

Dynamics control and instrumentation assignment 1

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TASK 1

A

- The open loop (non-feedback) systems contain of the active elements but in the closed loop system, there are both active and feedback elements and it is also called a feedback system.
- A closed loop control system considers the output and alters it to the desired condition. The control action in these systems is based on the output but an open loop control system acts completely on the basis of input and the output has no effect on the control action.
- Any changes on the gain's value in an open loop system, makes a larger impact on the output value compared to a closed loop system. In a closed loop system, there is a small change in output, by changing the value of the active elements gain, therefore it is a more reproducible system.
- In a closed loop system, there is a comparison element which compares the input signal with the feedback signal, subtract them and send an error signal to the active element.
- The structure of the closed loop system is more complex than an open loop system and has more components compared to the open loop system which leads to be more expensive.
- The accuracy and the reliability of the closed system is more than the open loop system.
- An open loop system responds faster than a closed loop system.

Open loop system

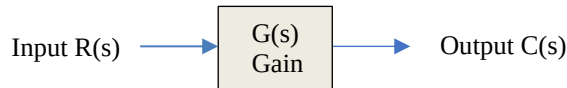


Figure 1. A General Open Loop system

An open loop system is a system that includes of input $R(s)$, transfer function $G(s)$ (which produces an output) and output $C(s)$.

The transfer function (T.F) in this system is calculated as:

$$\text{T.F} = G(s) = \frac{C(s)}{R(s)}$$

Because in an open loop system there is no knowledge of the output condition, it cannot correct itself in case of any errors happening, changing in the gain value or any disturbance on the system. This type of system is poorly equipped to handle disturbances or changes in the conditions. Therefore, the ability to complete the desired task will be reduced.

For example, if we have an amplifier with gain 20 with an input voltage of 1V. The output voltage would be equal to 20 V.

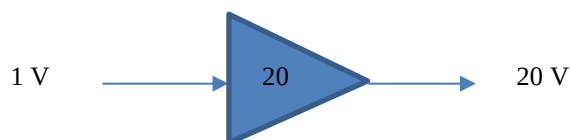


Figure 2. Schematic of a gain

Now if we 5% change the value of gain and increase it to 21V, the output would be equal to 21V.

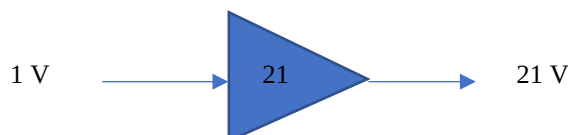


Figure 3. Schematic of a gain

Which shows the 5% change in gain results in %5 change in output;

$$\frac{21-20}{20} \times 100 = 5\% \text{ change}$$

Also, if there is any disturbance on the system,

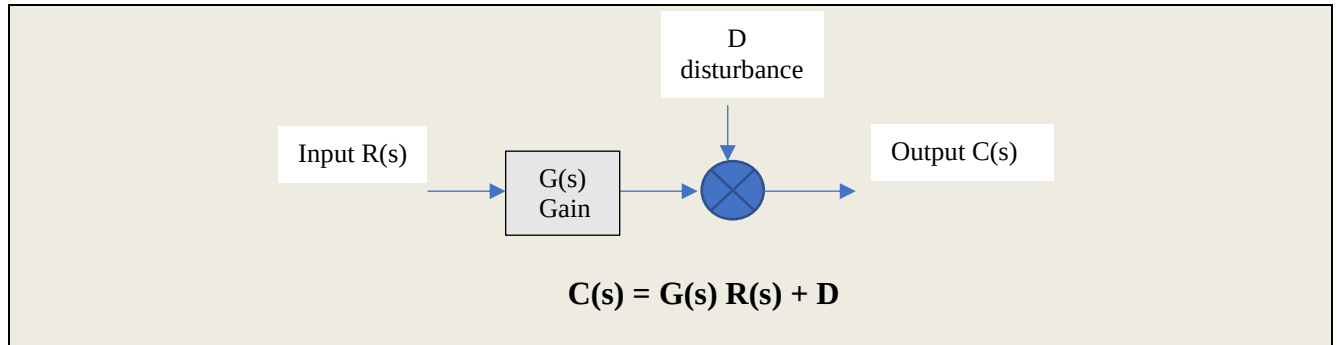


Figure 4. Disturbance after the gain

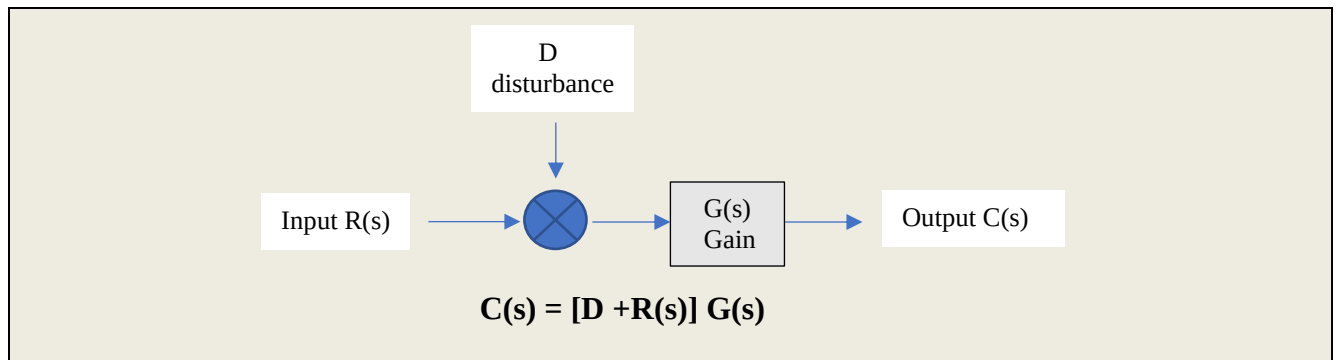


Figure 5. Disturbance before the gain.

In both cases, the output becomes greatly disturbed by any disturbances.

