2ai) Strain energy

$$U = \frac{1}{2} Q \delta$$

$$= \frac{1}{2} Q \gamma \delta s$$

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\gamma}$$

$$Q = \tau A \qquad \tau = \frac{Q}{A}$$

$$\therefore G = \frac{Q}{\gamma A} \quad \text{and} \quad \gamma = \frac{Q}{GA}$$

2 aii)

then

$$U = \frac{1}{2} Q \times \frac{Q}{AG} \times \delta s \qquad = \frac{1}{2} \times \frac{Q^2}{AG} \times \delta s$$

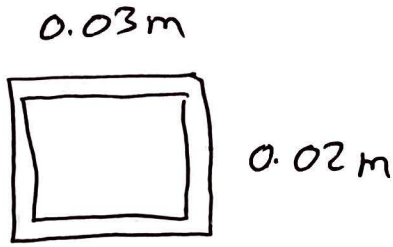
$$u = \frac{Q^2}{2AG} \times \delta s$$

from this total strain energy can be shown as

$$U = \frac{Q^2 L}{2AG}$$

Bryn Stubbings - 1702064

2bi)

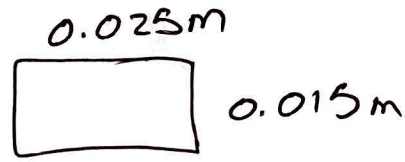


Thickness
25×10^{-3} m

Length
2.4 m

$$U = \frac{P^2 L}{2AE}$$

Internal dimensions



$$A = (0.03 \times 0.02) - (0.025 \times 0.015)$$

$$= 6 \times 10^{-4} - 3.75 \times 10^{-4}$$

$$= 2.25 \times 10^{-4} \text{ m}^2$$

$$U = \frac{(480 \times 10^6)^2 \times 2.4}{2 \times 2.25 \times 10^{-4} \times 190 \times 10^9}$$

$$= \frac{5.5 \times 10^{17}}{85.5 \times 10^6} \quad = 6.47 \times 10^9 \text{ J}$$

2 b ii) Potential energy $= Mgh$

$PE = mgh$

$U = \frac{P^2 L}{2AE}$

energy in tension

Assuming all Potential energy becomes strain energy

$$mgh = \frac{P^2 L}{2AE}$$

$$mgh = \frac{P^2 L}{2AE}$$

$$\therefore mgh = \frac{\sigma^2 AL}{2E}$$

$$h = \frac{\sigma^2 AL}{2Emg}$$ ← Volume ← weight

$$h = \frac{\sigma^2 AL}{2EW}$$

2biii)

$$h = \frac{\sigma^2 V}{2EW}$$

$$\sigma^2 = \frac{2EWh}{V} \qquad \sigma = \sqrt{\frac{2EWh}{V}}$$

$$\sigma = \frac{2 \times 190 \times 10^9 \times 20 \times 9.81 \times 0.45}{2.25 \times 10^{-4} \times 2.4}$$

$$\sigma = \frac{3.36 \times 10^{13}}{5.4 \times 10^{-4}} = 6.213 \times 10^{16} \text{ Pa}$$

2biv) As all parameters are the same except dimensions of bar / pipe from box to circle the area must be the same

~~V_1 must = V_2~~

~~8.25×10^{-4} =~~ first find inner diameter

$$2.25 \times 10^{-4} = \pi d_1^2 - \pi d_2^2$$

$$\pi d_2^2 = \pi d_1^2 - 2.25 \times 10^{-4}$$

2b) iv) $$d_2^2 = \frac{\pi d_1^2 - 2.25\times10^{-4}}{\pi}$$

$$d_2 = \sqrt{\frac{\pi d_1^2 - 2.25\times10^{-4}}{\pi}}$$

$$= \sqrt{\frac{\pi\, 0.02^2 - 2.25\times10^{-4}}{\pi}}$$

$$d_2 = 0.018\,m$$

$$\text{wall thickness} = \frac{d_1 - d_2}{2}$$

$$= \frac{0.02 - 0.018}{2}$$

$$= 0.0013\,m$$

4 a) $I_{xx} = 2(I_{xx}\ \text{C-Sections}) + 2(I_{xx}\ \text{Plates})$

$I_{xx} = (2 \times 26.2 \times 10^6) + 2 \times (I_g + Ah^2)$

$= 52.4 \times 10^6 + 2 \times \left(\left(\frac{300 \times 12^3}{12} \right) + (300 \times 12 \times 94^2) \right)$

$= 52.4 \times 10^6 + 63.7 \times 10^6$

$= 116.1 \times 10^6\ mm^4$

$I_{yy} = 2(I_{yy}\ \text{C-Sections}) + 2(I_{yy}\ \text{Plates})$

$I_{yy} = 2 \times (I_g + Ah) + 2\left(\frac{bd^3}{12} \right)$

$= 2 \times (1.57 \times 10^6 + 4400 \times 79) + 2 \times \left(\frac{12 \times 300^3}{12} \right)$

