

Q1: [30 MARKS] Differentiation

$$(i) \quad y = 8x^3 + 3$$

$$\frac{dy}{dx} = 24x^2 \quad [2]$$

$$(ii) \quad y = 4x^3 + 10x^{1/2} + 5/6$$

$$\begin{aligned} \frac{dy}{dx} &= 12x^2 + 5x^{-1/2} \\ &= 12x^2 + \frac{5}{\sqrt{x}} \quad [3] \end{aligned}$$

$$(iii) \quad y = 2x^{-5} + x^3 \quad [3]$$

$$\begin{aligned} \frac{dy}{dx} &= -10x^{-6} + 3x^2 \\ &= -\frac{10}{x^6} + 3x^2 \end{aligned}$$

$$(iv) \quad y = \sin(5x)$$

$$\frac{dy}{dx} = 5\cos(5x) \quad [2]$$

$$(v) \quad y = x^2 \cos x \text{ (Product Rule)}$$

$$\begin{aligned} \frac{dy}{dx} &= x^2(-\sin x) + \cos(x)2x \\ &= x[-x\sin x + 2\cos x] \quad [5] \end{aligned}$$

$$(vi) \quad y = (9x+3)^{10}$$

$$\begin{aligned} \frac{dy}{dx} &= 10(9x+3)^9 \cdot 9 \\ &= 90(9x+3)^9 \quad [3] \end{aligned}$$

(Chain Rule)

$$(vii) \quad y = x + 3\tan(x)$$

$$\frac{dy}{dx} = 1 + 3\sec^2(x) \quad [2]$$

$$1b \quad y = x^{-1} + \cos(3x)$$

$$\frac{dy}{dx} = -x^{-2} - 3\sin(3x)$$

$$\frac{d^2y}{dx^2} = +2x^{-3} - 9\cos(3x)$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 27\sin(3x)$$

$$\frac{d^4y}{dx^4} = +24x^{-5} + 81\cos(3x)$$

[4]

$$\underline{1c} \quad y = \frac{2x+1}{(3+x^2)}$$

$$\frac{dy}{dx} = \frac{(3+x^2)(2) - (2x+1)(2x)}{(3+x^2)^2}$$

$$= \frac{6+2x^2-4x^2-4x}{(3+x^2)^2}$$

$$= \frac{6-4x-2x^2}{(3+x^2)^2}$$

$$= \frac{2[3-2x-x^2]}{(3+x^2)^2}$$

$$= -2 \frac{[x^2+2x-3]}{(3+x^2)^2}$$

$$= -2 \frac{[x-1][x+3]}{(3+x^2)^2}$$

when $x=0$

$$\frac{dy}{dx} = \frac{-2(-1)(3)}{(3+0^2)^2}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

[6]

total
[30 marks]

Q.2: [30 MARKS] Integration

$$\begin{aligned} \text{(i)} \int (10x + x^3) dx &= \frac{10x^2}{2} + \frac{x^4}{4} + C \\ &= 5x^2 + \frac{x^4}{4} + C \end{aligned} \quad [3]$$

$$\begin{aligned} \text{(ii)} \int (2x + 3e^x) dx &= \frac{2x^2}{2} + 3e^x + C \\ &= x^2 + 3e^x + C \end{aligned} \quad [3]$$

$$\text{(iii)} \int \left(\frac{2}{x} + 4\right) dx = 2 \ln x + 4x + C \quad [3]$$

$$\begin{aligned} \text{(iv)} \int (x-1)(3x+4) dx &= \int (3x^2 + x - 4) dx \\ &= \frac{3x^3}{3} + \frac{x^2}{2} - 4x + C \\ &= x^3 + \frac{x^2}{2} - 4x + C. \end{aligned} \quad [3]$$

$$\begin{aligned} \text{(v)} \int_3^6 (3x^2 - 2x) dx &= \left[\cancel{3}x^3 - \cancel{2}x^2 \right]_3^6 \\ &= [(6)^3 - (6)^2] - [(3)^3 - (3)^2] \\ &= [216 - 36] - [27 - 9] \\ &= 180 - 18 \\ &= 162 \end{aligned} \quad [3]$$

$$2b.i) I = \int_0^1 \frac{e^x dx}{\sqrt{e^x + 1}}$$

let $u = e^x + 1 \Rightarrow u - 1 = e^x$
 $\frac{du}{dx} = e^x \Rightarrow du = e^x dx$

