

1809810 - SGEE5000

Assignment II

by David Thomas

FILE	16526_DAVID_THOMAS_1809810_-_SGEE5000_ASSIGNMENT_II_177684_1114115086.PDF (884.48K)		
TIME SUBMITTED	15-MAY-2020 03:09PM (UTC+0100)	WORD COUNT	7322
SUBMISSION ID	127790917	CHARACTER COUNT	47611

HNC Electrical and Electronic Engineering
SGEE 5000 – Electronics II
Active Filter Design

Author – David Mathew Thomas
Student ID - 1809810
Date – 19-05-2020
Word Count – TBC 2739

Table of Contents

1. Introduction, aims and objectives	6
2. Research into filter design	7
3. Design method, choices and calculations	12
3.1. 5 th Order Bessel – Low-pass Filter	12
3.2. 5 th Order Tschebyscheff – Low-pass Filter	15
3.3. 5 th Order Butterworth – Low-pass Filter	18
3.4. 3 rd Order Butterworth – High-pass Filter	21
3.5. 6 th Order Butterworth – High-pass Filter	23
3.6. 9 th Order Butterworth – High-pass Filter	25
4. Testing and improving the design	28
4.1. Low-pass Filter	28
4.2. High-pass Filter	32
5. Conclusions	36
6. Reference List & Bibliography	37
7. Appendix	39

Table of Figures

Figure 1 - Example of an Active Low-Pass Filter (incorporating resistors, capacitors and an op-amp.....	7
Figure 2 - Types of Active Filters.....	8
Figure 3 - Butterworth Filter.....	8
Figure 4 - Bessel Filter – Group Delay.....	9
Figure 5 - Example of a Noise Filter with signal pre and post.....	10
Figure 6 – Bessel Coefficients Table	12
Figure 7 - Bessel 5th Order Low-pass Filter	14
Figure 8 - Tschebyscheff Coefficients Table	15
Figure 9 - Tschebyscheff 5th Order Low-pass Filter	17
Figure 10 - Butterworth Coefficients Table	18
Figure 11 - Butterworth 5th Order Low-pass Filter	20
Figure 12 - Butterworth Coefficient Table	21
Figure 13 - Butterworth 3rd Order High-pass Filter	22
Figure 14 - Butterworth Coefficient Table.....	23
Figure 15 - Butterworth 6th Order High-pass Filter.....	24
Figure 16 - Butterworth Coefficient Table.....	25
Figure 17 - Butterworth 9th Order High-pass Filter.....	27
Figure 18 - Bessel 5th Order Filter Roll Off	28
Figure 19 - Butterworth 5th Order Filter Roll Off	28
Figure 20 - Tschebyscheff 5th Order Filter Roll Off	29
Figure 21 - Bessel 5th Order Low-pass Filter Logarithmic Frequency.....	29
Figure 22 - Butterworth 5th Order Low-pass Filter Logarithmic Frequency.....	30
Figure 23 - Tschebyscheff 5th Order Low-pass Filter Logarithmic Frequency	30
Figure 24 - Filter comparison.....	31
Figure 25 - Butterworth 3rd Order High-pass Filter Roll Off.....	32
Figure 26 - Butterworth 6th Order High-pass Filter Roll Off.....	32
Figure 27 - Butterworth 9th Order High-pass Filter Roll Off.....	33
Figure 28 – Butterworth 3rd Order High-pass Filter Logarithmic Frequency	33
Figure 29 - Butterworth 6th Order High-pass Filter Logarithmic Frequency	34
Figure 30 - Butterworth 9th Order High-pass Filter Logarithmic Frequency	34
Figure 31 - Butterworth High-pass Filter Comparison	35
Figure 32 - Butterworth 9th Order High-pass Filter.....	39

Table of Tables

Table 1 - Filter Summary Comparison	10
Table 2 - Bessel 5th Order – Calculated Low-pass Filter Components	13
Table 3 - Bessel 5th Order – Simulated Low-pass Filter Component Values	14
Table 4 - Tschebyscheff 5th Order – Calculated Low-pass Filter Components.....	16
Table 5 - Tschebyscheff 5th Order – Simulated Low-pass Filter Component Values.....	17
Table 6 - Butterworth 5th Order – Calculated Low-pass Filter Components.....	19
Table 7 - Butterworth 5th Order – Simulated Low-pass Filter Component Values.....	20
Table 8 - Butterworth 3rd Order – Calculated High-pass Filter Components.....	21
Table 9 - Butterworth 3rd Order – Simulated Low-pass Filter Component Values	22
Table 10 - Butterworth 6th Order – Calculated High-pass Filter Components.....	24
Table 11 - Butterworth 6th Order – Simulated Low-pass Filter Component Values	24
Table 12 - Butterworth 9th Order – Calculated High-pass Filter Components.....	26
Table 13 - Butterworth 9th Order – Simulated Low-pass Filter Component Values	27
Table 14 - Low-pass Filter Frequency Comparison	29
Table 15 - Low-pass Filter Logarithmic Frequency Comparison	30
Table 16 - Butterworth High-pass Filter Frequency Comparison.....	33
Table 17 - Butterworth High-pass Filter Logarithmic Frequency Comparison	34

Table of Equations

Equation 1 - Bessel 5th Order Stage 1	12
Equation 2 - Bessel 5th Order Stage 1	12
Equation 3 - Bessel 5th Order Stage 2 Capacitor 2 value	12
Equation 4 - Bessel 5th Order Stage 2 Resistor	12
Equation 5 - Bessel 5th Order Stage 2 R1 Value	13
Equation 6 - Bessel 5th Order Stage 2 R2 Value	13
Equation 7 - Bessel 5th Order Stage 3 Capacitor 2 value	13
Equation 8 - Bessel 5th Order Stage 3 Resistor	13
Equation 9 - Bessel 5th Order Stage 3 R1 Value	13
Equation 10 - Bessel 5th Order Stage 3 R2 Value	13
Equation 11 - Tschhebyscheff 5th Order Stage 1	15
Equation 12 - Tschhebyscheff 5th Order Stage 1	15
Equation 13 - Tschhebyscheff 5th Order Stage 2 Capacitor 2 value	15
Equation 14 - Tschhebyscheff 5th Order Stage 2 Resistor	15
Equation 15 - Tschhebyscheff 5th Order Stage 2 R1 Value	16
Equation 16 - Tschhebyscheff 5th Order Stage 2 R2 Value	16
Equation 17 - Tschhebyscheff 5th Order Stage 3 Capacitor 2 value	16
Equation 18 - Tschhebyscheff 5th Order Stage 3 Resistor	16
Equation 19 - Tschhebyscheff 5th Order Stage1 R1 Value	16
Equation 20 - Tschhebyscheff 5th Order Stage1 R2 Value	16
Equation 21 - Butterworth 5th Order Stage 1	18
Equation 22 - Butterworth 5th Order Stage 1	18
Equation 23 - Butterworth 5th Order Stage 2 Capacitor 2 value	18
Equation 24 - Butterworth 5th Order Stage 2 Resistor	18
Equation 25 - Butterworth 5th Order Stage 2 R1 Value	19
Equation 26 - Butterworth 5th Order Stage 2 R2 Value	19
Equation 27 - Butterworth 5th Order Stage 3 Capacitor 2 value	19
Equation 28 - Butterworth 5th Order Stage 3 Resistor	19
Equation 29 - Butterworth 5th Order Stage 3 R1 Value	19
Equation 30 - Butterworth 5th Order Stage 3 R2 Value	19
Equation 31 - Butterworth 3rd Order Stage 1 R1	21
Equation 32 - Butterworth 3rd Order Stage 2 R1	21
Equation 33 - Butterworth 3rd Order Stage 2 R2	21
Equation 34 - Butterworth 6th Order Stage 1 R1	23
Equation 35 - Butterworth 6th Order Stage 1 R1	23
Equation 36 - Butterworth 6th Order Stage 2 R1	23
Equation 37 - Butterworth 6th Order Stage 2 R2	23
Equation 38 - Butterworth 6th Order Stage 3 R1	24
Equation 39 - Butterworth 6th Order Stage 3 R2	24
Equation 40 - Butterworth 9th Order Stage 1 R1	25
Equation 41 - Butterworth 9th Order Stage 2 R1	25
Equation 42 - Butterworth 9th Order Stage 2 R2	25
Equation 43 - Butterworth 9th Order Stage 3 R1	26
Equation 44 - Butterworth 9th Order Stage 3 R2	26
Equation 45 - Butterworth 9th Order Stage 4 R1	26
Equation 46 - Butterworth 9th Order Stage 4 R2	26
Equation 47 - Butterworth 9th Order Stage 5 R1	26
Equation 48 - Butterworth 9th Order Stage 5 R2	26

1. Introduction, aims and objectives

The project involved the design and simulation of three types of filter; Bessel, Butterworth and Tschebyscheff with the aim of designing 5th order low-pass filters for Bessel, Butterworth and Tschebyscheff, as well as 3rd, 6th and 9th order Butterworth high-pass filters. Once designed, the objective of the project is to compare characteristics of each of the different 5th order filter types, along with those of the Butterworth high-pass filter at the various stages.