$= 3.84 \times 10^6 + 54 \times 10^6$

$= 57.84 \times 10^6\ mm^4$

$I_{yy} < I_{xx}$

the strut will tend to buckle about the y axis

4 b) $k = \sqrt{\frac{I}{A}}$ $I = 57.84 \times 10^6 \ mm^4$

$A = 2 \times 4400 + 2(300 \times 12)$

$A = 16{,}000 \ mm^2$

$k = \sqrt{\frac{57.84 \times 10^6}{16000}}$

$k = 60.12 \ mm$

4 c) Slenderness Ratio $= \frac{L}{k}$

$Sr = \frac{8}{60.12 \times 10^{-3}}$

$Sr = \cancel{0.133}$ 133.07

4 d) $P_e = \frac{\pi^2 EI}{L^2}$

$57.84 \times 10^6 \text{ mm}^4$
$= 57.84 \times 10^{-6} \text{ m}^4$

$$P_e = \frac{\pi^2 \times 200 \times 10^9 \times 57.84 \times 10^{-6}}{8^2}$$

$$P_e = 1.78 \times 10^6 \text{ N}$$

4 e) $P_R = \frac{\sigma_y A}{1 + a(L/k)^2}$

$$P_R = \frac{320 \times 10^6 \times \cancel{16 \times 10^{-3}}\ 0.016}{1 + \frac{1}{7500} \times 133.07^2}$$

$$P_R = 1.52 \times 10^6 \text{ N}$$

4f) $P_e = \frac{\pi^2 E I}{L^2}$

$I = 57.84 \times 10^6 + 2 \times \left(\frac{bd^3}{12} + Ah^2 \right)$

$= 57.84 \times 10^6 + 2 \left(\frac{224 \times 6^3}{12} + \left(224 \times 6 \times 153^2 \right) \right)$

$= 57.84 \times 10^6 + 62.9 \times 10^6$

$= 120.78 \times 10^6 \ mm^4$

$I = 120.78 \times 10^{-6} \ m^4$

$P_e = \frac{\pi^2 \times 200 \times 10^9 \times 120.78 \times 10^{-6}}{8^2}$

$P_e = 3.725 \times 10^6 \ N$

3a)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\cancel{\tau = \frac{TR}{J}} \quad T = \frac{\tau J}{R}$$

$$\sigma_1 - \sigma_3 = \sigma_y$$

$$325 \times 10^6 - -325 \times 10^6 = 650 \times 10^6$$

$$\sigma_y = 650 \times 10^6 \text{ Pa}$$

$$\text{Safety factor} = 6 = \frac{\sigma_y}{\text{Allowable working Stress}} \quad (\text{AWS})$$

$$\text{AWS} = \frac{\sigma_y}{6} = \frac{650 \times 10^6}{6}$$

$$\text{AWS} = 108.\dot{3} \times 10^6 \text{ Pa}$$

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi \times (37.5 \times 10^{-3})^4}{32} = 1.94 \times 10^{-7} \text{ m}^4$$

$$T = \frac{\tau J}{R}$$

$$T = \frac{108.\dot{3} \times 10^{6} \times 1.94 \times 10^{-7}}{3.5 \times 10^{-3}}$$

$$T = \cancel{560.3\ Nm}\ \cancel{598.6\ Nm}\ 600.3\ Nm$$

3b)

$$\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right) = 2\sigma_y^2$$

$$\sigma_1 = 325 \times 10^6$$

$$\sigma_2 = 0$$

$$\sigma_3 = -325 \times 10^6$$

$$2\sigma_y^2 = (325 \times 10^6)^2 + (325 \times 10^6)^2 + (-650 \times 10^6)^2$$

$$2\sigma_y^2 = 6.3375 \times 10^{17}$$

$$\sigma_y = \sqrt{\frac{6.3375 \times 10^{17}}{2}}$$

$$\sigma_y = 562.9 \times 10^6\ Pa$$

$$AWS = \frac{562.9 \times 10^6}{6}$$

$$AWS = 93.8 \times 10^6 \text{ pa}$$

$$T = \frac{93.8 \times 10^6 \times 1.94 \times 10^{-7}}{3.5 \times 10^{-3}}$$

$$T = \cancel{485.3 \text{ Nm}} \; 519.9 \text{ Nm}$$

3c) $\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right) = 2\sigma_y^2$

$\sigma_1 = 880 \times 10^6$

$\sigma_2 = 0$

$\sigma_3 = -880 \times 10^6$

$$\left((880 \times 10^6)^2 + (880 \times 10^6)^2 + (-1760 \times 10^6)^2\right) = 2\sigma_y^2$$

$$2\sigma_y^2 = 4.6464 \times 10^{18}$$

$$\sigma_y = \sqrt{\frac{4.6464 \times 10^{18}}{2}} = 1.52 \times 10^9 \text{ Pa}$$

3c) $T = \dfrac{\overset{552.7}{\cancel{1.52}} \times 10^{6} \times 6.136 \times 10^{-7}}{23.5 \times 10^{-3}}$

$T = 14.4 \times 10^{3}\ N$

$J = \dfrac{\pi \times (50 \times 10^{-3})^4}{32}$

$= 6.136 \times 10^{-7}$

$AWS = \dfrac{1.52 \times 10^{9}}{2.75}$

$= 552.7 \times 10^{6}$

3d) $\dfrac{14.4 \times 10^{3} - 519.9}{519.9} \times 100 = 2722$

an increase of 2722% is seen