

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

EXAMINATION PERIOD
MAY 2025

CYFNOD ARHOLIADAU
MAI 2025

HND ELECTRICAL AND ELECTRONIC ENGINEERING

MODULE CODE: GOEE4002

Dynamics Control and Instrumentation

TIME: 2 HOURS

ANSWER ALL FIVE QUESTIONS

Equipment or materials required – Calculators and a formulae booklet are allowed.

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Question One (20 marks)

A control system is being designed to regulate the temperature in a room. The system uses a heating element as the actuator, a temperature sensor as the feedback, and a controller that adjusts the heating element based on the sensor input.

- a. If the heating system in the room is designed as an open-loop system where the heating element is turned on for a fixed period, regardless of the room temperature.
- i. Using a simple block diagram, illustrate the open-loop system.

[4 marks]

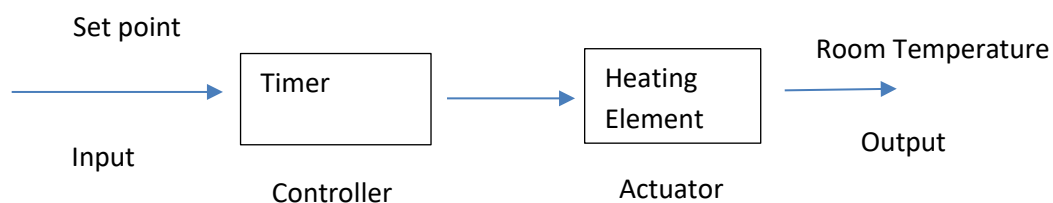


Figure 1

- ii. If the system experiences an external disturbance, such as a cold breeze entering the room, how will the open-loop system respond?

[1 mark]

In an open-loop system, if an external disturbance like a breeze enters the room, the system will not be able to adjust to the temperature change because there is no feedback mechanism to adjust its operation. It would simply continue its predetermined behaviour, possibly resulting in insufficient heating.

- b. In contrast, a closed-loop system adjusts the heating element based on the actual room temperature, using feedback from the temperature sensor. Using a simple block diagram, illustrate the feedback control loop for this closed-loop system, showing the components of the system.

[5 marks]

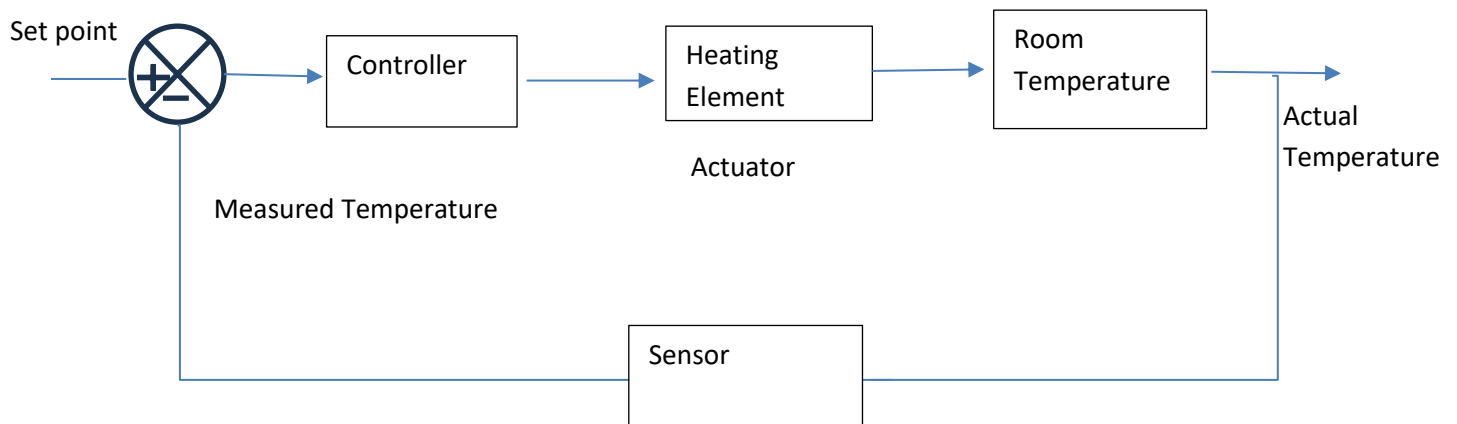


Figure 2

- c. The closed-loop system is designed with a proportional controller with gain, K_p , and the sensor has unity feedback. The heating element has a transfer function given by:

$$G(s) = \frac{1}{s + 2}$$

- i. Derive the closed-loop transfer function.