Initially research was undertaken to investigate the differences between the three main types of filter designs, Bessel, Butterworth and Tschebyscheff. Research was also carried out into how these filters are used in modern technology and how they are implemented and used in two specific control processes, to change and alter either phase or amplitude of a signal's frequency.

Initial design stages involved that of the 5th order low-pass filters and the one high-pass filter, which was designed with three different stages. Information would be taken for the respective filter types' coefficient table for the required calculations to be made. Each filter's component would be created from the calculations. With the components calculated, the circuits are then built, tested and analysed in Multisim.

2. Research into filter design

Filters are electrical circuits which allow or block components of a particular frequency. Early filter designs were of a passive nature, incorporating such elements as resistors, capacitors and inductors, however as the need to improve filter designs and network power quality became apparent, the concept of active filters was created. (Rashid, 2011)

As the name suggests, active filters contain an active element/component, most commonly in the form of an operational amplifier (op-amp), used alongside passive features such as resistors and capacitors. With inductors no longer required as part of the circuit, interference from magnetic field signals is removed. (Engineering_360, 2020)

Active filters provide an improved performance, with the presence of an output to input feedback resulting in a more stable signal and an increase in the variety of frequency response. (Engineering_360, 2020). The presence of feedback, as well as no need for inductors, also enables the use of lower component values, in particular capacitor values, meaning resultant filters are smaller and cheaper. (Engineering_360, 2020) Additionally, there is no insertion loss, and control over input and output impedance can be achieved through interstage isolation. (Elprocus, 2020)

Active filters can be grouped/classified in accordance with the frequencies that they block/allow; (Unknown, 2020)

- Low-pass filter: block/limit signals above a set (cut-off) frequency
- High-pass filter: block/limit signals below a set (cut-off) frequency
- Band-pass filter: block/limit signals outside a frequency range, or pass those within a frequency range
- Band-reject, band stop filter: block/limit signals within a frequency range

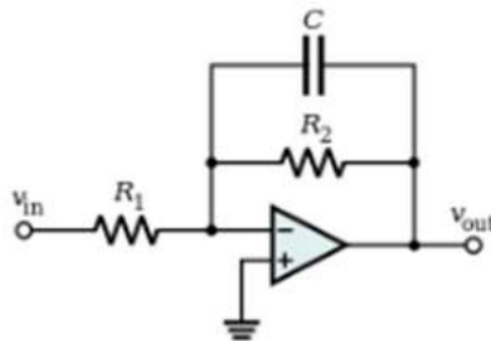


Figure 1 - Example of an Active Low-Pass Filter (incorporating resistors, capacitors and an op-amp

(Engineering_360, 2020)

There are four main active filter types, Butterworth, Tschebyscheff, Bessel and Elliptical, with Butterworth, Tschebyscheff and Bessel being the most common. No one filter type possess all the perfect characteristics, and as such in order to make the most appropriate filter choice for a particular application, a comparison of the characteristics, taking into account such factors as phase response, roll-off, passband gain, should be made. (Island, 2010)

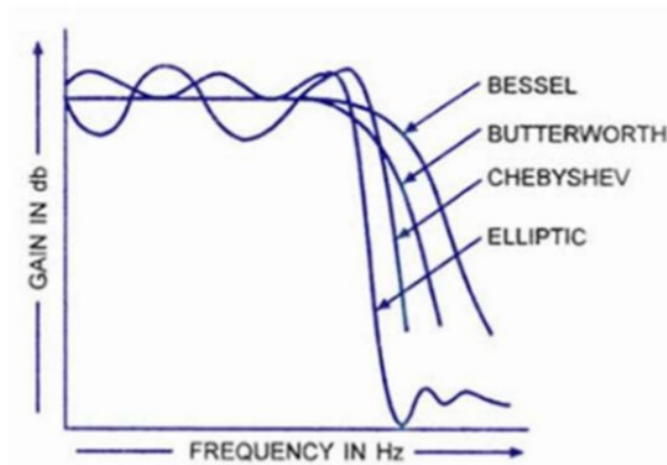


Figure 2 - Types of Active Filters

(Elprocus, 2020)

Butterworth Filters

The characteristics of a Butterworth filter lie between those of the Tschebyscheff and Bessel. (Circuitstoday, 2020) The most notable feature, and the one that gives rise to its alternative name, Maximally Flat filter, is that it exhibits a maximal flat frequency response in the passband i.e. up to the cut-off frequency. (Circuitstoday, 2020) No ripple can be seen. (Island, 2010)

There is a reasonable phase response provided by the Butterworth, being slightly non-linear, however a sharp cut-off near the cut-off frequency is seen, with a smooth moderate rate of roll-off (20dB/decade for each pole). (Island, 2010)

This filter is often used in applications related to audio processing. (Elprocus, 2020)

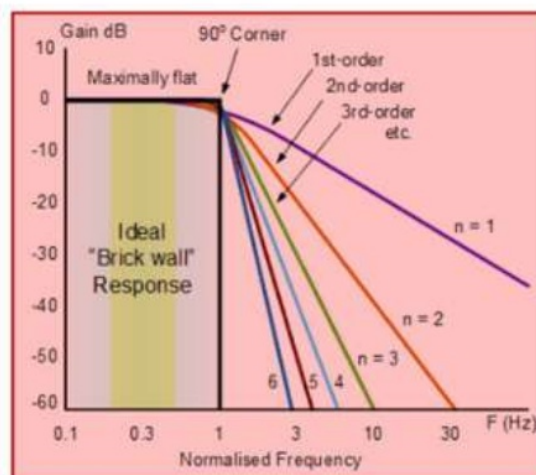


Figure 3 - Butterworth Filter

(Elprocus, 2020)

Tschebyscheff Filters

Two types of Tschebyscheff filters exist, type I and II. Type I is the most common and is described below. Type II is essentially the inverse of Type II with ripple occurring in the stopband. (Island, 2010)

Compared to the Butterworth, phase response is poor, the non-linear response resulting in pulse distortion from unequal delays in the passband. (Circuitstoday, 2020)

As seen with the Butterworth filter, the Tschebyscheff also demonstrates a significant phase shift approaching the cut-off frequency/passband, with an even sharper cut-off than that seen with the Butterworth. (Circuitstoday, 2020) However, the consequence of such a sharp cut-off, or fast roll-off, is that gain ripples (minima and maxima) appear in the passband i.e. below the cut-off frequency. Tschebyscheff filters are consequently also described as Equal Ripple filters. (Circuitstoday, 2020) As the steeper the roll-off, the greater the ripple gain (i.e. higher peak to peak), a fine balance has to be reached, this feature being adjustable at the stage of filter design. (Circuitstoday, 2020) These filters are not routinely employed for audio applications, being of far more use in applications requiring elimination of a wide range of frequencies outside of that desired in the passband. (Elprocus, 2020)

The sharper transition between pass and stopbands means absolute errors are much less than with the other filters, along with a much quicker execution speed. (Island, 2010)

Bessel Filters

The Bessel filter has the advantage of harbouring 'ideal phase characteristics', demonstrating a linear response across a wide frequency i.e. approaching close to the cut-off. (Circuitstoday, 2020)

Although the passband does not exhibit the flatness seen with the Butterworth filter, and the cut-off/transition from the stopband to the passband is only a gentle slope (Circuitstoday, 2020) i.e. less sharp than the Tschebyscheff filter, the Bessel filter has the advantage of an almost linear phase response meaning at zero frequency it maximises group delay, being constant in the passband. (Engineering_360, 2020) It also provides ideal square-wave transmission behaviour, square-wave to the input resulting in a square-wave output, which as the delay for all frequencies is the same, means no overshoot is seen. (Electronics-note, 2020) As a result, these filters are best suited to pulse applications where phase characteristics are an essential feature. (Elprocus, 2020)

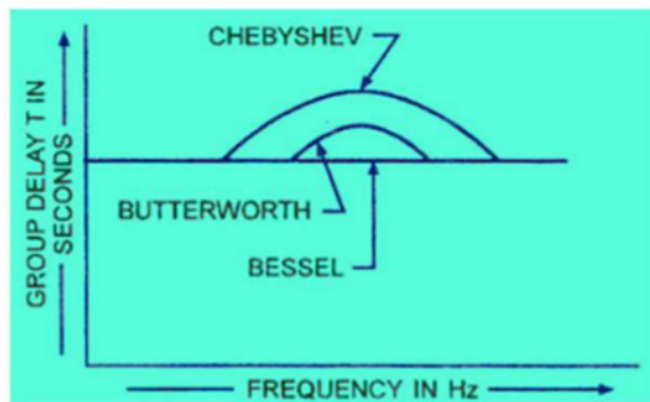


Figure 4 - Bessel Filter – Group Delay

(Elprocus, 2020)

Filter Comparison			
	Butterworth	Tschebyscheff	Bessel
Pass Band	Flat (no ripple)	Ripple	Flat (however begins to drop prior to the cut-off frequency)
Roll-off Rate	Moderate (smooth/monotonic)	Fastest/sharpest roll-off	Poor
Stopband-passband transition	Reasonable	Sharpest	Shallowest
Phase Response	Reasonable (slightly non-linear)	Least linear	Ideal (almost linear)
Group delay	Moderate	Least	Constant in the passband

Table 1 - Filter Summary Comparison

Filters are used/employed in a variety of modern electronic systems, enabling desired frequencies to be extracted from a signal where a number of other unwanted frequencies, relating to noise or distortion, may also be present. (Davies, 2017) Incorporation of the filter creates a network where changes to characteristics relating to frequency of the input signal e.g. amplitude, affects those of the resultant output. (Elprocus, 2020) The choice between active and passive filters, and the type to be employed e.g. high or low-pass, bandpass or band stop will depend on the needs of the application. Filters are used in biomedical instrumentation (interfacing sensors with data logging to smooth out the resultant trace i.e. remove unwanted noise), radio communication systems (suppression of noise by isolating the desired signal or reduction of interference at radio receivers from unwanted frequencies) and audio electronics (to amplify the audio signal) to name a few. (Elprocus, 2020)

A common use of filters is to remove noise in medical instrumentation, most control systems housing a first-order lag filter where there is an exponential approach of output to input value.