when $x=0$, $u = e^0 + 1 = 2$

$x=1$, $u = e^1 + 1$

$$\therefore I = \int_{u=2}^{1+e} \frac{du}{\sqrt{u}}$$

$$I = \int_2^{1+e} u^{-1/2} du$$

$$I = \left[\frac{u^{1/2}}{1/2} \right]_2^{1+e}$$

$$I = [2\sqrt{u}]_2^{1+e}$$

$$I = 2\sqrt{1+e} - 2\sqrt{2}$$

$$= \underline{1.0281 \text{ 4apl's}}$$

[5]

$$2c(ii) \quad I = \int x \cos(x) dx \quad (\text{PARTS})$$

$$\begin{aligned} I &= \sin(x) \cdot x - \int \sin(x) \cdot 1 dx \\ &= x \sin(x) - (-\cos(x)) + C \\ &= \underline{x \sin(x) + \cos(x) + C} \quad [5] \end{aligned}$$

$$\begin{aligned} (ii) \quad \int_1^e \ln x dx &= \int_1^e 1 \cdot \ln x dx \\ &= [x \ln x]_1^e - \int_1^e x \cdot \frac{1}{x} dx \\ &= e \ln e - \cancel{1 \ln 1}^0 - [x]_1^e \\ &= e \ln e - (e - 1) \\ &= e - e + 1 \\ &= 1. \quad [5] \end{aligned}$$

Q.3) [30 MARKS] Complex Numbers

$$\begin{aligned} \text{a. i)} \quad (3+2i)(1-i) &= 3+2i-3i-2i^2 \\ &= 3-i-2(-1) \\ &= 3-i+2 \\ &= 5-i \end{aligned} \quad [2]$$

$$\begin{aligned} \text{ii)} \quad \frac{3}{(2-i)} &= \frac{3}{(2-i)} \cdot \frac{(2+i)}{(2+i)} \\ &= \frac{3(2+i)}{4-\cancel{2i}+\cancel{2i}-i^2} \\ &= \frac{6+3i}{5} \\ &= \frac{6}{5} + \frac{3}{5}i \end{aligned} \quad [2]$$

$$\begin{aligned} \text{iii)} \quad \frac{1}{i^5} &= \frac{1}{i^2} \cdot \frac{1}{i^2} \cdot \frac{1}{i} \\ &= \frac{1}{(-1)} \cdot \frac{1}{(-1)} \cdot \frac{1}{i} \\ &= \frac{1}{i} \\ &= \frac{1}{i} \cdot \frac{i}{i} \\ &= \frac{i}{-1} \\ &= \underline{0-i} \end{aligned} \quad [2]$$

APPLIED ANALITI

$$\underline{3b} \quad z_1 = \frac{2-j}{2+j}$$

$$z_1 = \frac{(2-j)(2-j)}{(2+j)(2-j)}$$

$$z_1 = \frac{4 - 2j - 2j + j^2}{4 + 2j - 2j - j^2}$$

$$z_1 = \frac{3-4j}{5}$$

[3]

