



Complete, fold and seal
(see note 2 below)

SURNAME: Woods

FORENAME(S): Thomas

12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

List the numbers of the questions you have attempted	Office Use
A	39
B1	20
B2	20.
B4.	20
Total	99

(Gweler yr ochr arall am y fersiwn Cymraeg)

1.) $15(3x + 2) - 8(2 - 4x) = 125$

expand

$$45x + 30 - 16 + 32x = 125$$

$$77x + 14 = 125$$

$$77x = 125 - 14$$

~~$$77x = 125 - 14$$~~

$$77x = 111$$

$$x = \frac{111}{77}$$

$$x = 1.44 \quad \checkmark \quad (2dp)$$

5

2.)

i.)

$$4 \begin{pmatrix} 3 & -1 \\ 5 & 1 \end{pmatrix} - 3 \begin{pmatrix} 2 & -3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -4 \\ 20 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -9 \\ 6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 5 \\ 14 & 1 \end{pmatrix} \quad \checkmark$$

2

ii.)

BC

$$B = \begin{pmatrix} 2 & -3 \\ 2 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 4 & -3 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$C_{11} = (2 \times 4) + (-3 \times 2) = 2$$

$$C_{12} = (2 \times -3) + (-3 \times 3) = -15$$

$$C_{13} = (2 \times 2) + (-3 \times 4) = -8$$

$$C_{21} = (2 \times 4) + (1 \times 2) = 10$$

$$C_{22} = (2 \times -3) + (1 \times 3) = -3$$

$$C_{23} = (2 \times 2) + (1 \times 4) = 8$$

$$= \begin{pmatrix} 2 & -15 & -8 \\ 10 & -3 & 8 \end{pmatrix}$$

4

$$\text{iii.) } A^{-1} = (3 \times 1) - (5 \times -1) = 8$$

$$\therefore \frac{1}{A} = \frac{1}{8} \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{8} & \frac{1}{8} \\ -\frac{5}{8} & \frac{3}{8} \end{pmatrix} \checkmark$$

5

$$3.) \quad y = 5x^3 - \frac{3}{x^4} + 4\sqrt{x}$$

$$y = 5x^3 - 3x^{-4} + 4x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 15x^2 + 12x^{-5} + 2x^{-\frac{1}{2}} \checkmark$$

5

$$4.) \quad Z_1 = 5 - j4$$

$$Z_2 = 3 + j5$$

$$r = \sqrt{5^2 + 4^2}$$

$$r = 6.4 \checkmark$$

$$\phi = \tan^{-1}\left(\frac{4}{5}\right) \checkmark$$

$$\phi = 38.66^\circ \checkmark$$

$$\therefore$$

$$Z_1 = 6.4 \angle -38.66^\circ \checkmark$$

$$r = \sqrt{3^2 + 5^2}$$

$$r = 5.83 \checkmark$$

$$\phi = \tan^{-1}\left(\frac{5}{3}\right) \checkmark$$

$$\phi = 59^\circ \checkmark$$

$$\therefore$$

$$Z_2 = 5.83 \angle 59^\circ \checkmark$$

$$\begin{aligned}
 \text{i.) } Z_1 Z_2 &= (6.4 \times 5.83) \angle -38.66 + 59 \\
 &= \underline{37.31 \angle 20.34^\circ}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii.) } \frac{Z_1}{Z_2} &= \frac{6.4 \angle -38.66^\circ}{5.83 \angle 59^\circ} \\
 &= \frac{6.4}{5.83} \angle (-38.66 - 59) \\
 &= \underline{1.1 \angle -97.66^\circ}
 \end{aligned}$$

10

$$\text{5.) } \int_0^{\frac{\pi}{2}} (4 \cos 2x - 6 \sin 3x) dx$$

$$= \left[\frac{4}{2} \sin 2x + \frac{6}{3} \cos 3x \right]_0^{\frac{\pi}{2}}$$

8.

$$= \left[2 \sin(2 \times \frac{\pi}{2}) + 2 \cos(3 \times \frac{\pi}{2}) \right] - \left[2 \sin(2 \times 0) + 2 \cos(3 \times 0) \right]$$

$$= (0) - (2)$$

$$= -2$$

39
 40

Section B

1.) i.)

$$12V = 7I_1 + 8(I_1 + I_2)$$

$$12 = 7I_1 + 8I_1 + 8I_2$$

$$12 = 15I_1 + 8I_2 \quad (1)$$

$$8 = 14I_2 + 8(I_1 + I_2)$$

$$8 = 14I_2 + 8I_1 + 8I_2$$

$$8 = 8I_1 + 22I_2 \quad (2)$$

Cramer's Rule,

$$I_1 = \frac{\begin{vmatrix} 12 & 8 \\ 8 & 22 \\ 15 & 8 \\ 8 & 22 \end{vmatrix}}{\begin{vmatrix} 15 & 8 \\ 8 & 22 \end{vmatrix}} = \frac{(12 \times 22) - (8 \times 8)}{(15 \times 22) - (8 \times 8)} = \frac{200}{266} = 0.754$$

$$I_2 = \frac{\begin{vmatrix} 15 & 12 \\ 8 & 8 \\ 15 & 8 \\ 8 & 22 \end{vmatrix}}{\begin{vmatrix} 15 & 8 \\ 8 & 22 \end{vmatrix}} = \frac{(15 \times 8) - (8 \times 12)}{266} = \frac{24}{266} = 0.09$$

ii.)

