.

LET EACH MOUNT HAVE A
VERTICAL STIFFNESS $S = k \text{ N/m}$.

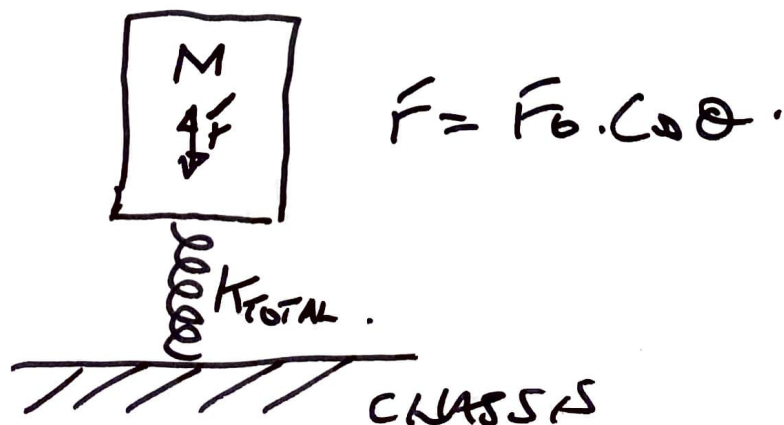
6. FIRST ORDER EXCITATION:
 SUPPOSE THE ENGINE IS AN
 UNBALANCED SINGLE CYLINDER WITH.
 A RESULTANT VERTICAL FORCE =

$$= F_0 \cos \theta \quad \text{--- (5)}$$

WHERE F_0 IS THE
 UNBALANCED AMPLITUDE



THESE FORCES ACT ON THE
 CRANKSHAFT MAIN BEARINGS
 WITHIN THE ENGINE & CAN BE
 DYNAMICALLY REPRESENTED AS BELOW:



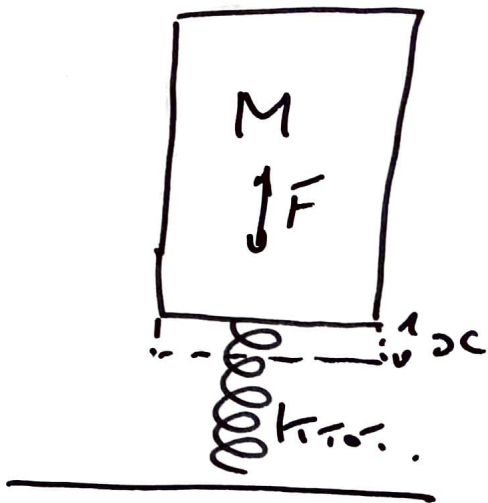
7.

AS THE CLUTSHAFT ROTATES, THE ROTATION ANGLE INCREASES WITH TIME FROM ZERO AT THE START AND INCREASES BY 2π RADIAN PER REVOLUTION.

IF THE ANGULAR VELOCITY IS ω RAD/SEC, θ CAN BE GIVEN BY: $\theta = \omega t$.

$$\therefore F = \underline{F_0 \cos \omega t} \quad \text{--- (6)}$$

IF THE ENGINE MOVES DOWN A SMALL DISTANCE x BECAUSE OF THE INTERNAL APPLIED FORCE THE TOTAL EFFECTIVE SPRING FROM THE RUBBER MOUNTS WILL BE COMPRESSED LEADING TO A RESTING FORCE AS BELOW:



Q EQUATION OF MOTION BECOMES

$$F - k_{\text{spring}} \cdot x = M \cdot \frac{d^2 x}{dt^2} \quad \text{--- (7)}$$

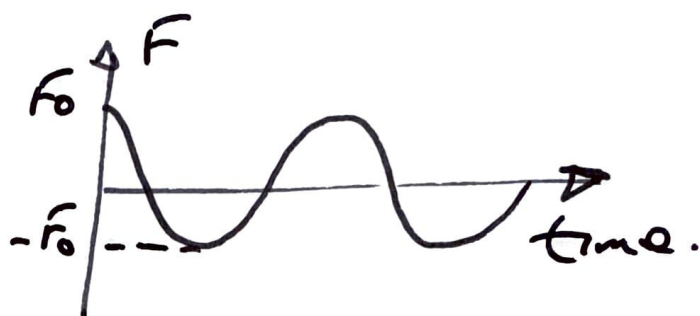
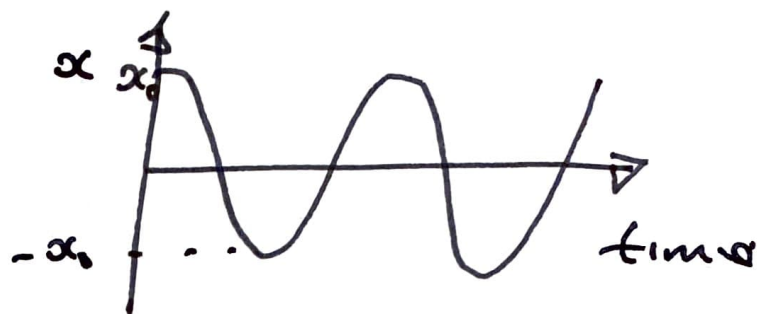
BECAUSE THE INPUT FORCE
VARIES CYCLICALLY WITH TIME
AS $F = F_0 \cdot \cos \omega t$