Closed loop system:

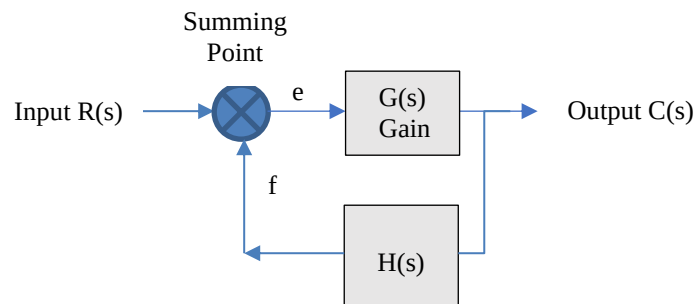


Figure 6. A General close Loop system

Closed loop system also known as a feedback control system is a control system which uses an open loop system as its forward path ($G(s)$ transfer function) but has one or more feedback loops or paths with another transfer function $H(s)$ between its output and its input. It means that some portion of the output is returned “back” to the input to form part of the systems excitation.

Closed-loop systems are designed to automatically achieve and maintain the desired output by comparing it with the actual condition. It uses a summing point (comparison element) between the feedback f and the input $R(s)$ and generates a positive or negative error signal (e) which the controller responds too. The total transfer function for the negative feedback would be;

$$T.F = \frac{G(s)}{1 + G(s)H(s)}$$

And for the positive feedback will be:

$$T.F = \frac{G(s)}{1 - G(s)H(s)}$$

Because a closed-loop system has some knowledge of the output condition, via $H(s)$ it is better design to handle any system disturbances or changes in the conditions. For example, if there is a closed loop system with negative feedback and $G(s)=100$ and $H(s) = 0.1$ then the transfer function would be equal to:

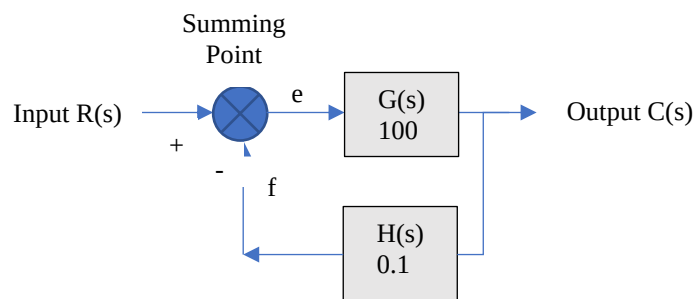


Figure 7. A closed loop system with gain of 100 and $H(s)$ equal to 0.1.

$$T.F = \frac{100}{1+(100 \times 0.1)} = 9.09$$

By increasing 10% to the value of the active component, $G(s) = 110$, the value of the transfer function will be:

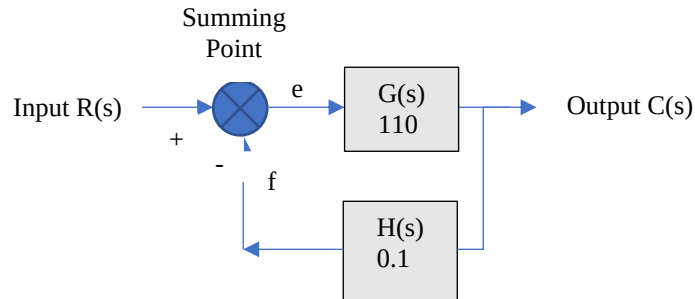


Figure 7. A closed loop system with gain of 110 and $H(s)$ equal to 0.1.

$$T.F = \frac{110}{1+(110 \times 0.1)} = 0.77\%$$

That means, that by 10% change in the value of the active component, it will be just 0.77% change in the overall performance of the system. Therefore, a feedback system is a more reproducible and reliable system compare to an open loop system.

In the closed loop system, the disturbance will be reduced by the factor of H .

- **Set point**

Setpoint is the target value at which a controller attempts to maintain the process variable.

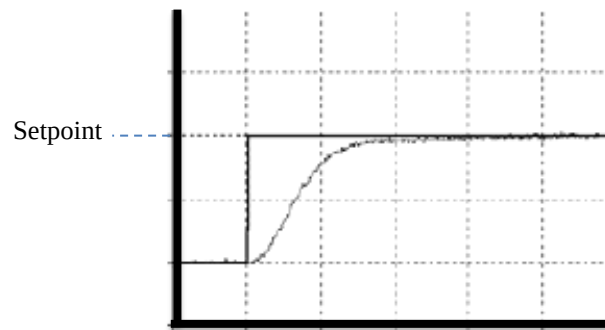


Figure 8. Schematic of a setpoint.

- **Process variable**

The parameters or quantities that we wish to control at the correct limit are called Process Variables. They are the current measured value of a particular part of a process which is continuously being monitored or controlled.

- **Plant**

A plant is a system to be controlled. In the control system, the plant has one or more outputs and one or more inputs. Sensors measure the plant outputs and actuators drive the plant inputs.

- **Settling Time**

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value.

- **Steady state error**

Steady-state error is a property of the input/output response for a linear system and is defined as the difference between the desired value and the actual value of a system output in the limit as time goes to infinity. In general, a good control system will be one that has a low steady-state error.

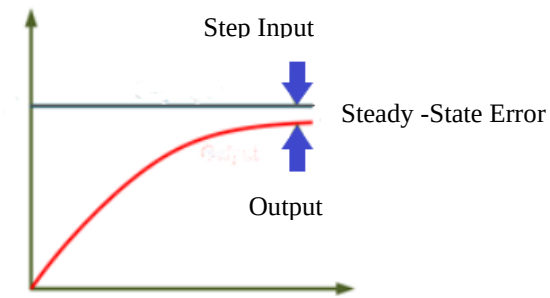


Figure 9. Schematic of a steady state error.

- **PID**

A PID (proportional integral derivative) controller is an instrument which is used in control applications to regulate the process variables, such as temperature, flow, pressure and speed. PID controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controller.

TASK 2

A

In the concept of control system, stability is a critical issue throughout the design process. The stability allows the system to reach the steady-state and remain in that state for that particular input even after variation in the parameters of the system. Therefore, it is an important property of a control system.

We can use the transfer function to check the stability and the response of a system for any given input. Basically, we can consider any system's response as some fraction of polynomial in S as a numerators and polynomial in S as a dominator.

$$\frac{Y(s)}{X(s)} = \frac{bs^m + b_1s^{m-1} + \dots + b_{m-1}s + b_m}{a_0s^n + a_1s^{n-1} + \dots + a_{n-1}s + a_n}$$

The above transfer function represents n^{th} plant. We can determine if a system is stable or unstable by looking at the roots of this polynomial dominator. The roots of nominator define the zeros and the roots of denominator define the poles of the transfer function.