$$12 = 15I_1 + 8I_2 \quad (1) \quad \times 8$$

$$8 = 8I_1 + 22I_2 \quad (2) \quad \times 15$$

$$96 = 120I_1 + 64I_2 \quad (3)$$

$$120 = 120I_1 + 330I_2 \quad (4)$$

Subtract eq 3 from eq 4

$$24 = 266I_2$$

$$I_2 = \frac{24}{266}$$

$$I_2 = 0.09$$

Sub I_2 into eq 2

$$8 = 8I_1 + 22(0.09)$$

$$8 = 8I_1 + 1.98$$

$$8I_1 = 8 - 1.98$$

$$8I_1 = 6.02$$

$$I_1 = \frac{6.02}{8}$$

$$I_1 = 0.75$$

$$\frac{20}{20}$$

2.)

$$\therefore \int_0^3 \left(\frac{1}{(3x+5)^4} \right) dx$$

$$\text{let } u = 3x + 5$$

$$\frac{du}{dx} = 3$$

$$\therefore dx = \frac{1}{3} du$$

$$\therefore \int_0^3 \left(\frac{1}{(3x+5)^4} \right) dx = \int_0^3 \frac{1}{u^4} \cdot \frac{1}{3} du$$

$$= \int_0^3 \frac{1}{3} u^{-4} du = \left[\frac{1}{3} \frac{u^{-3}}{-3} \right]_0^3 = \left[\frac{(3x+5)^{-3}}{-9} \right]_0^3$$

$$= \left[\frac{((3 \times 3) + 5)^{-3}}{-9} \right] - \left[\frac{((3 \times 0) + 5)^{-3}}{-9} \right]$$

$$= (-4.05 \times 10^{-5}) - (-8.89 \times 10^{-4}) = 8.485 \times 10^{-4}$$

4.) i.)

$$a.) \quad y = \frac{2e^{6x}}{\sin 3x}$$

$$\text{let } u = 2e^{6x}$$

$$\frac{du}{dx} = 12e^{6x}$$

$$\text{and let } v = \sin 3x$$

$$\frac{dv}{dx} = 3 \cos 3x$$

$$\therefore \frac{dy}{dx} = \frac{(\sin 3x)(12e^{6x}) - (2e^{6x})(3 \cos 3x)}{(\sin 3x)^2}$$

$$\frac{dy}{dx} = \frac{12e^{6x} \sin 3x - 6e^{6x} \cos 3x}{(\sin 3x)^2}$$

$$b.) \quad y = 6 \cos(3x - 5)$$

$$\text{let } u = 3x - 5$$

$$\frac{du}{dx} = 3$$

$$\text{and let } v = 6 \cos u$$

$$\frac{dv}{du} = -6 \sin u$$

$$\therefore \frac{dy}{dx} = (3)(-6 \sin u)$$

$$= -18 \sin u$$

$$u = 3x - 5$$

$$\therefore \frac{dy}{dx} = -18 \sin(3x - 5)$$

4.) ii.) $y = 3x^3 + 4x^2 - 2x - 5$

$$\frac{dy}{dx} = 9x^2 + 8x - 2$$

at a stationary point $\frac{dy}{dx} = 0$

$$\therefore 9x^2 + 8x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(9)(-2)}}{2(9)}$$

$$x = \frac{-8 \pm \sqrt{136}}{18}$$

$$x = 0.2 \quad \checkmark \quad \text{or} \quad x = -1.09 \quad \checkmark$$

at $x = 0.2$

$$y = 3(0.2)^3 + 4(0.2)^2 - 2(0.2) - 5$$

$$y = -5.216 \quad \checkmark$$

at $x = -1.09$

$$y = 3(-1.09)^3 + 4(-1.09)^2 - 2(-1.09) - 5$$

$$y = -1.95 \quad \checkmark$$

coordinates for stationary points are

$$(0.2, -5.216), (-1.09, -1.95)$$

$$\frac{d^2y}{dx^2} = 18x + 8 \quad \checkmark$$

$$\text{at } x = 0.2$$

$$\frac{d^2y}{dx^2} = 18(0.2) + 8 = 11.6 \text{ +ve so } \text{minimum.}$$

$$\text{at } x = -1.09$$

$$\frac{d^2y}{dx^2} = 18(-1.09) + 8 = -11.62 \text{ -ve so } \text{maximum.}$$

2.) ii.)

$$\frac{8}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$8 = A(x+2) + B(x+1)$$

$$\text{let } x+2 = 0$$

$$\therefore x = -2$$

$$8 = A(-2+2) + B(-2+1)$$

$$8 = B(-1)$$

$$-B = 8$$

$$B = -8$$

$$\text{let } x+1 = 0$$

$$\therefore x = -1$$

$$8 = A(-1+2) + B(-1+1)$$

$$8 = A(1) +$$

$$A = 8$$

$$\therefore \frac{8}{(x+1)(x+2)} = \frac{8}{(x+1)} - \frac{8}{(x+2)}$$

$$\int_1^4 \frac{8}{(x+1)} - \frac{8}{(x+2)} = \left[8 \ln(x+1) - 8 \ln(x+2) \right]_1^4$$

10
20
20

$$= \left[8 \ln(4+1) - 8 \ln(4+2) \right] - \left[8 \ln(1+1) - 8 \ln(1+2) \right]$$

~~$$= (-1.46) - (-3.24)$$~~

~~$$= 1.78$$~~

$$= (-1.4586) - (-3.2437)$$

$$= 1.7851$$

20
20