[4 marks]

$$T_{cl}(s) = \frac{K_p G(s)}{1 + K_p G(s)}$$

Substitute:

$$G(s) = \frac{1}{s + 2}$$

$$T_{cl}(s) = \frac{K_p \left(\frac{1}{s + 2} \right)}{1 + K_p \left(\frac{1}{s + 2} \right)}$$

$$T_{cl}(s) = \frac{\left(\frac{K_p}{s + 2} \right)}{\left(\frac{(s + 2) + K_p}{s + 2} \right)}$$

$$T_{cl}(s) = \frac{K_p}{s + (2 + K_p)}$$

- ii. Determine steady state temperature response T_{ss} , if the desired temperature is changed by a step input of $T_{set} = 30^\circ\text{C}$.

[6 marks]

The closed-loop transfer function is:

$$T_{cl}(s) = \frac{K_p}{s + (2 + K_p)}$$

The Laplace Transform of a step input $T_{set} = 30^\circ\text{C}$ is:

$$R(s) = \frac{30}{s}$$

The closed-loop output:

$$Y(s) = T_{cl}(s) \cdot R(s)$$

Substitute the transfer function and the step input:

$$Y(s) = \frac{K_p}{s + (2 + K_p)} \cdot \frac{30}{s}$$

$$Y(s) = \frac{30K_p}{s(s + (2 + K_p))}$$

To find the steady-state response, we use the final value theorem:

$$T_{ss} = \lim_{s \rightarrow 0} s \cdot Y(s)$$

Substituting for $Y(s)$:

$$T_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{30K_p}{s(s + (2 + K_p))}$$

$$T_{ss} = \lim_{s \rightarrow 0} \frac{30K_p}{s + (2 + K_p)}$$

$$T_{ss} = \frac{30K_p}{2 + K_p}$$

Question Two (20 marks)

A block of mass $m=5\text{kg}$ is placed on a rough horizontal surface. The coefficient of friction between the block and the surface is $\mu=0.3$. A force $F = 20\text{N}$ is applied horizontally to the block.

- a. Using Newton's second law, calculate the acceleration of the block. Assume that the applied force is greater than the force of friction and that frictional force is kinetic.

[4 marks]

$$F_f = \mu \cdot m \cdot g$$

$$F_f = 0.3 \times 5 \times 9.81 = 14.715\text{N}$$

$$F_{net} = ma$$

$$F - F_f = ma$$

$$20 - 14.715 = 5a$$

$$a = \frac{5.285\text{N}}{5\text{kg}}$$

$$a = 1.057\text{ m/s}^2$$

- b. The block is subjected to a PID controller that adjusts the applied force F to maintain the block's speed at a constant value of 2m/s . The transfer function of the system, representing the relationship between the force and velocity is given by:

$$G(s) = \frac{1}{ms + b}$$

- i. If $b = 0.5\text{Ns/m}$ is the damping coefficient, use the transfer function to derive the steady state applied force to maintain the block's velocity at 2m/s in the presence of friction.

[4 marks]

The damping force is given by:

$$F_d = -bv$$

$$F_d = -0.5\text{Ns/m} \times 2\text{ m/s} = -1\text{N}$$

$$F_{ss} = F_f + |F_d|$$

$$F_{ss} = 14.715 + 1 = 15.715\text{N}$$

- ii. Calculate $G(0)$ and explain its significance in determining the steady state-applied force.