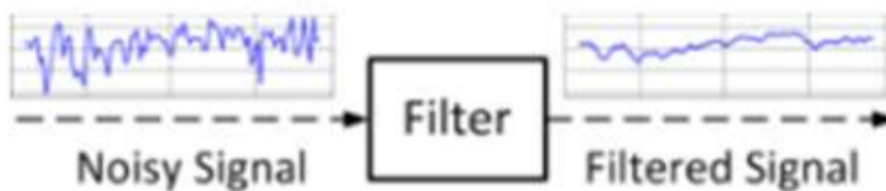


Figure 5 - Example of a Noise Filter with signal pre and post

(Smuits, 2016)

Most commonly this will be a low-pass filter attenuating higher (fast-changing) frequencies often associated with noise in measurement signals and allowing lower (slow-changing) frequencies. (Smuits, 2016) However, this is not always the case. Incorporation of filters for example in ECG measurement, to remove unwanted noise/signal frequencies from the ECG trace incorporates both low and high-pass filters. Low-pass filters enable the removal/attenuation of higher frequencies/interference associated with muscle movement, whereas high-pass filter remove those unwanted lower frequencies associated with respiration, to produce a cleaner, more informative trace. (Watford, 2014)

Another example of filters in practice is that used in audio systems. In a car audio system, a mixture of high-pass, low-pass and anti-aliasing filters may be found dependant on whether the application is for rejection of DC components, a sub-woofer, or at the ADC input respectively. The incorporation of an op-amp i.e. an active filter enables the audio signal to be amplified whilst maintaining the frequency response, further stability provided with the addition of a capacitor. (Instruments, 2019) Anti-aliasing filters are often a low-pass filter, so called as they act to reduce aliasing issues where high-frequency components are seen as low frequency aliases once sampled by the control system. By placing the filter before the control system, these high frequency signals are removed. Ideally chosen filters incorporate a flat passband and sharp cut-off. (Smuits, 2016), (Lai, 2003) Similarly, audio filters may be used to remove electrical noise associated with live music, where to remove/limit feedback from amplification equipment, a band-stop filter to impede the narrow frequency band which would result in feedback, may be incorporated. A high-pass filter sited between the microphone (source) and board removes low frequency signals which rarely carry sound of any value. (Wentz, 2018)

3. Design method, choices and calculations

3.1. 5th Order Bessel – Low-Pass Filter

- To design a 5th order Bessel filter with a cut-off frequency of 180Khz.

Each stage of the filter has to be designed.

$$r1 = \frac{a1}{2\pi f_c C1}$$

Equation 1 - Bessel 5th Order Stage 1

n	i	a _i	b _i	k _i = f _{ci} / f _c	Q _i
1	1	1.0000	0.0000	1.000	—
2	1	1.3817	0.6180	1.000	0.58
3	1	0.7560	0.0000	1.323	—
	2	0.9996	0.4772	1.414	0.69
4	1	1.3397	0.4889	0.978	0.52
	2	0.7743	0.3890	1.797	0.81
5	1	0.6656	0.0000	1.502	—
	2	1.1402	0.4128	1.184	0.56
	3	0.6216	0.3245	2.138	0.92

Figure 6 – Bessel Coefficients Table

(Kugelstadt, 2008)

Stage 1

For the calculation, ai = 0.6656 and bi = 0. Using capacitor value of 1nF:

$$r1 = \frac{0.6656}{2\pi \times 180 \times 10^3 \times 1 \times 10^{-9}} = 588.5\Omega$$

Equation 2 - Bessel 5th Order Stage 1

Stage 2

For the calculation, ai = 1.1402 and bi = 0.4128 Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(0.4128)}{1.1402^2} = 1.27nF$$

Equation 3 - Bessel 5th Order Stage 2 Capacitor 2 value

Taking c2 = 1.3nF, values of R1 and R2 for stage 2 can be calculated.

$$R = \frac{ai C2 \mp \sqrt{ai^2 C2^2 - 4(bi c1 c2)}}{4 \pi f_c C1 C2}$$

Equation 4 - Bessel 5th Order Stage 2 Resistor

$$R1 = \frac{1.1402 \times 1.3 \times 10^{-9} - \sqrt{(1.1402)^2 (1.3 \times 10^{-9})^2 - 4(0.4128 \times 1 \times 10^{-9} \times 1.3 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 1.3 \times 10^{-9}} = R1 = 427.6\Omega$$

Equation 5 - Bessel 5th Order Stage 2 R1 Value

$$R2 = \frac{1.1402 \times 1.3 \times 10^{-9} + \sqrt{(1.1402)^2 (1.3 \times 10^{-9})^2 - 4(0.4128 \times 1 \times 10^{-9} \times 1.3 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 1.3 \times 10^{-9}} = R2 = 580.5\Omega$$

Equation 6 - Bessel 5th Order Stage 2 R2 Value

Stage 3

For the calculation, ai = 0.6216 and bi = 0.3245. Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(0.3245)}{0.6216^2} = 3.36nF$$

Equation 7 - Bessel 5th Order Stage 3 Capacitor 2 value

Taking c2 = 3.4nF, values of R1 and R2 for stage 3 can be calculated.

$$R = \frac{ai C2 \mp \sqrt{ai^2 C2^2 - 4(bi c1 c2)}}{4 \pi f c C1 C2}$$

Equation 8 – Bessel 5th Order Stage 3 Resistor

$$R1 = \frac{0.6216 \times 3.4 \times 10^{-9} - \sqrt{(0.6216)^2 (3.4 \times 10^{-9})^2 - 4(0.3245 \times 1 \times 10^{-9} \times 3.4 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 3.4 \times 10^{-9}} = R1 = 244.8\Omega$$

Equation 9 - Bessel 5th Order Stage 3 R1 Value

$$R2 = \frac{0.6216 \times 3.4 \times 10^{-9} + \sqrt{(0.6216)^2 (3.4 \times 10^{-9})^2 - 4(0.3245 \times 1 \times 10^{-9} \times 3.4 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 3.4 \times 10^{-9}} = R2 = 304.9\Omega$$

Equation 10 - Bessel 5th Order Stage 3 R2 Value

Bessel 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	588.5Ω	1nF	N/A
Stage 2	427.6Ω	580.1Ω	1nF	1.3nF
Stage 3	244.8Ω	304.9Ω	1nF	3.4nF

Table 2 - Bessel 5th Order – Calculated Low-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Bessel 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	590.0Ω	1nF	N/A
Stage 2	430.0Ω	580.0Ω	1nF	1.3nF
Stage 3	245.0Ω	300.0Ω	1nF	3.4nF

Table 3 - Bessel 5th Order – Simulated Low-pass Filter Component Values

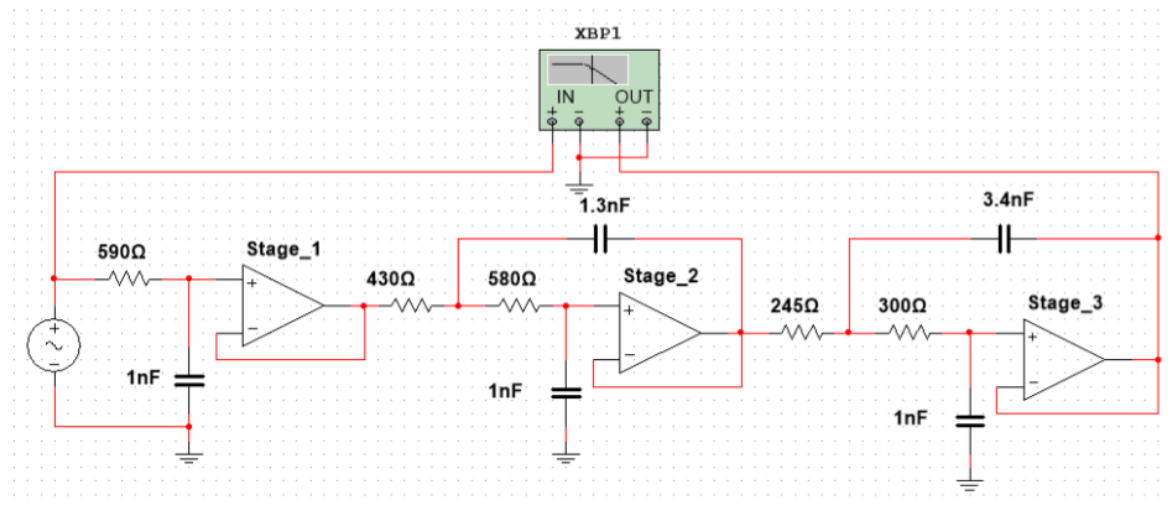


Figure 7 - Bessel 5th Order Low-pass Filter

3.2. 5th Order Tschhebyscheff – Low-pass Filter

- To design a 5th Order Tschhebyscheff filter with a cut-off frequency of 180Khz. Each stage of the filter has to be designed.