$$z_2 = \frac{3+j}{1-2j}$$

$$z_2 = \frac{(3+j)(1+2j)}{(1-2j)(1+2j)}$$

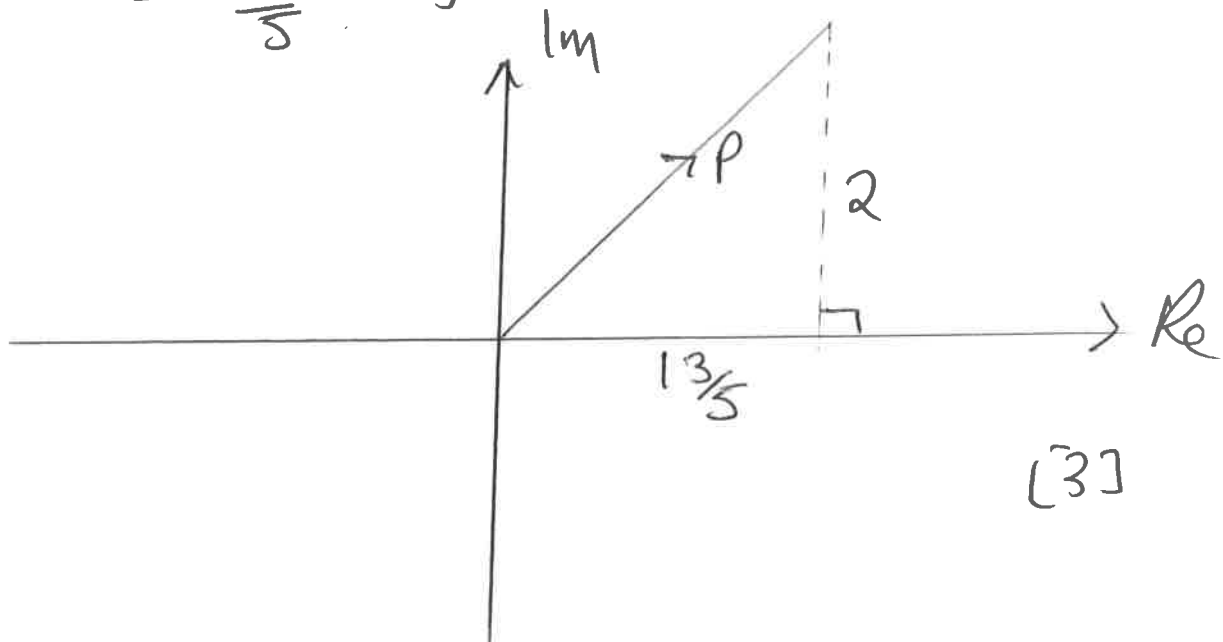
$$= \frac{3+j+6j-2j^2}{1-2j+2j-4j^2}$$

$$= \frac{5+7j}{5}$$

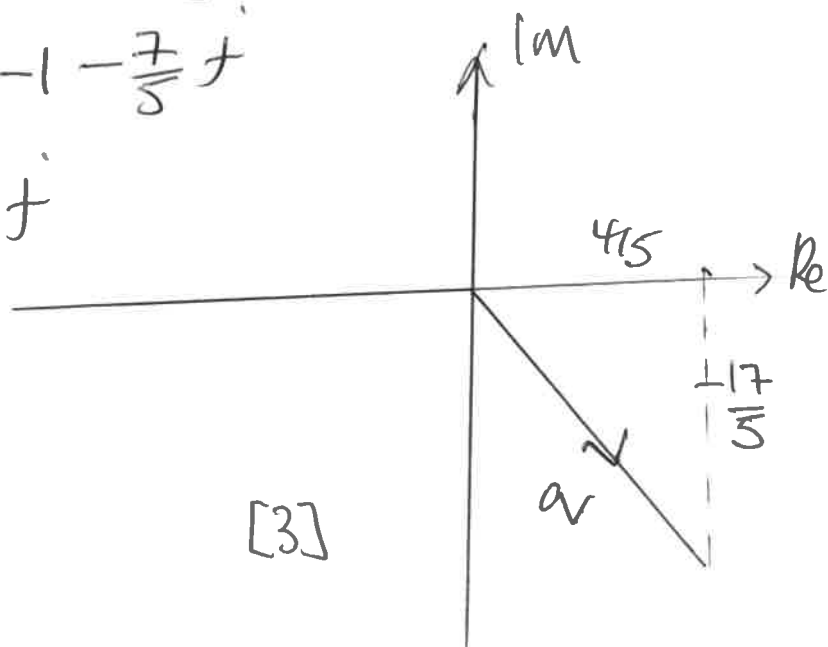
$$\underline{z_2 = 1 + \frac{7}{5}j}$$

[3]

$$\begin{aligned}
 \text{C.ii)} \quad p &= z_1 + 2z_2 \\
 &= \frac{3}{5} - \frac{4}{5}j + 2\left(1 + \frac{7}{5}j\right) \\
 &= \frac{3}{5} - \frac{4}{5}j + 2 + \frac{14}{5}j \\
 &= \frac{13}{5} + \frac{10}{5}j \\
 &= \frac{13}{5} + 2j
 \end{aligned}$$



$$\begin{aligned}
 \text{(ii)} \quad q &= 3z_1 - z_2 \\
 q &= 3\left[\frac{3}{5} - \frac{4}{5}j\right] - \left[1 + \frac{7}{5}j\right] \\
 q &= \frac{9}{5} - \frac{12}{5}j - 1 - \frac{7}{5}j \\
 q &= \frac{4}{5} - \frac{17}{5}j
 \end{aligned}$$