THE RESPONSE x MUST ALSO
VARY WITH THE SAME CYCLIC
FREQUENCY AND IS GIVEN THE
AMPLITUDE x_0 SUCH THAT

$$x = x_0 \cdot \cos \omega t$$

--- / ZERO.

9.



THE EQUATION OF MOTION CAN
BE WRITTEN:

$$F_0 \cos \omega t - \frac{kx_0 \cos \omega t}{\text{TOTAL}} = M \frac{d^2(x_0 \cos \omega t)}{dt^2}$$

— (8)

$$= M x_0 \frac{d^2(\cos \omega t)}{dt^2}$$

TO SOLVE THIS, PUT
 $x = x_0 \cos \omega t$

10.

$$\frac{dx}{dt} = -\omega x_0 \sin \omega t$$

$$\frac{d^2 x}{dt^2} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = -\omega^2 x_0 \cos \omega t$$

— (9)

PUTTING (9) INTO (8) GIVES:

$$F_0 \cos \omega t - \frac{k_{\text{TOTAL}} x_0 \cos \omega t}{\text{TOTAL}} = -M \omega^2 x_0 \cos \omega t$$

— (10)

DIVIDE EQUATION (10) THROUGHOUT BY $\cos \omega t$ TO GIVE:

$$F_0 - \frac{k_{\text{TOTAL}} x_0}{\text{TOTAL}} = -M \omega^2 x_0$$

— (11)

REARRANGING GIVES:

$$F_0 = \frac{k_{\text{TOTAL}} x_0}{\text{TOTAL}} - M \omega^2 x_0$$

$$(k_{\text{TOTAL}} - M \omega^2)$$

11.

$$F_0 = (k_{\text{total}} - M\omega^2) \cdot x_0$$

$$x_0 = \frac{F_0}{(k_{\text{total}} - M\omega^2)} \quad \text{--- (12)}$$

TAKE A FURTHER EXAMPLE
IF THE CRANKSHAFT WAS UNDERBALANCED
AND BALANCED ONLY THE ROTATIONAL
PART OF THE PISTON / CON ROD ASSEMBLY.

THE RESULTANT VERTICAL INERTIA
FORCE AMPLITUDE would now be

$$F_0 = M_{\text{rec}} \cdot R \omega^2$$

$$\text{OR } F_0 = M_{\text{rec}} \cdot l \omega^2$$

--- (13)

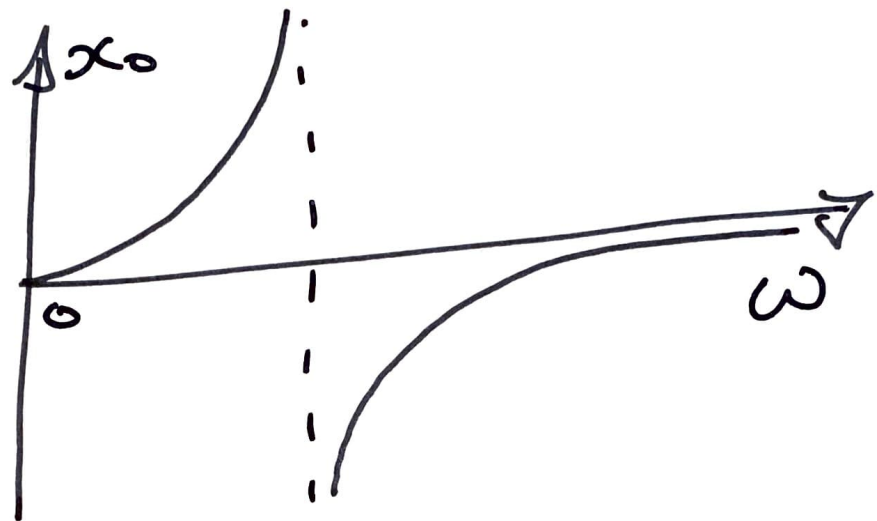
SUBSTITUTING THIS IN EQUATION (12)

GIVES:

$$x_0 = \frac{M_{\text{rec}} \cdot R \omega^2}{(k_{\text{total}} - M\omega^2)} \quad \text{--- (14)}$$

12.