$$r1 = \frac{a1}{2\pi fcC1}$$

Equation 11 - Tschhebyscheff 5th Order Stage 1

n	i	a _i	b _i	k _i = f _{ci} / f _c	Q _i
1	1	1.0000	0.0000	1.000	—
2	1	1.0650	1.9305	1.000	1.30
3	1	3.3496	0.0000	0.299	—
	2	0.3559	1.1923	1.396	3.07
4	1	2.1853	5.5339	0.557	1.08
	2	0.1964	1.2009	1.410	5.58
5	1	5.6334	0.0000	0.178	—
	2	0.7620	2.6530	0.917	2.14
	3	0.1172	1.0686	1.500	8.82

Figure 8 - Tschhebyscheff Coefficients Table

(Kugelstadt, 2008)

Stage 1

For the calculation, ai = 5.6334 and bi = 0. Using capacitor value of 1nF:

$$r1 = \frac{5.6334}{2\pi \times 180 \times 10^3 \times 1 \times 10^{-9}} = 4.98K\Omega$$

Equation 12 – Tschhebyscheff 5th Order Stage 1

Stage 2

For the calculation, ai = 0.7620 and bi = 2.6530. Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(2.6530)}{0.7620^2} = 18.15nF$$

Equation 13 - Tschhebyscheff 5th Order Stage 2 Capacitor 2 value

Taking c2 = 20nF, values of R1 and R2 for stage 2 can be calculated.

$$R = \frac{ai C2 \mp \sqrt{ai^2 C2^2 - 4(bi c1 c2)}}{4 \pi fc C1 C2}$$

Equation 14 - Tschhebyscheff 5th Order Stage 2 Resistor

$$R1 = \frac{0.7620 \times 20 \times 10^{-9} - \sqrt{(0.7620)^2 (20 \times 10^{-9})^2 - 4(2.6530 \times 1 \times 10^{-9} \times 20 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 20 \times 10^{-9}} = R1 = 221.6\Omega$$

Equation 15 - Tschebyscheff 5th Order Stage 2 R1 Value

$$R2 = \frac{0.7620 \times 20 \times 10^{-9} + \sqrt{(0.7620)^2 (20 \times 10^{-9})^2 - 4(2.6530 \times 1 \times 10^{-9} \times 20 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 20 \times 10^{-9}} = R2 = 452.2\Omega$$

Equation 16 - Tschebyscheff 5th Order Stage 2 R2 Value

Stage 3

For the calculation, ai = 0.1172 and bi = 1.0686 Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(1.0686)}{0.1172^2} = 311.2nF$$

Equation 17 - Tschebyscheff 5th Order Stage 3 Capacitor 2 value

Taking c2 = 315nF, values of R1 and R2 for stage 3 can be calculated.

$$R = \frac{ai C2 \mp \sqrt{ai^2 C2^2 - 4(bi c1 c2)}}{4 \pi f c C1 C2}$$

Equation 18 - Tschebyscheff 5th Order Stage 3 Resistor

$$R1 = \frac{0.1172 \times 315 \times 10^{-9} - \sqrt{(0.1172)^2 (315 \times 10^{-9})^2 - 4(1.0686 \times 1 \times 10^{-9} \times 315 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 315 \times 10^{-9}} = R1 = 46.11\Omega$$

Equation 19 - Tschebyscheff 5th Order Stage1 R1 Value

$$R2 = \frac{0.1172 \times 315 \times 10^{-9} + \sqrt{(0.1172)^2 (315 \times 10^{-9})^2 - 4(1.0686 \times 1 \times 10^{-9} \times 315 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 315 \times 10^{-9}} = R2 = 57.5\Omega$$

Equation 20 - Tschebyscheff 5th Order Stage1 R2 Value

Tschebyscheff 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	4.98KΩ	1nF	N/A
Stage 2	221.6Ω	452.2Ω	1nF	20nF
Stage 3	46.1Ω	57.51Ω	1nF	315nF

Table 4 - Tschebyscheff 5th Order – Calculated Low-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Tschebyscheff 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	5.0K Ω	1nF	N/A
Stage 2	220 Ω	450 Ω	1nF	20nF
Stage 3	45 Ω	60.0 Ω	1nF	315nF

Table 5 - Tschebyscheff 5th Order – Simulated Low-pass Filter Component Values

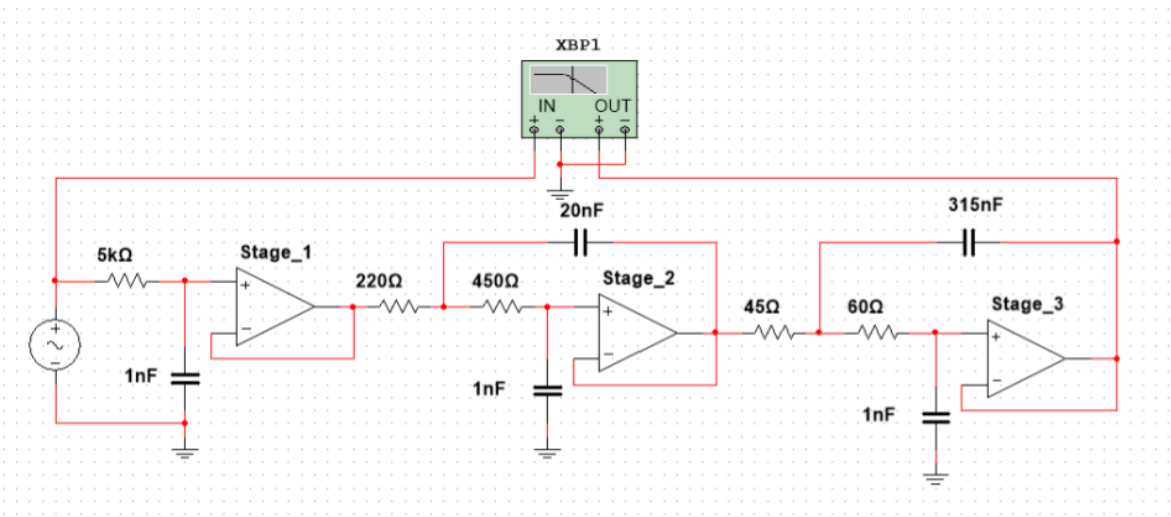


Figure 9 - Tschebyscheff 5th Order Low-pass Filter

3.3. 5th Order Butterworth – Low-pass Filter

- To design a 5th order Butterworth filter with a cut-off frequency of 180Khz. Each stage of the filter has to be designed.

$$r1 = \frac{a1}{2\pi f_c C1}$$

Equation 21 - Butterworth 5th Order Stage 1

n	i	a _i	b _i	k _i = f _{ci} / f _c	Q _i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62

Figure 10 - Butterworth Coefficients Table

(Kugelstadt, 2008)

Stage 1

For the calculation, a_i = 1.0000 and b_i = 0. Using capacitor value of 1nF:

$$r1 = \frac{1}{2\pi \times 180 \times 10^3 \times 1 \times 10^{-9}} = 884.2\Omega$$

Equation 22 - Butterworth 5th Order Stage 1

Stage 2

For the calculation, a_i = 1.6180 and b_i = 1.0000. Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(1)}{1.6180^2} = 1.53nF$$

Equation 23 - Butterworth 5th Order Stage 2 Capacitor 2 value

Taking c2 = 2nF, values of R1 and R2 for stage 2 can be calculated.

$$R = \frac{a_i C2 \mp \sqrt{a_i^2 C2^2 - 4(b_i c1 c2)}}{4 \pi f_c C1 C2}$$

Equation 24 - Butterworth 5th Order Stage 2 Resistor

$$R1 = \frac{1.6180 \times 2 \times 10^{-9} - \sqrt{(1.6180)^2 (2 \times 10^{-9})^2 - 4(1.0000 \times 1 \times 10^{-9} \times 2 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 2 \times 10^{-9}} = R1 = 367.8\Omega$$

Equation 25 - Butterworth 5th Order Stage 2 R1 Value

$$R2 = \frac{1.6180 \times 2 \times 10^{-9} + \sqrt{(1.6180)^2 (2 \times 10^{-9})^2 - 4(1.0000 \times 1 \times 10^{-9} \times 2 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 2 \times 10^{-9}} = R2 = 1062.8\Omega$$

Equation 26 - Butterworth 5th Order Stage 2 R2 Value

Stage 3

For the calculation, ai = 0.6180 and bi = 1.0000. Using capacitor value of 1nF:

$$c2 = \frac{c1 \times 4b}{a^2} = c2 = \frac{1 \times 10^{-9} \times 4(1.0000)}{0.6180^2} = 10.5nF$$