To check the stability of a system, these poles values need to be plotted in a pole-zero diagram. This S-plane diagram has two axes, the horizontal (σ), which represents the real numbers and the vertical axis ($j\omega$) which represents the imaginary numbers. The poles of the system are represented by X and the zeros by O and we can recall S as;

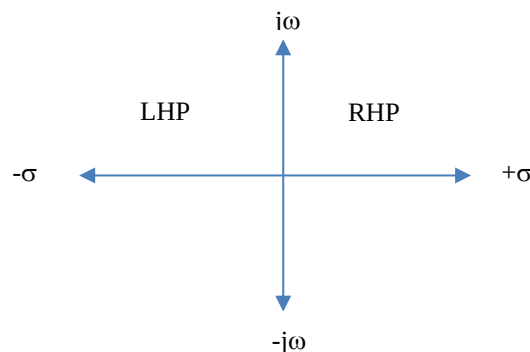


Figure 10. Pole-Zero diagram. The vertical axis ($j\omega$) represents the imaginary numbers and the horizontal axis (σ) represents the real numbers. The poles in the LHP side shows the stability of the system.

$$S = \sigma + j\omega$$

If the positions of all poles of the system are on the left-hand plane (LHP) the system is called a stable system and if they lie in right - hand plane (RHP), the system is an unstable system. If the poles lie on the imaginary axis, $j\omega$, then the system is called, marginally stable.

If our system has a more complicated transfer function, for example a 5th or 6th order polynomial denominator, then there are two ways to find the roots.

- Routh Hurwitz
- Root Locus

Routh Hurwitz

In the Routh Hurwitz way, stability criterion is having one necessary condition and one sufficient condition for stability. If any control system does not satisfy the necessary condition, then we can say that the control system is unstable. But, if the control system satisfies the necessary condition, then it may or may not be stable. So, the sufficient condition is helpful for knowing whether the control system is stable or not.

The necessary condition is that the signs of all coefficients of the characteristic denominator polynomial $D(s)$ should be the same. If one of them has a different sign, you can state that this system is unstable.

$$D(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots + a_n s^0$$

Where “a” are the coefficients of the S term. To find the roots we need to use the Routh array method. In this method, there is no need to calculate the roots of the characteristic equation. Fill the first two rows of the Routh array with the coefficients of the characteristic polynomial as mentioned in the table below. Start with the coefficient of s^n which is the highest S order and continue up to the s^0 . Fill the remaining rows of the Routh array with the elements as mentioned in the table below. Continue this process till you get the first column element of row s^0 .

First Column

s^n	a_0	a_2	a_4	
s^{n-1}	a_1	a_3	a_5	
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$	$b_3 = \dots$	
....	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	$c_2 = \frac{b_1 a_5 - b_3 a_1}{b_1}$	$c_3 = \dots$	
.....				
s^0	a_n			

Now, find the number of the sign changes in the first column of the Routh table. The number of sign changes in the first column of the Routh table gives the number of roots of characteristic equation that exist in the right half of the 's' plane and the control system is unstable.

Root Locus

The Root locus is the locus of the roots of the characteristic equation by varying system gain K from zero to infinity. K is the linear gain of the system and also;

$$G(s)H(s) = \frac{N(s)}{D(s)}$$

where, N(s) is the numerator of the and D(s) is the denominator of the polynomial.

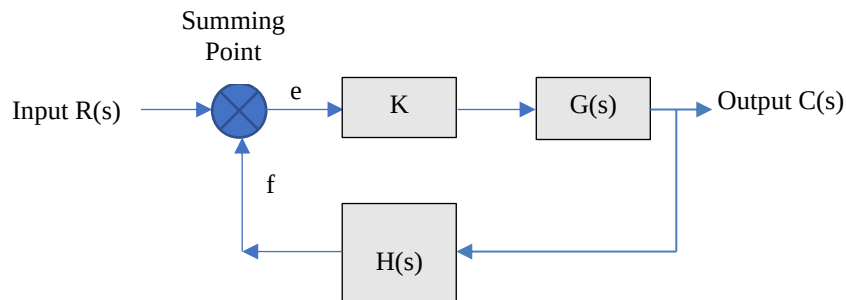


Figure 11. A schematic of a closed loop with varying K gain.

We can re- draw this block as:

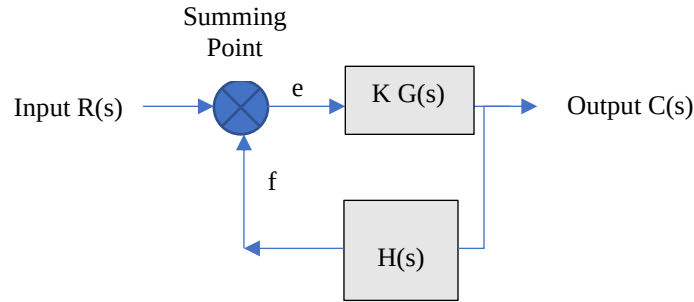


Figure 12. A schematic of a closed loop.

$$T.F = \frac{KG(s)}{1+KG(s)H(s)}$$

We are looking for the poles here, zeros come from $N(s)$ and poles come from $D(s)$.

$$1 + KG(s)H(s) = 1 + \frac{N(s)}{D(s)} = 0$$

$$D(s) + KN(s) = 0$$

We are going to vary the K from zero to infinity. If $K = 0$ then $D(s) = 0$. That means, the closed loop poles are equal to open loop poles when K is zero.

If $K \rightarrow \infty$ then $N(s) = 0$ which it means the closed loop poles are equal to the open loop zeros when K is infinity. From above two cases, we can conclude that the root locus branches start at open loop poles and end at open loop zeros. The number of branches is equal to the number of closed loop poles. Angle condition is used to know whether the point exist on root locus branch or not.

$$G(s)H(s) = -1 + j0$$

And the phase angle of $G(s)H(s)$ is:

$$\angle G(s)H(s) = (2n + 1)\pi$$

And the magnitude is:

$$|G(s)H(s)| = \sqrt{-1^2 + (0)^2} = 1$$

So, the magnitude condition is that the point at which the magnitude of the open loop transfer function is one.