[4 marks]

The transfer function of the system is given by:

$$G(s) = \frac{1}{ms + b}$$

At steady state, $s = 0$

The transfer function at steady state is given by:

$$G(0) = \frac{1}{m(0) + b}$$

$$b = 0.5 \text{Ns/m}$$

$$G(0) = \frac{1}{0.5} = 2 \text{m/Ns}$$

$G(0)$ shows us that the system's response to the applied force in steady state is 2m/Ns , which tells us how much velocity we get for each unit of applied force, considering the damping factor.

- c. Assume that the block is connected by a string to another block of mass, $m_2 = 8 \text{kg}$, which hangs vertically off the edge of the surface. The string passes over a pulley. The system is controlled by a feedback loop that adjusts the tension in the string to keep the blocks moving at a constant acceleration of 2m/s^2 . Write down the equations of motion for both blocks and derive the required tension T using Newton's second law.

[4 marks]

For Block 1 (on the surface):

Using Newton's second law:

$$F - F_f - T = m_1 \cdot a$$

$$F - 14.715 - T = 5 \text{kg} \times 2 \text{m/s}^2$$

$$F - T = 24.715$$

For block 2(hanging vertically):

$$m_2 g - T = m_2 \cdot a$$

$$8 \times 9.81 - T = 8 \times 2$$

$$T = 62.48N$$

Substituting for T into the first equation:

$$F - 62.48 = 24.715$$

$$F = 87.195N$$

- d. Consider that the position of Block 1(the one on the surface) is measured using a position sensor which provides a voltage output that is proportional to the position of the block, with a scaling factor of $0.01V/m$. If the block moves at a constant velocity of $2m/s$, calculate the voltage output of the sensor after 5 seconds.

[4 marks]

$$x = vt$$

$$x = 2 \text{ m/s} \times 5s = 10m$$

The voltage output is related to the position by the scaling factor:

$$V = \text{Scaling factor} \times \text{Position}$$

$$V = 0.01 V/m \times 10m$$

$$V = 0.1V$$

Question Three (20 marks)

A unity feedback system has the following open loop transfer function:

$$G(s) = \frac{(s + 3)}{(s + 4)(s^2 + 3s + 6)}$$

- a. Draw the poles and zeros diagram, then:

[8 marks]

For zeros:

$$s + 3 = 0$$

$$s = -3$$

Poles:

$$(s + 4)(s^2 + 3s + 6)$$

$$s + 4 = 0$$

$$s = -4$$

$$s^2 + 3s + 6 = 0$$

Solving for s:

$$s = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$s = \frac{-3 \pm \sqrt{-15}}{2}$$

$$s = \frac{-3 \pm j\sqrt{15}}{2}$$

$$s = -\frac{3}{2} + \frac{j\sqrt{15}}{2}$$

And

$$s = \frac{-3}{2} - \frac{j\sqrt{15}}{2}$$

Zeros:

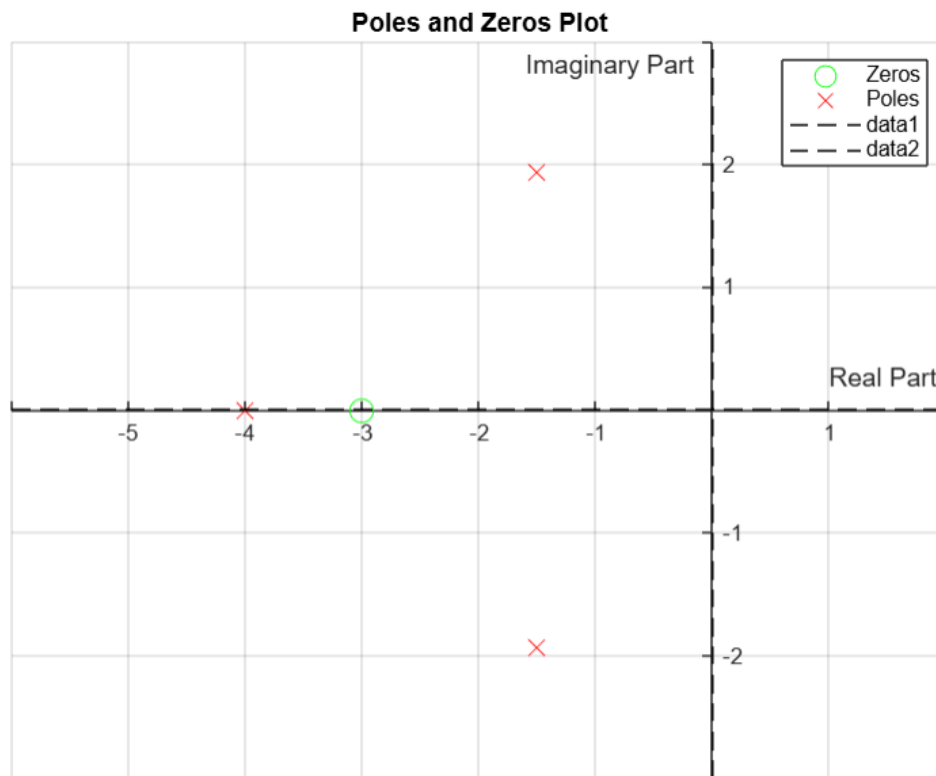
$$s = -3$$

Poles:

$$s = -4$$

$$s = -1.5 + \frac{j\sqrt{15}}{2}$$

$$s = -1.5 - \frac{j\sqrt{15}}{2}$$



b. Evaluate the value of $G(s)$ at the point $s = -2 + j$.

[12 marks]

To evaluate $G(s)$ at $s = -2 + j$

Substitute $s = -2 + j$ into the transfer function:

$$G(s) = \frac{(s + 3)}{(s + 4)(s^2 + 3s + 6)}$$

Numerator:

$$(-2 + j) + 3 = 1 + j$$

Denominator first term:

$$\begin{aligned} s + 4 \\ (-2 + j) + 4 = 2 + j \end{aligned}$$

Second term:

$$s^2 + 3s + 6:$$

$$(-2 + j)^2 + 3(-2 + j) + 6$$

$$(-2 + j)^2 = 4 - 4j - 1 = 3 - 4j$$

$$3(-2 + j) + 6 = -6 + 3j + 6 = 3j$$

$$s^2 + 3s + 6 = 3 - 4j + 3j = 3 - j$$

$$G(s) = \frac{1+j}{(2+j)(3-j)}$$

Multiplying across the brackets in the denominator:

$$(2+j)(3-j) = 6 - 2j + 3j + 1 = 7 + j$$

$$G(s) = \frac{1+j}{7+j}$$

$$G(s) = \frac{(1+j)(7-j)}{(7+j)(7-j)}$$

$$G(s) = \frac{7 + 7j - j + 1}{49 + 1} = \frac{8 + 6j}{50}$$

$$G(s) = 0.16 + j0.12$$

Question Four (20 marks)

For the given system block diagram:

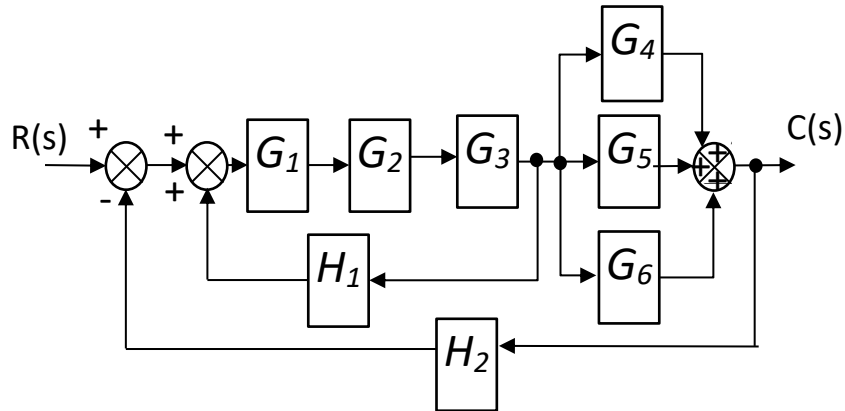
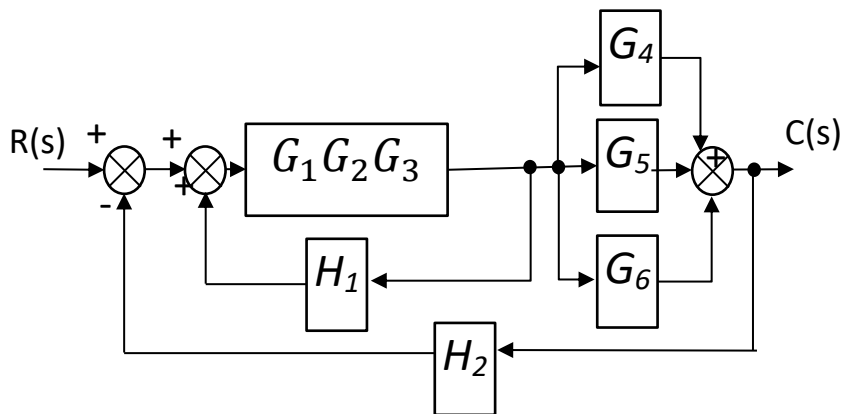