Equation 27 – Butterworth 5th Order Stage 3 Capacitor 2 value

Taking c2 = 15nF, values of R1 and R2 for stage 3 can be calculated.

$$R = \frac{ai C2 \mp \sqrt{ai^2 C2^2 - 4(bi c1 c2)}}{4 \pi f c C1 C2}$$

Equation 28 - Butterworth 5th Order Stage 3 Resistor

$$R1 = \frac{0.6180 \times 15 \times 10^{-9} - \sqrt{(0.6180)^2 (15 \times 10^{-9})^2 - 4(1.0000 \times 1 \times 10^{-9} \times 15 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 15 \times 10^{-9}} = R1 = 123.1\Omega$$

Equation 29 - Butterworth 5th Order Stage 3 R1 Value

$$R2 = \frac{0.6180 \times 15 \times 10^{-9} + \sqrt{(0.6180)^2 (15 \times 10^{-9})^2 - 4(1.0000 \times 1 \times 10^{-9} \times 15 \times 10^{-9})}}{4 \times \pi \times 180 \times 10^3 \times 1 \times 10^{-9} \times 15 \times 10^{-9}} = R2 = 423.3\Omega$$

Equation 30 - Butterworth 5th Order Stage 3 R2 Value

Butterworth 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	884.2Ω	1nF	N/A
Stage 2	367.8Ω	1.06KΩ	1nF	2nF
Stage 3	123.1Ω	423.3Ω	1nF	15nF

Table 6 - Butterworth 5th Order – Calculated Low-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Butterworth 5 th Order – Low-pass Filter Components				
	R1 Value	R2 Value	C1 Value	C2 Value
Stage 1	N/A	885.0Ω	1nF	N/A
Stage 2	370.0Ω	1.0KΩ	1nF	2nF
Stage 3	125.0Ω	425.0Ω	1nF	15nF

Table 7 - Butterworth 5th Order – Simulated Low-pass Filter Component Values

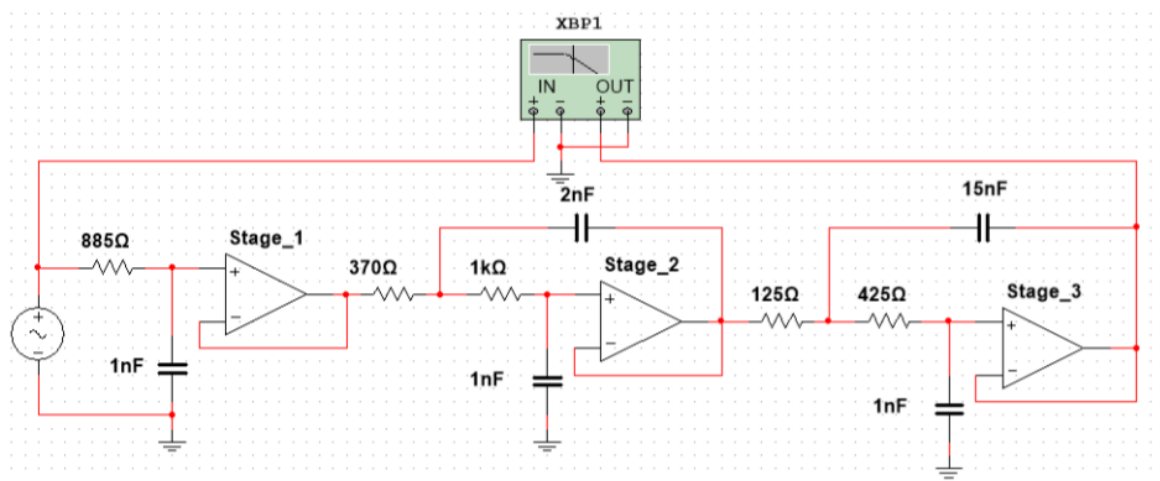


Figure 11 - Butterworth 5th Order Low-pass Filter

3.4. 3rd Order Butterworth – High-pass Filter

- To design a 3rd order Butterworth high-pass filter with a cut-off frequency of 180Khz.

Each stage of the filter has to be individually designed.

For each stage of the design a 5nF capacitor shall be used. Values of a₁ and b₁ coefficients are from the illustration below.

n	i	a _i	b _i	k _i = f _{Ci} / f _C	Q _i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62

Figure 12 - Butterworth Coefficient Table

(Kugelstadt, 2008)

Stage 1

For the calculation, a₁ = 1.0000 and b₁ = 0. Using capacitor value of 5nF;

$$R1 = \frac{1}{2 \pi f_c a_1 c_1} = \frac{1}{2 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 176.8\Omega$$

Equation 31 - Butterworth 3rd Order Stage 1 R1

Stage 2

For the calculation, a₁ = 1.0000 and b₁ = 1.0000. Using capacitor value of 5nF:

$$R2 = \frac{1}{\pi f_c a_2 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 353.7\Omega$$

Equation 32 - Butterworth 3rd Order Stage 2 R1

$$R3 = \frac{a_2}{4 \pi f_c b_2 C} = \frac{1}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 88.4\Omega$$

Equation 33 - Butterworth 3rd Order Stage 2 R2

Butterworth 3 rd Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	176.8Ω	N/A	5nF
Stage 2	353.7Ω	88.4Ω	5nF

Table 8 - Butterworth 3rd Order – Calculated High-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Butterworth 3 rd Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	180.0Ω	N/A	5nF
Stage 2	355.0Ω	90.0Ω	5nF

Table 9 - Butterworth 3rd Order – Simulated Low-pass Filter Component Values

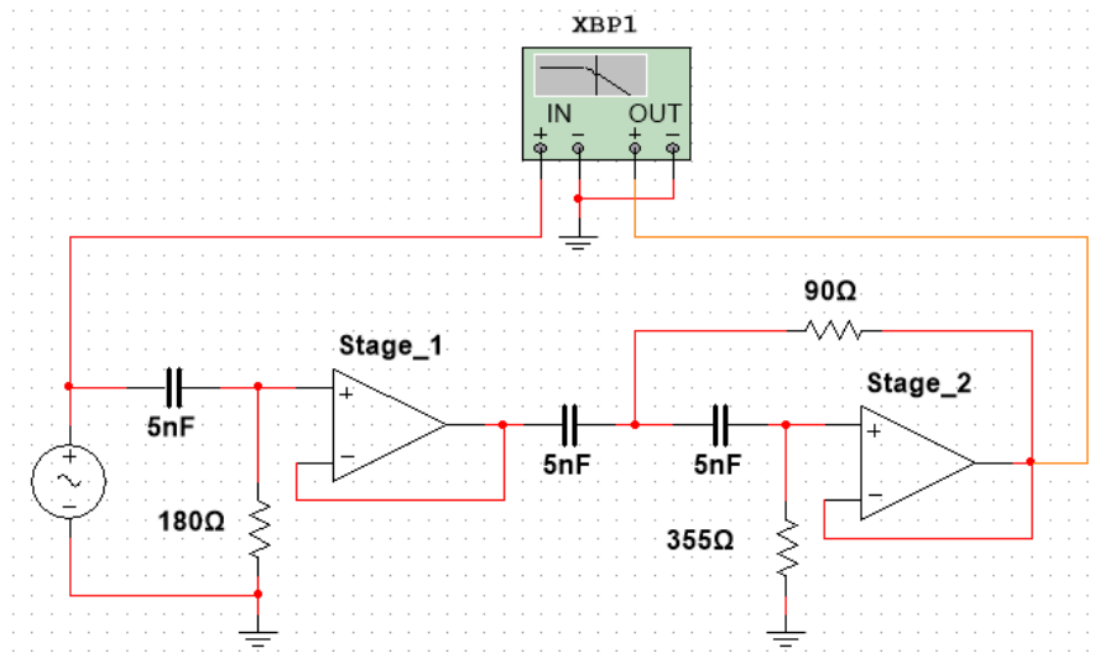


Figure 13 - Butterworth 3rd Order High-pass Filter

3.5. 6th Order Butterworth – High-pass Filter

- To design a 6th order Butterworth high-pass filter with a cut-off frequency of 180Khz. Each stage of the filter has to be individually designed. For each stage of the design a 5nF capacitor shall be used. Values of a1 and b1 coefficients used are from the illustration below.

n	i	a _i	b _i	k _i = f _{Ci} / f _C	Q _i
1	1	1.0000	0.0000	1.000	—
2	1	1.4142	1.0000	1.000	0.71
3	1	1.0000	0.0000	1.000	—
	2	1.0000	1.0000	1.272	1.00
4	1	1.8478	1.0000	0.719	0.54
	2	0.7654	1.0000	1.390	1.31
5	1	1.0000	0.0000	1.000	—
	2	1.6180	1.0000	0.859	0.62
	3	0.6180	1.0000	1.448	1.62
6	1	1.9319	1.0000	0.676	0.52
	2	1.4142	1.0000	1.000	0.71
	3	0.5176	1.0000	1.479	1.93

Figure 14 - Butterworth Coefficient Table

(Kugelstadt, 2008)

Stage 1

For the calculation, ai = 1.9319 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f_c a1 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1.9319 \times 5 \times 10^{-9}} = 183.1 \Omega$$