TASK 3

A

When a force is applied on this material, it goes under stress:

$$\sigma = \frac{\text{Force}}{\text{Area}}$$

Therefore, the length of the material will change (it will be either longer or shorter). This deformation in the shape is both compression or tension and is called strain:

$$\varepsilon = \frac{\Delta L}{L}$$

which has a relation with stress by:

$$\sigma = E \text{ (Young's modulus)} \varepsilon$$

and can be measured using strain gauge. Strain gauge device contains of resistors and is sensitive to any small changes in the geometry of the object, where changing the length (strain) in the object will lead the change in its resistors.

In order to measure the strain (which we can then measure the stress of the object) with a strain gauge, it must be connected to an electric circuit which is capable of measuring the changes in resistance corresponding to strain.

The resistors in a Wheatstone bridge without any strain gauge have the same value of the resistance:

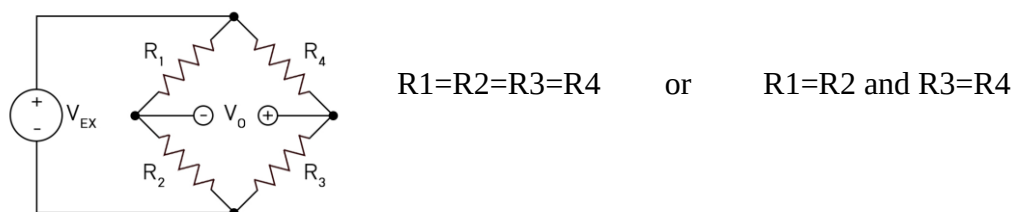


Figure 13. Wheatstone bridge. (Google pictures).

Therefore, the output voltage will show no potential difference. However, if one of these resistors are replaced with a strain gauge then the bridge will become unbalanced, and a voltage

will exist at the output terminals. The output voltage of the Wheatstone bridge circuit is expressed in millivolts.

Strain gauge transducers usually employ one to four strain gauge elements electrically connected to form a Wheatstone bridge circuit (See figure 14).

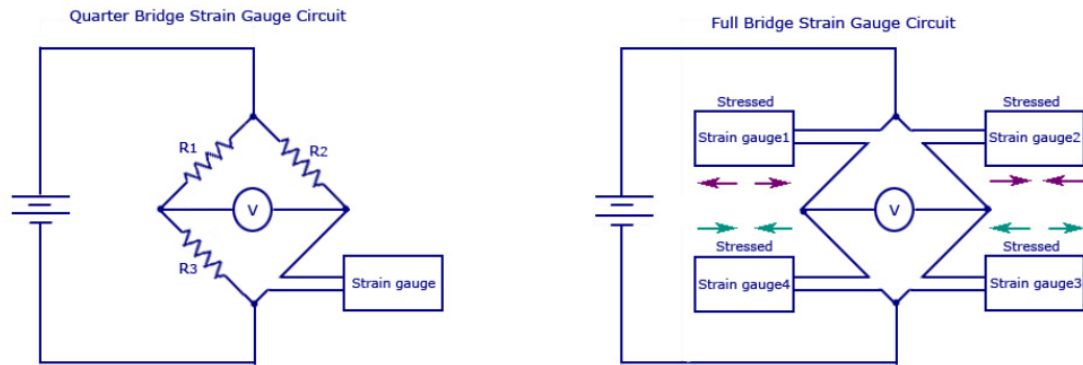


Figure 14. A schematic of a quarter (Left) and a full (Right) Wheatstone bridge strain gauge circuit (taken from InstrumentationToday.com)

Any small change in the resistance of the strain gauge will result the bridge unbalance and making it suitable for the detection of strain. If R_g (unstrained resistance) is the only active strain gauge, a small change in R_g (the difference between the strain and unstrained resistance) will result in an output voltage from the bridge. If the gauge factor is GF, the strain measurement is related to the change in R_g as follows:

$$\text{Gauge Factor (GF)} = \text{The rate of } \frac{\text{Change in Resistance}}{\text{Change in } L} = \frac{\frac{\Delta R}{R}}{\varepsilon}$$

$$GF = \frac{\Delta R_g}{\varepsilon R_g} \quad \therefore \quad \varepsilon = \frac{\Delta R_g}{GF \cdot R_g}$$

$$\frac{V_0}{V_{EX}} = \frac{-GF \cdot \varepsilon}{4} \left(\frac{1}{1 + GF \cdot \frac{\varepsilon}{2}} \right)$$

Where V_{EX} and V_0 are the input and output voltage, respectively.

The factors that affect the quality of measurements in this system, can be summarised as:

- **Temperature variation**

As this item is 20 meters away of its instrumentation, one of the most affective factors in the measurements would be the temperature variation. It can change the gauge resistance and causing variation in a measurable strain. There are a few ways to solve this problem for example, by using a dummy strain gauge or using a full bridge strain gauge, it is possible to eliminate the temperature affect.

Temperature also can alter the properties of the object which the strain gauge is attached to it. Differences in expansion coefficients between the gauge and base materials may cause dimensional changes in the sensor element. Therefore, a temperature compensation circuit would be needed.

- **Noise**

As the output signal is a very low-level voltage signal, any unwanted noise from other electrical devices affect the quality of measurement. For example, Capacitive coupling caused by the lead wires too close to AC power cables or ground currents are potential error sources in strain measurement. Other error sources may include magnetically induced voltages when the lead wires pass through variable magnetic fields, parasitic (unwanted) contact resistances of lead wires, insulation failure, and thermocouple effects at the junction of dissimilar metals. The sum of such interferences can result in significant signal degradation. Therefore, shielding the wires and guarding the instrumentation itself will minimize these effects.

- **Lead wire resistance**

The resistances of the wires which are in series connection with the strain gauge (R_{wire1} and R_{wire2}) contribute some value of resistance to the strain gauge's resistance (R_g) and lead to a false reading. This effect cannot be completely eliminated in this configuration but it is possible to minimise it by adding a third wire, connecting the right side of the voltmeter directly to the upper wire of the strain gauge (See figure 15).