3b

$$z = (1+2i)(1+i)$$

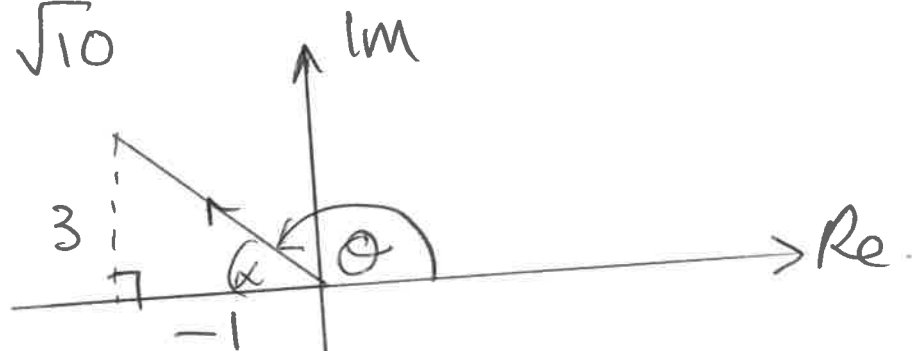
$$z = 1+2i+i+2i^2$$

$$z = 1+3i-2$$

$$z = -1+3i$$

$$|z| = \sqrt{(-1)^2 + (3)^2}$$

$$|z| = \sqrt{10}$$



$$\tan \alpha = \frac{3}{1} \Rightarrow \alpha = 71.57^\circ \text{ (rad)}$$

$$\therefore \theta + \alpha = 180^\circ$$

$$\theta + 71.57^\circ = 180^\circ$$

$$\theta = 108.43^\circ$$

$$z = r [\cos \theta + i \sin \theta]$$

$$z = \sqrt{10} [\cos(108.43^\circ) + i \sin(108.43^\circ)]$$

$$z = \sqrt{10} \angle 108.43^\circ$$

[6]

3c: let $z = \frac{3+i}{3-i}$

$$z = \frac{(3+i)(3+i)}{(3-i)(3+i)}$$

$$z = \frac{9+3i+3i+i^2}{9-3i+3i-i^2}$$

$$z = \frac{8+6i}{10}$$

$$z = \frac{2[4+3i]}{10}$$

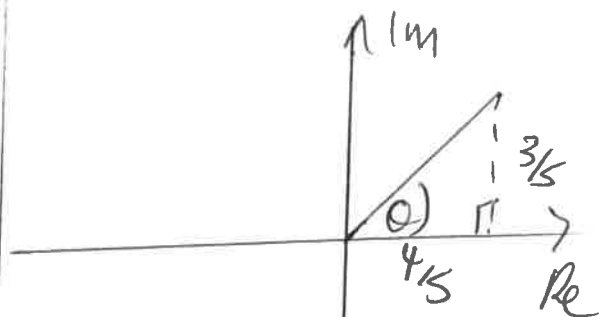
$$z = \frac{4+3i}{5}$$

$$|z| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}$$

$$|z| = \sqrt{\frac{16}{25} + \frac{9}{25}}$$

$$|z| = \sqrt{\frac{25}{25}}$$

$$\underline{|z| = 1}$$



$$\tan \theta = \frac{3/5}{4/5}$$

$$\tan \theta = 3/4$$

$$\theta = \tan^{-1}(3/4)$$

$$\theta = 36.87^\circ$$

$$\underline{\text{Arg}(z) = \theta = 36.87^\circ}$$

$$z = 1[\cos \theta + i \sin \theta]$$

$$z = 1[\cos(36.87^\circ) + i \sin(36.87^\circ)]$$

$$\underline{z = 1 \angle 36.87^\circ}$$

Q. 4: 10 marks (Logarithms)

a i) $4^{2x} = 3^{x+1}$

$$\log_{10} 4^{2x} = \log_{10} 3^{x+1}$$

$$2x \log_{10} 4 = (x+1) \log_{10} 3$$

$$2x \log_{10} 4 = x \log_{10} 3 + \log_{10} 3$$

$$2x \log_{10} 4 - x \log_{10} 3 = \log_{10} 3$$

$$x [2 \log_{10} 4 - \log_{10} 3] = \log_{10} 3$$

$$x = \frac{\log_{10} 3}{[2 \log_{10} 4 - \log_{10} 3]}$$

$$x = 0.65628 \dots$$

$$\underline{x = 0.66 \text{ (2dp)}}$$

[4]

a ii) $2 \ln x = \ln(4x-4)$

$$\ln x^2 = \ln(4x-4)$$

$$\Rightarrow x^2 = 4x-4$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$\Rightarrow \underline{x = -2 \text{ (twice)}}$$

[2]

$$\begin{aligned} 4b \text{ i)} \quad \log(x^4 y z) &= \log x^4 + \log y + \log z \\ &= 4 \log x + \log y + \log z \quad (2) \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad \ln\left(\frac{x^4 y}{z}\right) &= \ln(x^4 y) - \ln(z) \\ &= \ln x^4 + \ln y - \ln z \\ &= 4 \ln x + \ln y - \ln z. \quad (?) \end{aligned}$$