Equation 34 - Butterworth 6th Order Stage 1 R1

$$R2 = \frac{a1}{4 \pi f_c b1 C} = \frac{1.9313}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 170.8 \Omega$$

Equation 35 - Butterworth 6th Order Stage 1 R1

Stage 2

For the calculation, ai = 1.4142 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f_c a2 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1.4142 \times 5 \times 10^{-9}} = 250.1 \Omega$$

Equation 36 - Butterworth 6th Order Stage 2 R1

$$R2 = \frac{a2}{4 \pi f_c b2 C} = \frac{1.4142}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 125 \Omega$$

Equation 37 - Butterworth 6th Order Stage 2 R2

Stage 3

For the calculation, ai = 0.5176 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f c a3 C} = \frac{1}{\pi \times 180 \times 10^3 \times 0.5176 \times 5 \times 10^{-9}} = 683.3\Omega$$

Equation 38 - Butterworth 6th Order Stage 3 R1

$$R2 = \frac{a3}{4 \pi f c b3 C} = \frac{0.5176}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 45.8\Omega$$

Equation 39 - Butterworth 6th Order Stage 3 R2

Butterworth 6 th Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	183.1Ω	170.8Ω	5nF
Stage 2	250.1Ω	125.0Ω	5nF
Stage 3	683.3Ω	45.8Ω	5nF

Table 10 - Butterworth 6th Order – Calculated High-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Butterworth 6 th Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	185.0Ω	170.0Ω	5nF
Stage 2	250.0Ω	125.0Ω	5nF
Stage 3	685.0Ω	45.0Ω	5nF

Table 11 - Butterworth 6th Order – Simulated Low-pass Filter Component Values

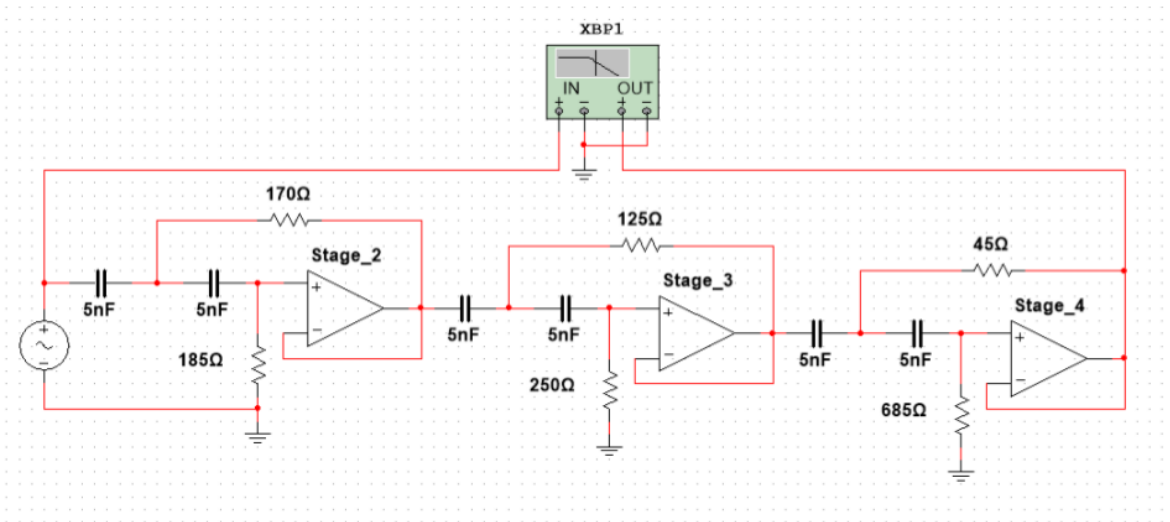


Figure 15 - Butterworth 6th Order High-pass Filter

3.6. 9th Order Butterworth – High-pass Filter

- To design a 9th Order Butterworth high-pass filter with a cut-off frequency of 180Khz.

Each stage of the filter has to be individually designed.
For each stage of the design a 5nF capacitor shall be used. Values of a1 and b1 coefficients are taken from the illustration below.

n	i	a _i	b _i	k _i = f _{ci} / f _c	Q _i
7	1	1.0000	0.0000	1.000	—
	2	1.8019	1.0000	0.745	0.55
	3	1.2470	1.0000	1.117	0.80
	4	0.4450	1.0000	1.499	2.25
8	1	1.9616	1.0000	0.661	0.51
	2	1.6629	1.0000	0.829	0.60
	3	1.1111	1.0000	1.206	0.90
	4	0.3902	1.0000	1.512	2.56
9	1	1.0000	0.0000	1.000	—
	2	1.8794	1.0000	0.703	0.53
	3	1.5321	1.0000	0.917	0.65
	4	1.0000	1.0000	1.272	1.00
	5	0.3473	1.0000	1.521	2.88

Figure 16 - Butterworth Coefficient Table

(Kugelstadt, 2008)

Stage 1

For the calculation, a_i = 1.0000 and b_i = 0. Using capacitor value of 5nF:

$$R1 = \frac{1}{2 \pi f_c a_1 c_1} = \frac{1}{2 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 176.8 \Omega$$

Equation 40 - Butterworth 9th Order Stage 1 R1

Stage 2

For the calculation, a_i = 1.8794 and b_i = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f_c a_2 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1.8794 \times 5 \times 10^{-9}} = 188.2 \Omega$$

Equation 41 - Butterworth 9th Order Stage 2 R1

$$R2 = \frac{a_2}{4 \pi f_c b_2 C} = \frac{1.8794}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 166.2 \Omega$$

Equation 42 - Butterworth 9th Order Stage 2 R2

Stage 3

For the calculation, ai = 1.5321 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f c a3 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1.5321 \times 5 \times 10^{-9}} = 230.8 \Omega$$

Equation 43 - Butterworth 9th Order Stage 3 R1

$$R2 = \frac{a3}{4 \pi f c b3 C} = \frac{1.5321}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 135.5 \Omega$$

Equation 44 - Butterworth 9th Order Stage 3 R2

Stage 4

For the calculation, ai = 1.0000 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f c a4 C} = \frac{1}{\pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 353.7 \Omega$$

Equation 45 - Butterworth 9th Order Stage 4 R1

$$R2 = \frac{a4}{4 \pi f c b4 C} = \frac{1}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 88.4 \Omega$$

Equation 46 - Butterworth 9th Order Stage 4 R2

Stage 5

For the calculation, ai = 0.3473 and bi = 1.0000. Using capacitor value of 5nF:

$$R1 = \frac{1}{\pi f c a5 C} = \frac{1}{\pi \times 180 \times 10^3 \times 0.3473 \times 5 \times 10^{-9}} = 1018.4 \Omega$$

Equation 47 - Butterworth 9th Order Stage 5 R1

$$R2 = \frac{a5}{4 \pi f c b5 C} = \frac{0.3473}{4 \times \pi \times 180 \times 10^3 \times 1 \times 5 \times 10^{-9}} = 30.7 \Omega$$

Equation 48 - Butterworth 9th Order Stage 5 R2

Butterworth 9 th Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	176.8.1Ω	N/A	5nF
Stage 2	188.2Ω	166.2Ω	5nF
Stage 3	230.5Ω	135.5Ω	5nF
Stage 4	353.7Ω	88.4Ω	5nF
Stage 5	1018.4Ω	30.7Ω	5nF

Table 12 - Butterworth 9th Order – Calculated High-pass Filter Components

With component values for each stage of the filter calculated, values are rounded to the nearest practical and suitable value to enable a realistic model build and simulation.

Butterworth 9 th Order – High-pass Filter Components			
	R1 Value	R2 Value	Capacitor Value
Stage 1	175.0Ω	N/A	5nF
Stage 2	190.0Ω	165.0Ω	5nF
Stage 3	230.0Ω	135.0Ω	5nF
Stage 4	350.0Ω	90.0Ω	5nF
Stage 5	1.0KΩ	30.0Ω	5nF

Table 13 - Butterworth 9th Order – Simulated Low-pass Filter Component Values

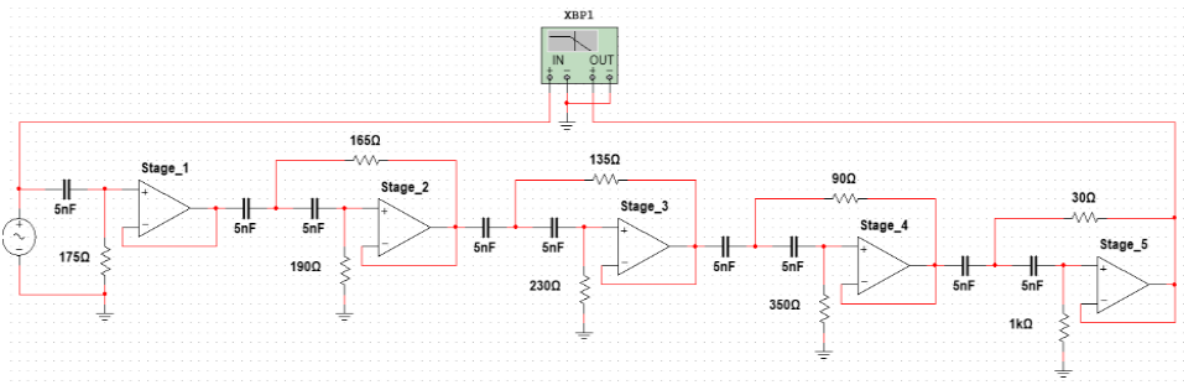


Figure 17 - Butterworth 9th Order High-pass Filter

(See Appendix for larger image)

4. Testing and improving the design

4.1. Low-pass Filter

With each of the filter stages' components calculated and circuits built, testing can take place for each version of the 5th order low-pass filter, enabling comparison.