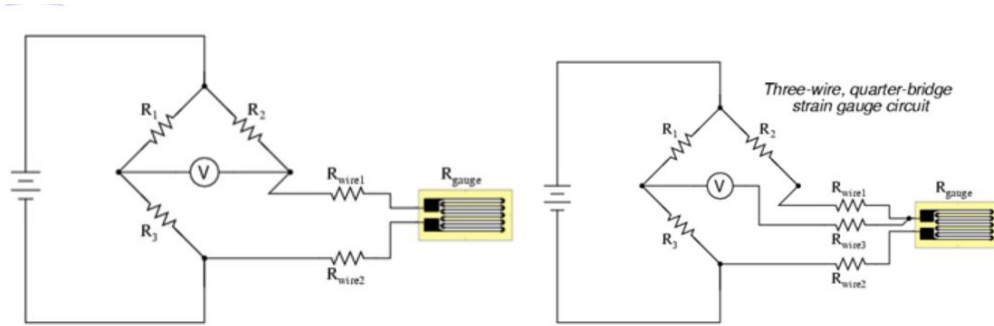


Figure 15. Eliminating the effect of the wire resistance in a quarter bridge strain gauge (Left) using the third wire (Right). (Taken from the sensorland.com)

- **Humidity**

Humidity can break the insulation between the gauge and the ground point. It also causes electro-chemical corrosion of gauge wire due to electrolysis.

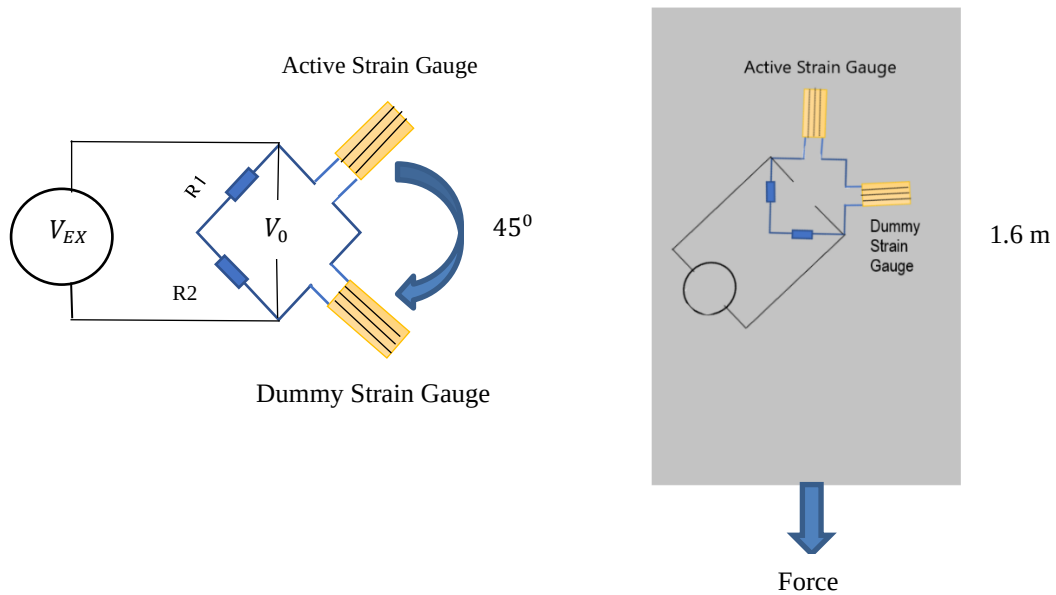


Figure 16. A sketch of a measurement system. It is a quarter bridge strain gauge circuit contains an active and a dummy gauge.

One of the sensors we can use is the quarter bridge strain gauge circuit with temperature compensation. It contains of input voltage (V_{EX}), Resistors $R1$ and $R2$, an active strain gauge and a dummy strain gauge. The active strain must be in a position and orientation so as to be exposed to physical strain and the dummy gauge, is isolated from all mechanical stress and acts as a temperature compensation device.

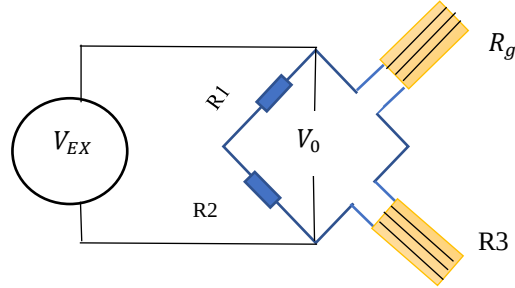


Figure 17. A schematic of a quarter bridge strain gauge system.

With maximum load of 5KN:

$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{5000}{0.06 \times 0.008} = 1042000 \text{ N/m}^2$$

$$\sigma = E \text{ (Young's modulus)} \varepsilon \quad \therefore \quad \varepsilon = \frac{\sigma}{E} = \frac{1042000}{10.3 \times 10^9} = 1.012 \times 10^{-4}$$

For example, If the input voltage is $V_{EX} = 10 \text{ V}$ and $GF = 2$ then the output voltage will be:

$$\frac{V_0}{V_{EX}} = \frac{-GF \cdot \varepsilon}{4} \left(\frac{1}{1 + GF \cdot \frac{\varepsilon}{2}} \right)$$

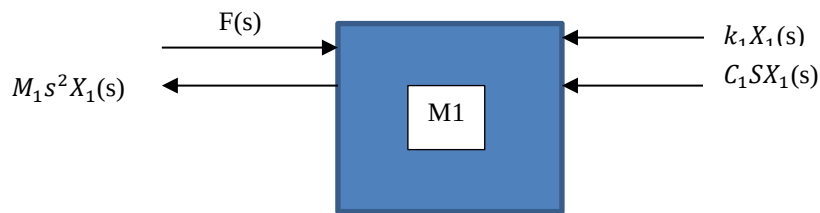
$$V_0 = (V_{EX}) \frac{-GF \cdot \varepsilon}{4} \left(\frac{1}{1 + GF \cdot \frac{\varepsilon}{2}} \right) = 10 \times \frac{-2 \times 1.012 \times 10^{-4}}{4} \left(\frac{1}{1 + 2 \times \frac{1.012 \times 10^{-4}}{2}} \right) =$$

$$V_0 = 5.06 \times 10^{-5} \text{ V}$$

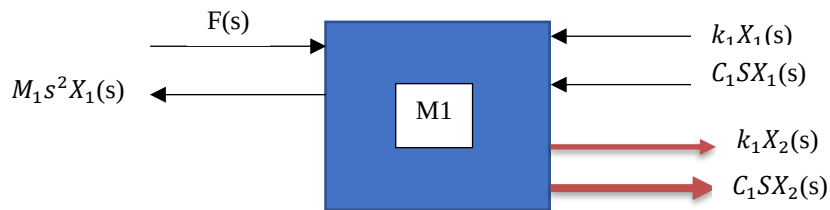
TASK 4

A

M2 is fixed and M1 is moving to the right, the free body diagram would be:

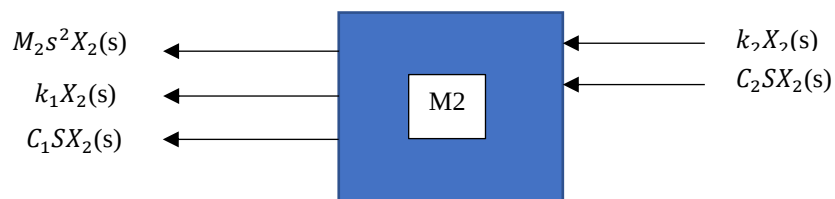


Now, M_1 is fixed and M_2 is moving to the right;



Eq-1	$F(s) + k_1X_2(s) + C_1SX_2(s) - M_1s^2X_1(s) - k_1X_1(s) - C_1SX_1(s) = 0$
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Now, M_1 is fixed and M_2 is moving to the right:



If M_2 is fixed and M_1 is moving to the right, then the total forces on M_2 will be:

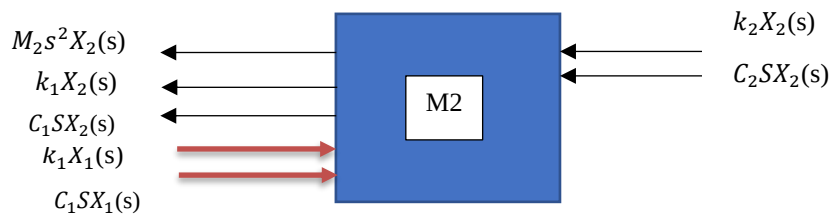


Figure 18. Free body diagram of two masses and springs and dampers.

Eq-2	$k_1X_1(s) + C_1SX_1(s) - M_2s^2X_2(s) - k_2X_2(s) - C_2SX_2(s) - k_1X_2(s) - C_1SX_2(s) = 0$
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$$F(s) + k_1X_2(s) + C_1SX_2(s) - M_1s^2X_1(s) - k_1X_1(s) - C_1SX_1(s) = 0$$

$$F(s) + X_1(s) [-M_1S^2 - k_1 - C_1S] + X_2(s) [k_1 + C_1S] = 0$$

$$\text{From Eq-2 } \therefore X_1(s) [k_1 + C_1S] = X_2(s) [M_2s^2 + k_2 + C_2S + k_1 + C_1S]$$

$$X_1(s) = \frac{X_2(s)[M_2s^2 + k_2 + C_2S + k_1 + C_1S]}{[k_1 + C_1S]}$$

$$F(s) + \frac{X_2(s)[M_2s^2 + k_2 + C_2S + k_1 + C_1S][-M_1S^2 - k_1 - C_1S]}{[k_1 + C_1S]} + X_2(s) [k_1 + C_1S] = 0$$

$$\text{T.F} = \frac{X_2(s)}{F(s)} = \text{Transfer Function} =$$

$$\frac{[k_1 + C_1S]}{S^4 (M_1M_2) + S^3 (C_1 M_2 + 2C_1M_1 + S^2 (M_2k_1 + M_1k_2 + C_1C_2 + k_1M_1) + S (k_2C_1 + k_1C_2) + (2k_1k_2 - k_1^2))}$$

B

$M_1 = 1.5 \text{ kg}$, $M_2 = 1 \text{ kg}$, $k_1 = 1$, $k_2 = 2$, $c_1 = 0.3$, $c_2 = 0.5$.