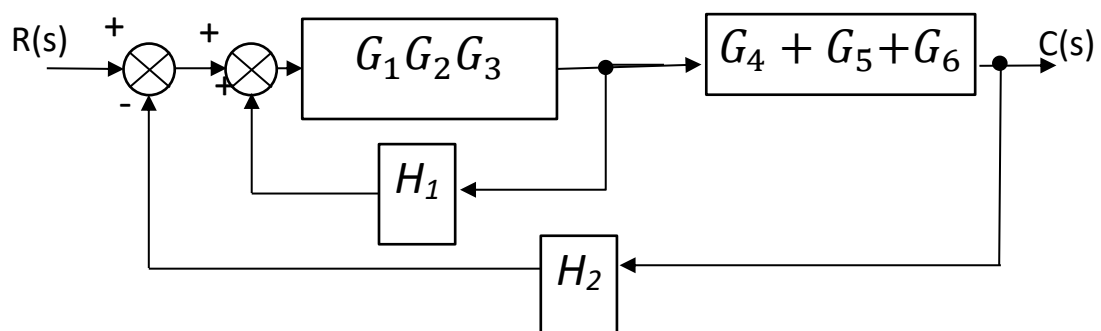
Figure 1

Determine the transfer function ($\frac{Y(s)}{R(s)}$) of the system using block diagram reduction method and draw all the reduction steps.

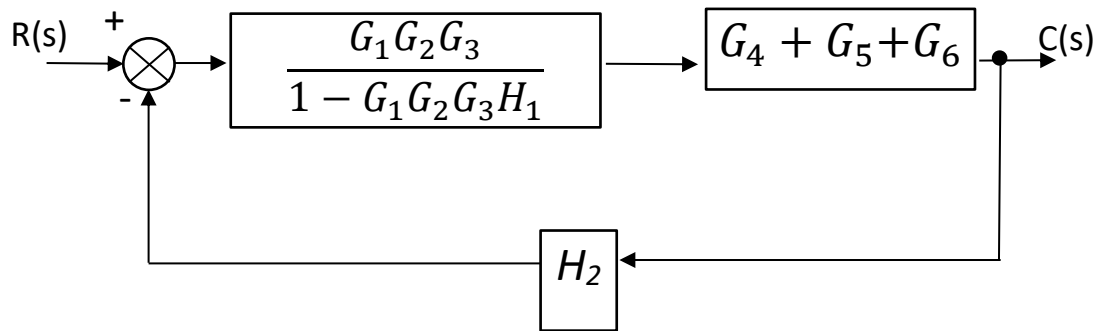
Reduce the 3 directly connected blocks in cascade into a single block:



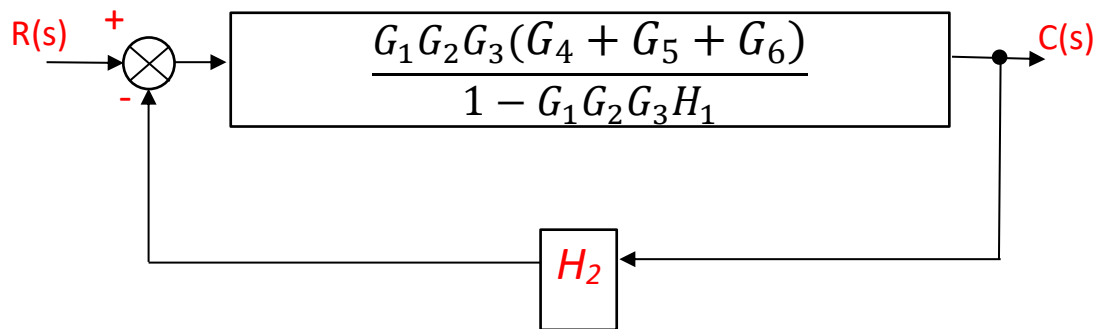
Reduce the blocks connected in parallel:



Simplifying the internal closed-loop system, the overall gain is given by:



Reducing the two blocks in series:



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3(G_4 + G_5 + G_6)}{1 - G_1G_2G_3H_1}}{1 + \left[\frac{G_1G_2G_3(G_4 + G_5 + G_6)}{1 - G_1G_2G_3H_1} \right] H_2}$$

Simplifying:

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3(G_4 + G_5 + G_6)}{(1 - G_1G_2G_3H_1) + G_1G_2G_3(G_4 + G_5 + G_6)H_2}$$

Question Five (20 marks)

An RLC circuit has resistance, $R = 150\Omega$ inductance, $L = 2H$, and capacitance, $C = 25\mu F$.

The current in the circuit is given by;

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{CL}$$

- a. What is the natural frequency of the circuit?

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{CL}$$

Second order linear homogeneous differential equation:

$$\frac{d^2 \theta_0}{dt^2} + 2\zeta \omega_n \frac{d\theta_0}{dt} + \omega_n^2 \theta_0 = b_1 \omega_n^2 \theta_0$$

Comparing coefficients

The natural frequency is given by:

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\omega_n = \frac{1}{\sqrt{2 \times 25 \times 10^{-6}}} = 141.421 \text{ rad/s}$$

[4 marks]

- b. Determine whether the system overdamped, critically damped or underdamped?

[4 marks]

$$\frac{R}{L} = 2\zeta \omega_n$$

$$\frac{150}{2} = 2\zeta \times 141.421 \text{ rad/s}$$

Solving for ζ :

$$\zeta = \frac{75}{282.84} = 0.265$$

Since $\zeta < 1$ the system is underdamped.

- c. What is the damped frequency of oscillation of the circuit?

[4 marks]

The damped frequency is given by:

$$\begin{aligned}\omega_d &= n\sqrt{1 - \zeta^2} \\ \omega_d &= 141.421\sqrt{1 - 0.265^2} \\ \omega_d &= 136.366 \text{ rad/s}\end{aligned}$$

d. Calculate the time constant τ for the circuit.

[4marks]

$$\begin{aligned}\tau &= \frac{1}{\zeta\omega_n} \\ \tau &= \frac{1}{0.265 \times 141.421} \\ \tau &= 0.02668 \text{ s}\end{aligned}$$

e. If the applied voltage is

$$V(t) = V_0$$

Where $V_0 = 10V$ is constant, find the steady- state current, i_{ss} of the circuit?

[4 marks]

When the applied voltage is constant, $V(t) = V_0$, the circuit reaches a steady state where the transient behaviour settles. The inductor behaves like a short circuit and the capacitor behaves like an open circuit. The current in the circuit is governed by the resistor alone, since the voltage across the inductor and capacitor no longer changes.

We therefore use Ohm's Law:

$$\begin{aligned}i_{ss} &= \frac{V_0}{R} \\ i_{ss} &= \frac{10}{150} = 0.0667 \text{ A} = 66.7 \text{ mA}\end{aligned}$$

Good luck

End of the exam