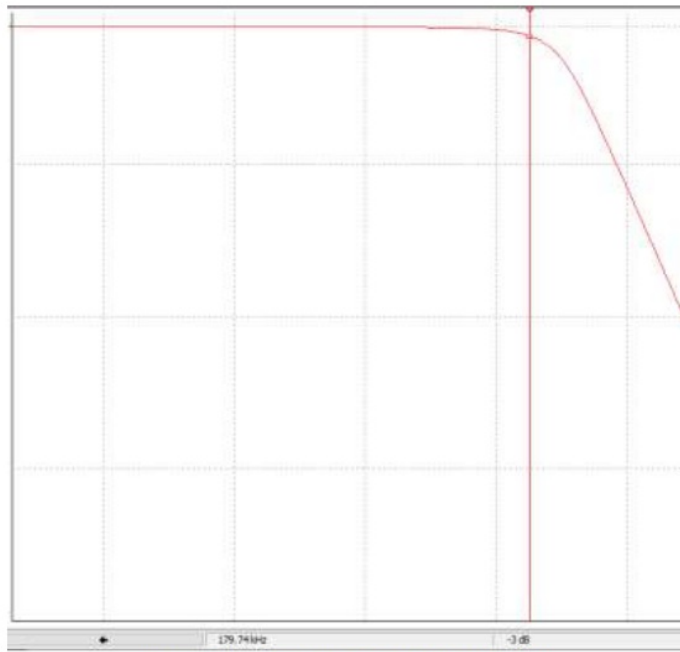


Figure 18 - Bessel 5th Order Filter Roll Off

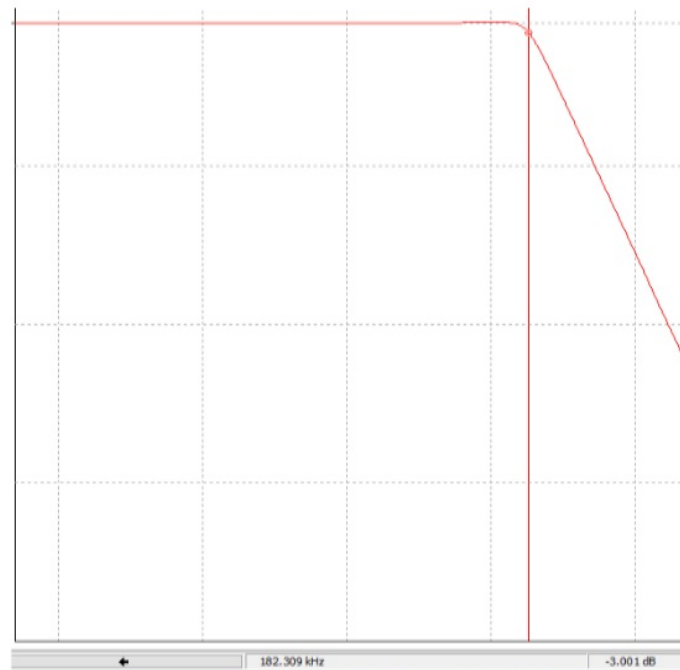


Figure 19 - Butterworth 5th Order Filter Roll Off

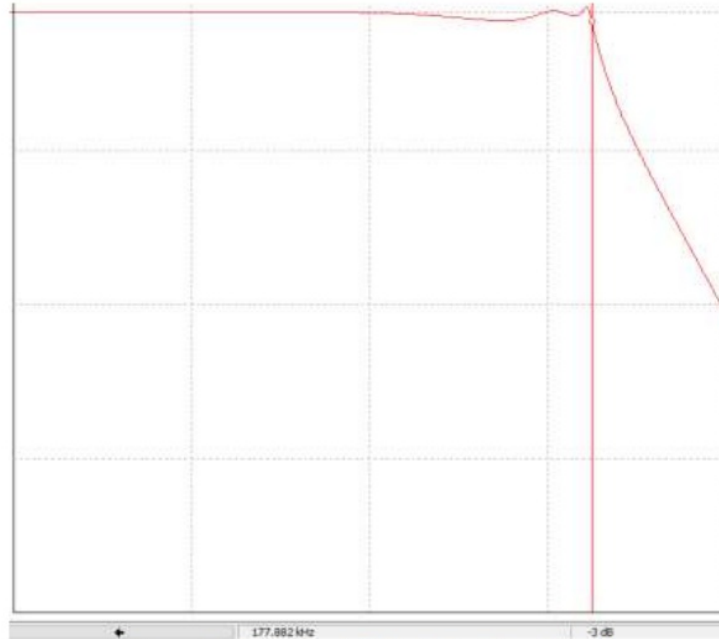


Figure 20 - Tschebyscheff 5th Order Filter Roll Off

5th Order Filter Comparison		
Filter Type	DB Roll Off	Measured Frequency at -3dB Roll Off
Bessel	-3.000 Db	179.740 Khz
Butterworth	-3.001 Db	182.309 Khz
Tschebyscheff	-3.000 Db	177.882 Khz