PLOTTING A GRAPH OF THE
DISPLACEMENT AMPLITUDE x_0 GIVES
FOLLOWING



At Angular velocity $\omega = 0$
 $x_0 = 0$

When THE DENOMINATOR IS ZERO

$$\text{ie when } (k_{\text{TOTAL}} - M\omega^2) = 0 \quad \text{--- (1)}$$

THE THEORETICAL AMPLITUDE IS
INFINITE BECAUSE YOU ARE
DIVIDING BY ZERO.

13.

THIS IS CALLED THE
 RESONANCE AND WITHOUT IN REALITY/
 THE RESONANCE WILL BE DAMPED
 AND x_0 WILL NOT BE INFINITE,
 THE VALUE OF ω AT THIS
 POINT IS THE ANGULAR RESONANT
 FREQUENCY & IS GIVEN BY:

$$M \omega_{\text{RES.}}^2 = k_{\text{TOTAL}}$$

$$\omega_{\text{RES}} = \sqrt{\frac{k_{\text{TOTAL}}}{M}} \quad \text{--- (16)}$$

THIS FREQUENCY IS SET UP TO BE
 BELOW THE IDLE SPEED OF THE
 ENGINE SO AS NOT TO BE
 EXCITED IN A REAL CAR.

14.

SECOND ORDER EXCITATION.

SUPPOSE IT IS AGAIN A SINGLE CYLINDER WITH VERTICAL SECOND ORDER FORCE

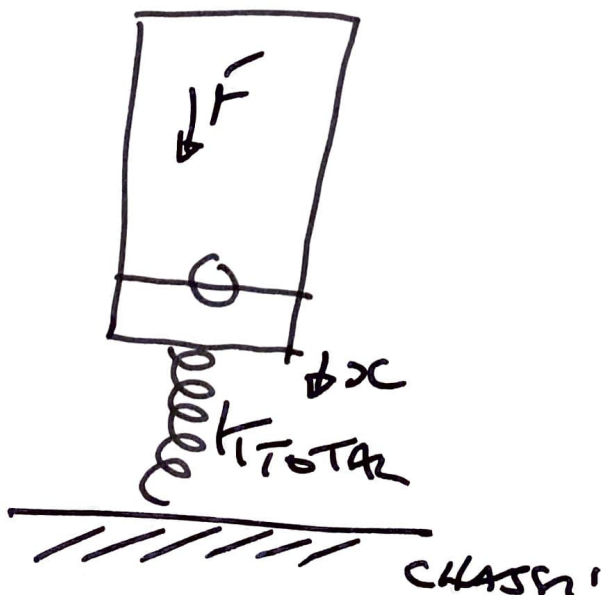
$$F = \frac{R}{L} M_{\text{rec}} R \omega^2 \cos 2\theta \quad (17)$$

$$\text{BUT } \theta = \omega t$$

WE CAN WRITE EQUATION (17) AS

$$F = \frac{R}{L} M_{\text{rec}} R \omega^2 \cos 2\omega t$$

THE SCHEMATIC DIAGRAM IS THE SAME AS BELOW:



15.

EQUATION OF MOTION BECOMES:

$$F - k_{\text{spring}} \cdot x = M \cdot \frac{d^2 x}{dt^2}$$

$$\frac{R}{L} M \omega^2 \cos 2\omega t$$

$$- k_{\text{spring}} \cdot x = M \cdot \frac{d^2 x}{dt^2} \quad (18)$$

SINCE THE FORCE IS VARYING AT A FREQUENCY 2ω , THE DISPLACEMENT MUST ALSO BE VARYING AT THE SAME FREQUENCY.

$$\text{PUT } x = x_0 \cdot \cos 2\omega t. \quad (19)$$

$$\frac{dx}{dt} = -2\omega x_0 \cdot \sin 2\omega t$$

$$\frac{d^2 x}{dt^2} = -4\omega^2 \cdot x_0 \cdot \cos 2\omega t \quad (20)$$

PUTTING INTO EQUATION (18) GIVES:

16.

$$\frac{R}{L} M_{\text{rec}} R \omega^2 \cos 2\omega t - k_{\text{TOT}} x_0 \cos 2\omega t = -4M\omega^2 x_0 \cos 2\omega t \quad (21)$$

Dividing everywhere by $\cos 2\omega t$ gives

$$\frac{R}{L} M_{\text{rec}} R \omega^2 - k_{\text{TOT}} x_0 = -4M\omega^2 x_0$$

$$\text{And } x_0 = \frac{\frac{R}{L} M_{\text{rec}} R \omega^2}{(k_{\text{TOT}} - 4M\omega^2)}$$

(22)

Resonant frequency is

Now when

$$(k_{\text{TOT}} - 4M\omega^2) = 0 \quad (23)$$

$$\text{i.e. } \omega^2 = \frac{k_{\text{TOT}}}{4M}$$

17.

$$\omega = \sqrt{\frac{k_{TOT.}}{4M}}$$

$$\omega_{res.} = \frac{1}{2} \sqrt{\frac{k_{TOT.}}{M}}$$

(24).

You may be initially surprised that this frequency seems to be half of the frequency calculated for first order. But it is not because the second order frequency is actually 2ω rad/sec making the actual vibration frequency at resonance the same.

The RPM this occurs at is given by

$$RPM = \frac{\omega_{res.} \times 60}{2\pi}$$

(25)

18)

WHEN IT IS A SECOND ORDER
EXCITATION THE RRM OF
RESONANCE WILL BE HALF THAT
WHEN IT IS A FIRST ORDER
EXCITATION.

==

THE ABOVE IS THE THEORY/
WE WILL NOW GO THROUGH
AN EXAMPLE OF EACH

① FIRST ORDER EXCITATION

ASSUME RESULTANT

VERTICAL INERTIA FORCE

$$= M_{\text{rec}} \cdot R \omega^2 (\cos t)$$

$$M_{\text{rec}} = 0,3 \text{ kg}$$

$$R = 0,04 \text{ m.}$$

19

ASSUME ENGINE/GEARBOX MASS

$$M = 100 \text{ kg.}$$

ASSUME MAXIMUM MOUNT
DEFLECTION OVER EXTREME '3g'
BUMPS HAS TO BE 10 mm.

$$\therefore \quad \underbrace{100 \times 3 \times 9.81}_{\text{SHOCK FORCE}} = \underbrace{k_{\text{TOTAL}} \cdot 10}_{\text{RESTING FORCE}}$$

$$\begin{aligned} k_{\text{TOTAL}} &= \frac{100 \times 3 \times 9.81}{10} \\ &= 294.3 \text{ N/mm} \\ &= \underline{\underline{294.3 \times 10^3 \text{ N/m.}}} \end{aligned}$$

DISPLACEMENT AMPLITUDE IS GIVEN

$$\text{B/ } x_0 = \frac{M_{\text{REC}} \cdot \omega^2}{(k_{\text{TOTAL}} - M \omega^2)}$$

$$X_0 = \frac{0,3 \times 0,04 \times \omega^2}{(294,3 \times 10^3 - 100 \omega^2)}$$

— (26)

RESONANCE IS WHEN

$$\omega^2 = \frac{294,3 \times 10^3}{100}$$

$$\omega = \sqrt{\frac{294,3 \times 10^3}{100}}$$

$$\omega_{res} = \underline{\underline{5412 \text{ Rads/sec}}}$$

RPM OF RESONANCE =

$$= \frac{5412 \times 60}{2\pi}$$

$$= \underline{\underline{518 \text{ RPM}}}$$

$$x_0 = \frac{0,3 \times 0,04 \times \omega^2}{(294,3 \times 10^3 - 100 \omega^2)}$$

→ (26)

Resonance is when

$$\omega^2 = \frac{294,3 \times 10^3}{100}$$

$$\omega = \sqrt{\frac{294,3 \times 10^3}{100}}$$

$$\omega_{res} = \underline{\underline{5412 \text{ Rads/sec}}}$$

RPM of Resonance =

$$= \frac{5412 \times 60}{2\pi}$$

$$= \underline{\underline{518 \text{ RPM.}}}$$

21.