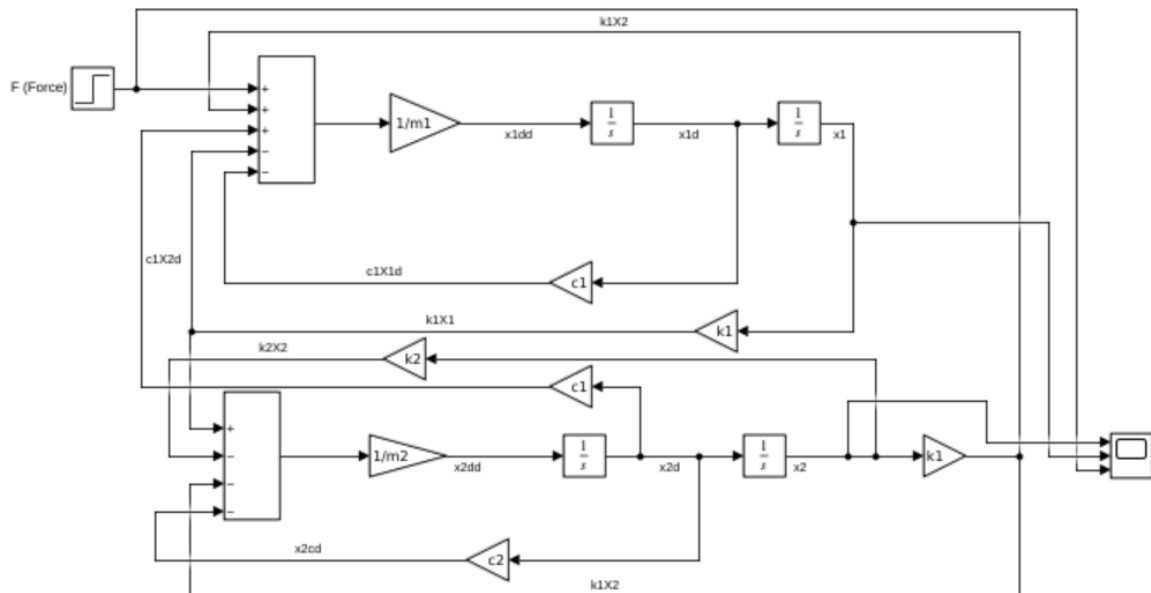
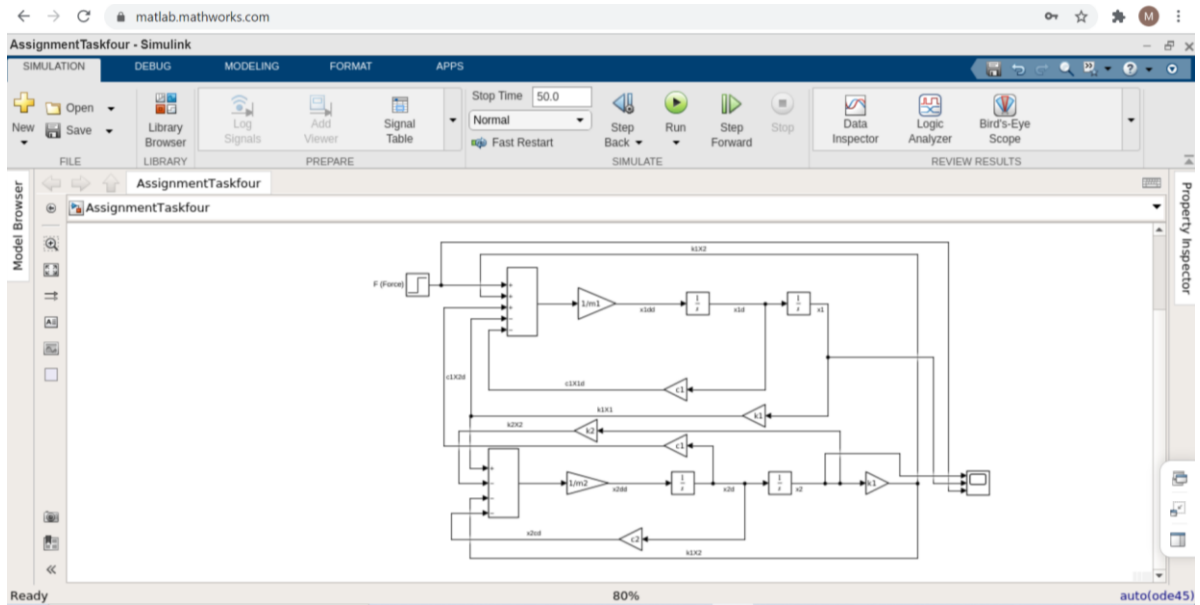


Figure 19. Simulink model of a two masses system.

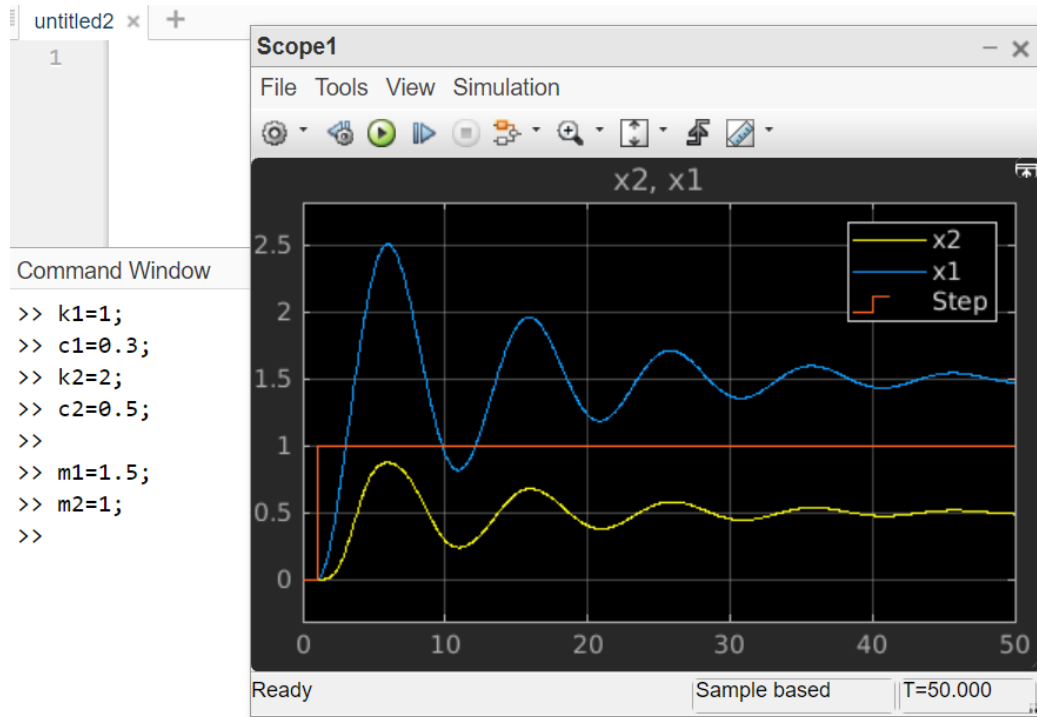


Figure 20. The scope of the two masses spring-damper Simulink model with parameters of: ($K_1=1$, $k_2=2$, $C_1=0.3$, $C_2=0.5$, $m_1=1.5$ and $m_2=1$) with stop time 50.

In this model, X_1 is the displacement of the M_1 and X_2 is the displacement of the M_2 . This scope shows the oscillations of the two masses because of the springs and dampers. As expected, the M_1 has a larger movement compared to the M_2 .

To make it more accurate, the maximum step size is 0.01 which means that there are calculations on every 0.01 seconds and the stop time is 50. The scope in the below shows the increase of the step time to 100.