AT HIGH RPM

$$100 \omega^2 \gg 294,3 \times 10^3$$

& x_0 CAN BE APPROXIMATED

TO:

$$x_0 \approx \frac{0,3 \times 0,04 \times \omega^2}{-100 \omega^2}$$

THE ω^2 CANCELS GIVING

$$x_0 \approx \frac{-0,3 \times 0,04}{100}$$

$$\approx -0,00012 \text{ m}$$

$$\approx -0,12 \text{ mm.}$$

THIS IS THE DISPLACEMENT
THE MOUNTS SEE & TOTAL
FORCE TRANSMITTED TO CHASSIS
IS GIVEN BY:

22.

$$\begin{aligned} F &= F_{\text{reson}} \times 0,00012 \\ &= 294,3 \times 10^3 \times 0,00012 \\ &\approx \underline{\underline{35,3 \text{ N}}} \end{aligned}$$

ORIGINAL FIRST ORDER FORCE
AMPLITUDE AT 6000 RPM

$$\begin{aligned} &= M_{\text{res}} \cdot R \omega^2 \\ &= 0,3 \times 0,04 \times \left(\frac{2\pi \times 6000}{60} \right)^2 \\ &= \underline{\underline{4739 \text{ N}}} \end{aligned}$$

YOU CAN SEE A VERY BIG
REDUCTION.

ALSO NOTE THAT ABOVE
THE RESONANT FREQUENCY, THE
SIGN OF x_0 GOES NEGATIVE.

THIS IS CORRECT.

23.

BELOW RESONANCE THE FORCE 'PUSHES' AND CAUSES MOTION.

ABOVE RESONANCE THE FORCE IS ENERGETIC / 'PULLING' AT END OF EACH OSCILLATION TO NOW STOP THE MOTION AND BRING IT BACK AND NOT 'OVERSHOOT'. THIS IS A COMPLICATED THING TO GRASP.

SECOND ORDER EXCITATION EXAMPLE

ASSUME THE SAME MASS & SPRING STIFFNESS. ASSUME COEFF LENGTH IS 160mm.

SECOND ORDER VERTICAL FORCE

$$= \frac{R}{L} M e e R \omega^2 \cos \omega t$$

$$= \frac{0,04}{0,16} \times 0,3 \times 0,04 \omega^2 \cos \omega t$$

$$= 0,003 \omega^2 \cos \omega t.$$

24.

DISPLACEMENT AMPLITUDE = 3 mm:

$$x_0 = \frac{0,003 \omega^2}{(k_{total} - 4M\omega^2)}$$

RESONANCE IS WHEN

$$4M\omega^2 = k_{total}$$

$$\omega = \frac{1}{2} \sqrt{\frac{k_{total}}{M}}$$

$$= \frac{1}{2} \sqrt{\frac{294,3 \times 10^3}{100}}$$

$$\omega_{res.} = \underline{\underline{27,1 \text{ Rads/sec.}}}$$

RPM AT RESONANCE

$$= \frac{27,1}{2\pi} \times 60$$

$$\sim \underline{\underline{258 \text{ RPM}}}$$

BECAUSE EXCITATION RESONANCE
IS SECOND ORDER IT IS
STILL THE SAME RESONANCE

25.

AS FIRST ORDER BUT AT
HALF THE RPM.

AT HIGH RPM

$$4M\omega^2 \gg K_{TOTAL}$$

$$\begin{aligned} \delta_0 &\approx - \frac{0,003 \cancel{\text{m}}^2}{4 \times 100 \cancel{\text{N}}^2} \\ &= - \frac{0,003}{400} \\ &= 7,5 \times 10^{-6} \text{ m.} \\ &= 7,5 \times 10^{-3} \text{ mm.} \end{aligned}$$

GIVING TOTAL TRANSMITTED
FORCE AMPLITUDE TO CHASSIS

$$\begin{aligned} &= - K_{TOTAL} \times 7,5 \times 10^{-6} \\ &= - \underline{\underline{2,2 \text{ N.}}} \end{aligned}$$



25.

AS FIRST ORDER BUT AT
HALF THE RPM.

AT HIGH RPM

$$4M\omega^2 \gg k_{TOTAL}$$

Q

$$x_0 \approx - \frac{0,003 \cancel{m}}{4 \times 100 \cancel{N}}$$

$$= - \frac{0,003}{400}$$

$$= 7,5 \times 10^{-6} \text{ m.}$$

$$= 7,5 \times 10^{-3} \text{ mm.}$$

GIVING TOTAL TRANSMITTED
FORCE AMPLITUDE TO CHASSIS

$$= - k_{TOTAL} \times 7,5 \times 10^{-6}$$

$$= - \underline{\underline{2,2 \text{ N.}}}$$