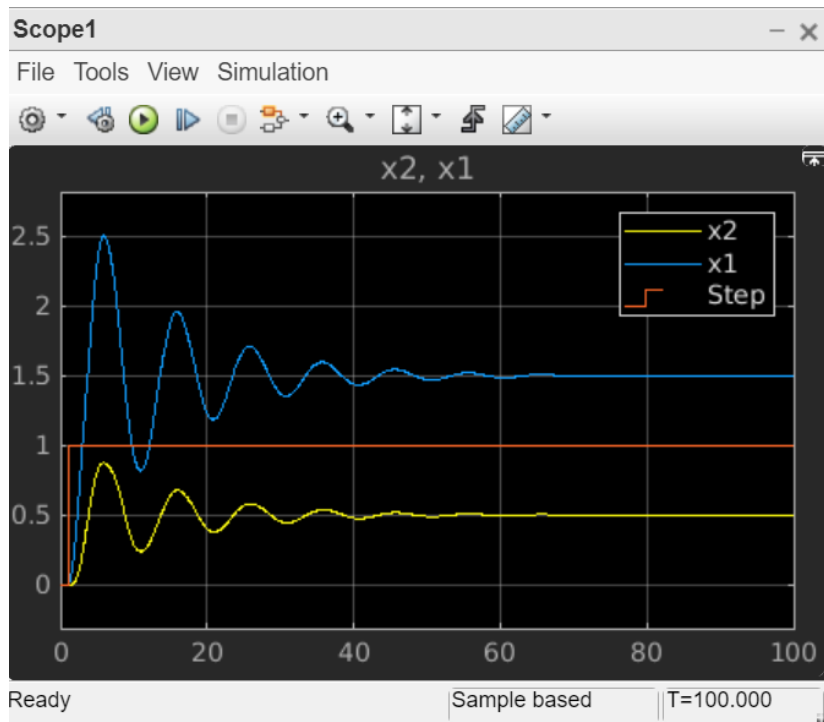


Figure 21. The scope of the two masses spring-damper Simulink model with parameters of: ($K_1=1$, $k_2=2$, $C_1=0.3$, $C_2=0.5$, $m_1=1.5$ and $m_2=1$) with stop time 100.

This scope shows how the amplitude of both masses decreases by time and they go through an underdamped response within a decaying envelope. The more underdamped the system, the more oscillations.

If the K increases, the response amplitude decreases, because the stiffer spring reduces the initial velocity of the mass in decreasing amount of time. In below scope, increasing the k_1 and k_2 to 3 and 4 respectively, we can see the reduction in amplitude and increasing in oscillations.

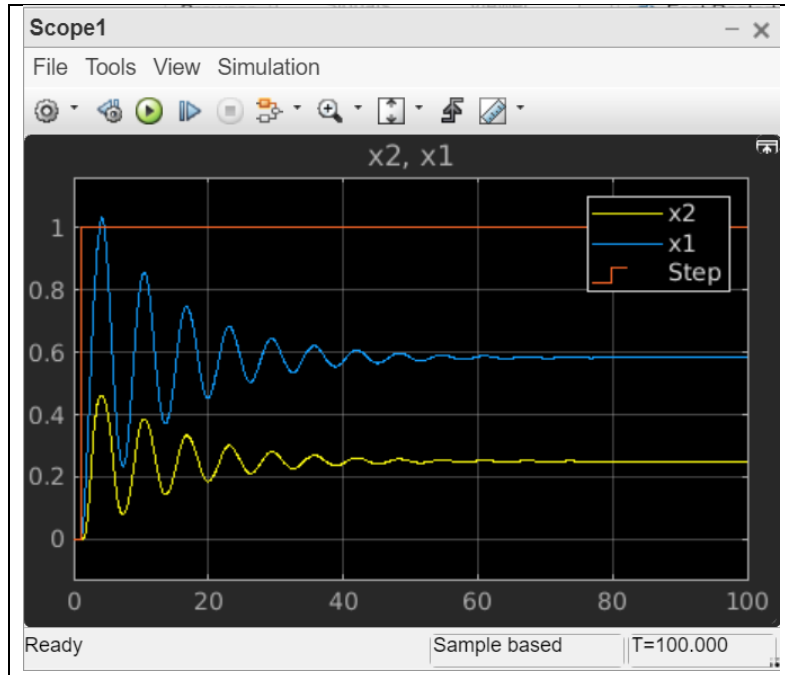


Figure 22. The scope of the two masses spring-damper Simulink model with parameters of: ($K_1=3$, $k_2=4$, $C_1=0.3$, $C_2=0.5$, $m_1=1.5$ and $m_2=1$) and stop time 100.

The below scope shows the reducing of the k_1 and k_2 to 0.9 and 0.95, respectively. The steady state part of the X_2 output is closer to the force (input) in this scope.

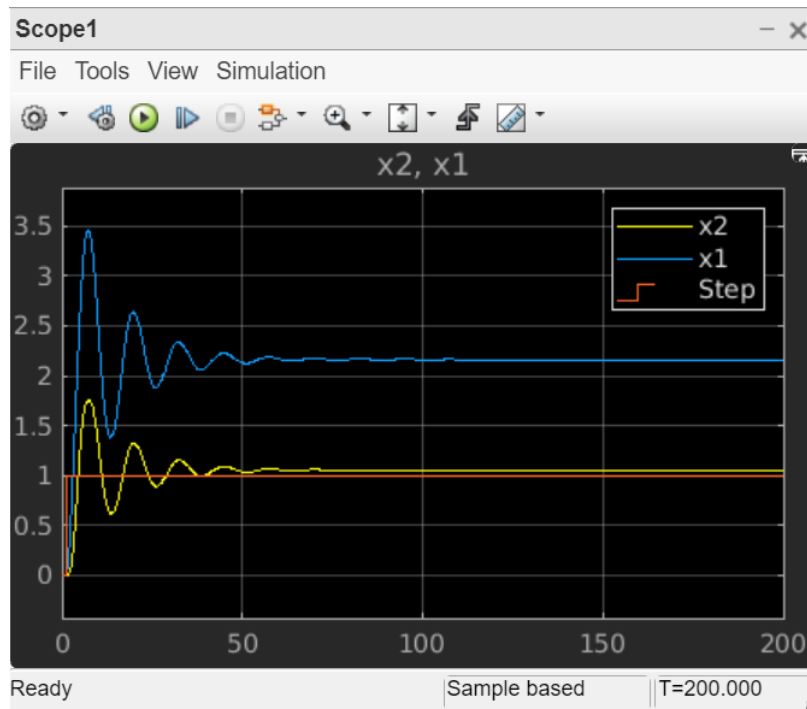


Figure 23. The scope of the two masses spring-damper Simulink model with parameters of: ($K_1=0.9$, $k_2=0.95$, $C_1=0.3$, $C_2=0.5$, $m_1=1.5$ and $m_2=1$) and stop time of 100.

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