Table 14 - Low-pass Filter Frequency Comparison

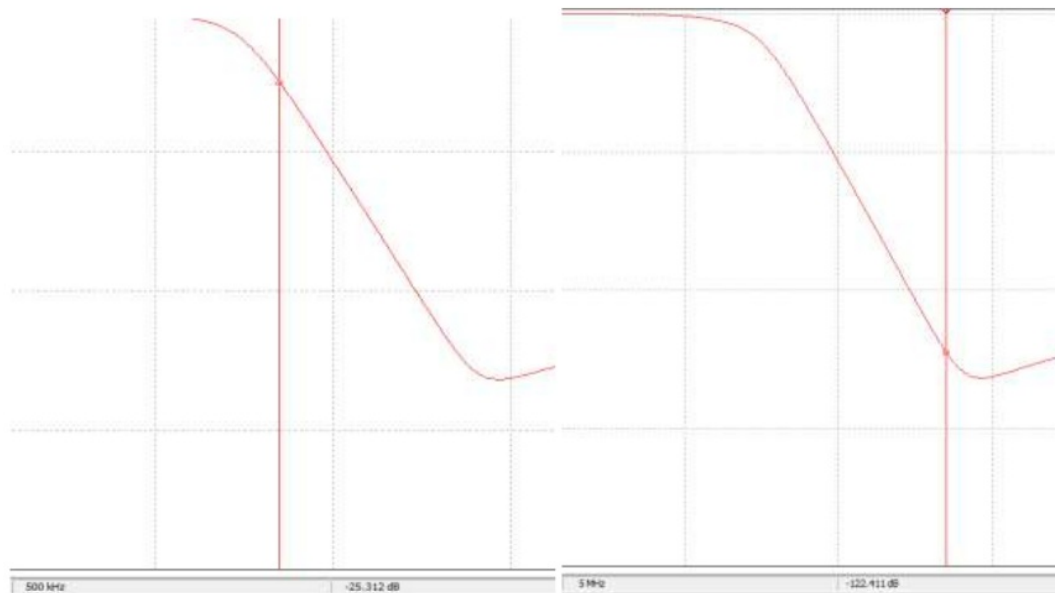


Figure 21 - Bessel 5th Order Low-pass Filter Logarithmic Frequency

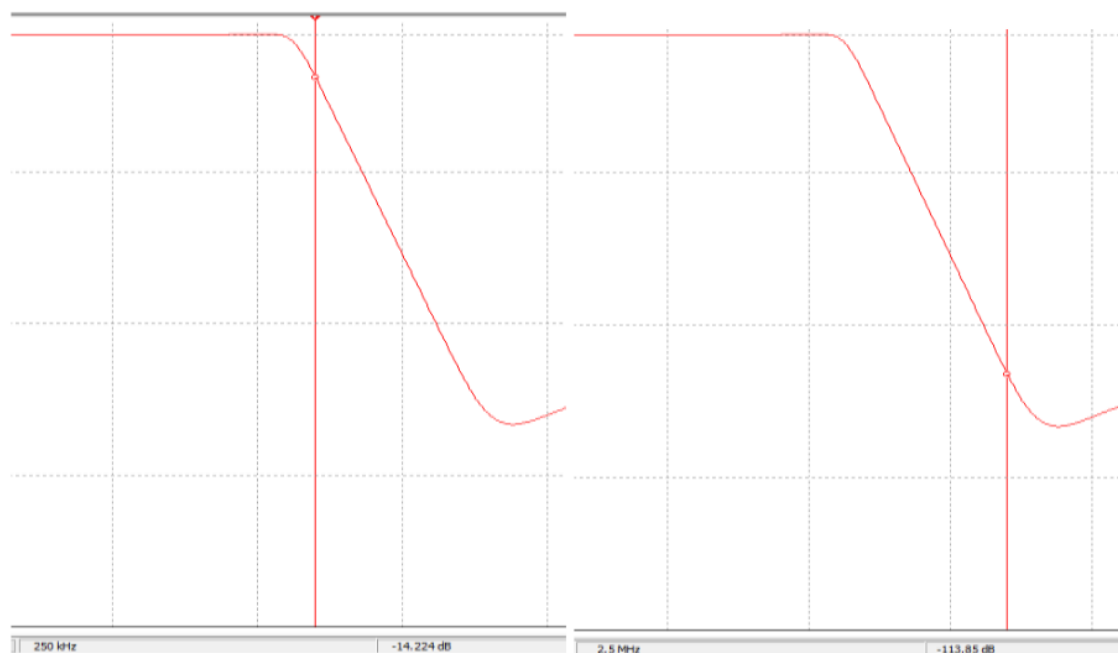


Figure 22 - Butterworth 5th Order Low-pass Filter Logarithmic Frequency

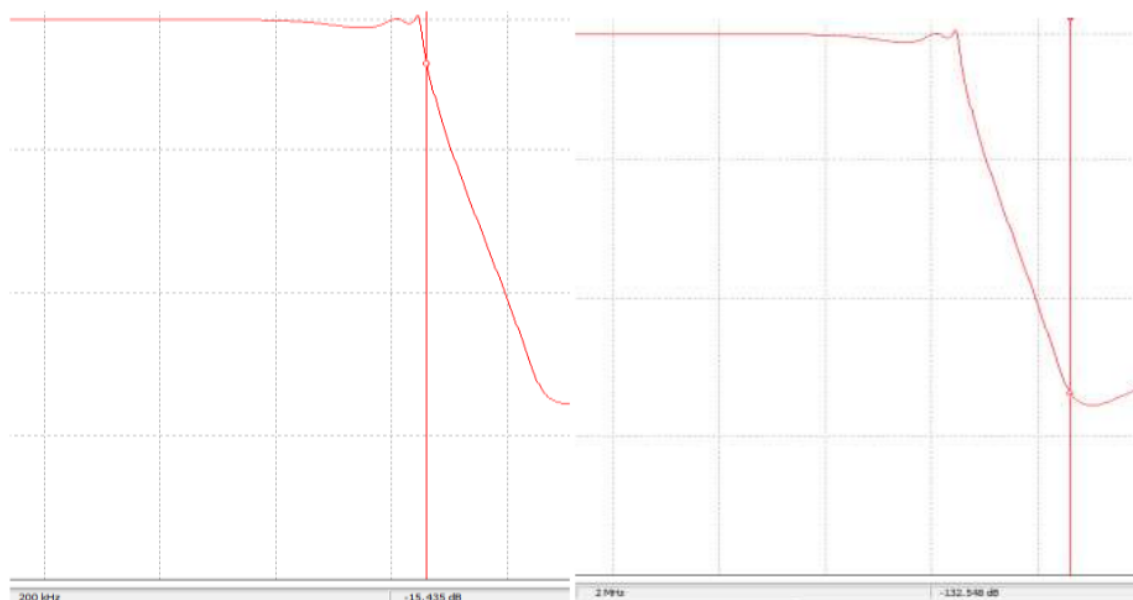


Figure 23 - Tschebyscheff 5th Order Low-pass Filter Logarithmic Frequency

Low-pass Filter Comparison					
Filter Type	Logarithmic Frequency High		Logarithmic Frequency Low		Rate of Roll off
Bessel	500 Khz	-25.321 dB	5.0 MHz	-122.411 dB	-97.090 dB
Butterworth	250 Khz	-14.224 dB	2.5 MHz	-113.850 dB	-99.626 dB
Tschebyscheff	200 Khz	-15.435 dB	2.0 MHz	-132.548 dB	-117.113 dB

Table 15 - Low-pass Filter Logarithmic Frequency Comparison

As illustrated in the images and tables above, there are notable differences between the Bessel, Butterworth and Tschebyscheff filters. Measuring the frequency of each filter at the -3.0dB roll-off, the Bessel filter is measuring at the required frequency of 180Khz, while the Butterworth is marginally above the required value by 2.3Khz and the Tschebyscheff marginally below by 2.1Khz.

As the filter is 5th order, the expected roll-off rate is 100dB, at 20dB per decade. dB/decade is used to measure rate of roll-off as it is measured at multiples of ten, or decades. The input of the circuit will fall by one tenth while the output will multiply by ten, at each stage of the filter. (Yerzley, 2019). Measuring the rate of roll-off for each filter, the Butterworth filter has the closest desired rate, measured at -99.6dB, while the Bessel is marginally below the expected value at -97.1dB, and the Tschebyscheff above at -117.1dB.

The image below succinctly illustrates the information gathered from the images and tables above, showing the variation in the filter's passbands and rate of roll-off. The Butterworth has the flattest passband, while in comparison the Tschebyscheff has a lot of oscillation at the passband. The Bessel whilst not flat as the Butterworth, is significantly more so that the Tschebyscheff. The rate of roll-off also varies, Bessel having the shallowest transition from pass to stopband, Tschebyscheff the steepest, and Butterworth a moderate transition.

The differences in filters mean that their applications are varied;

- Bessel filters are used in radio frequency, audio applications and electronic applications requiring a flat group or phase delay, resulting in the shape of the signal being preserved in the passband. (Electronics-notes, 2020)
- Butterworth filters are used in data converting applications in addition to high quality audio applications. The filter is used in these applications ripples of magnitude. (Administrator, 2019)
- Tschebyscheff filters are used in radio frequency applications where ripples in the passband are not of concern to the application as the steep roll off is required. (Electronics-notes, 2020)

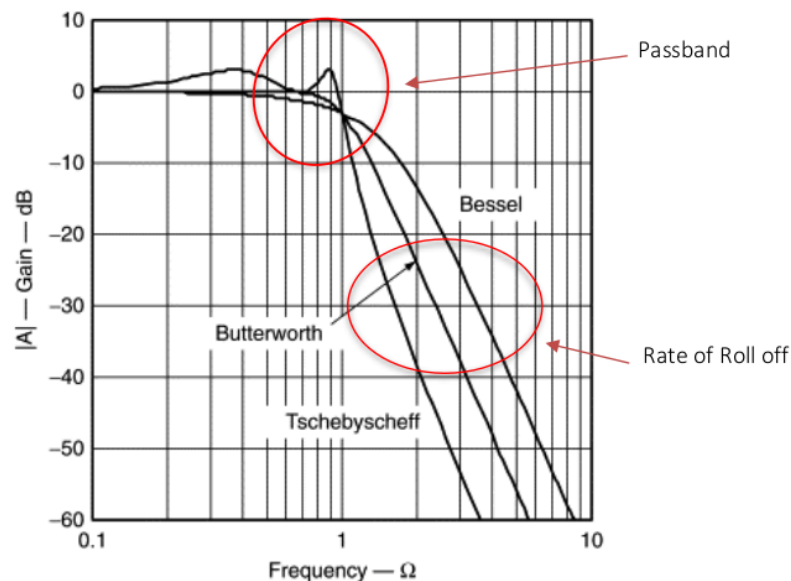


Figure 24 - Filter comparison

(Sciencedirect, 2020)

4.2. High-pass Filter

With each of the filter stages' components calculated and circuits built, testing can take place for each order of the Butterworth high-pass filter to enable comparison.

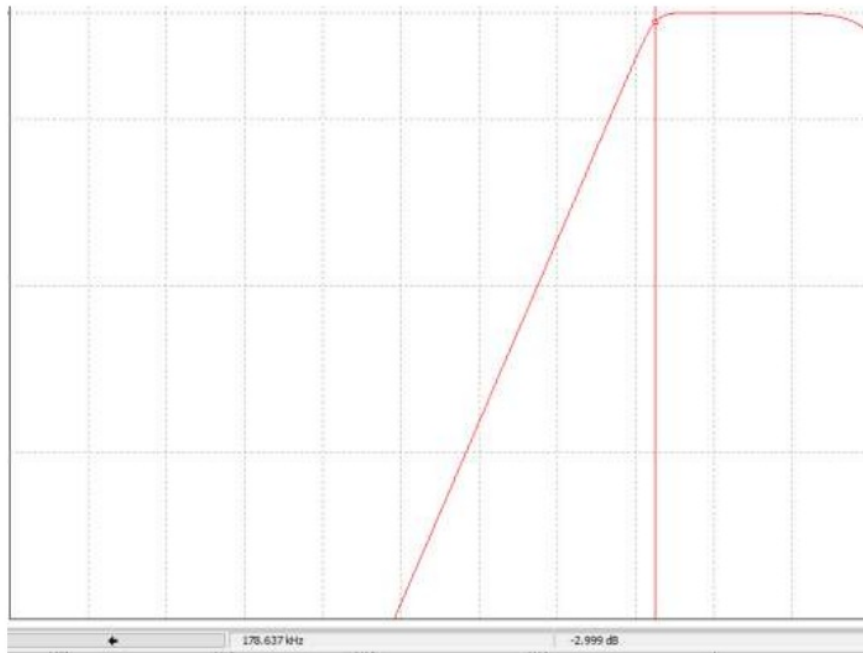


Figure 25 - Butterworth 3rd Order High-pass Filter Roll Off

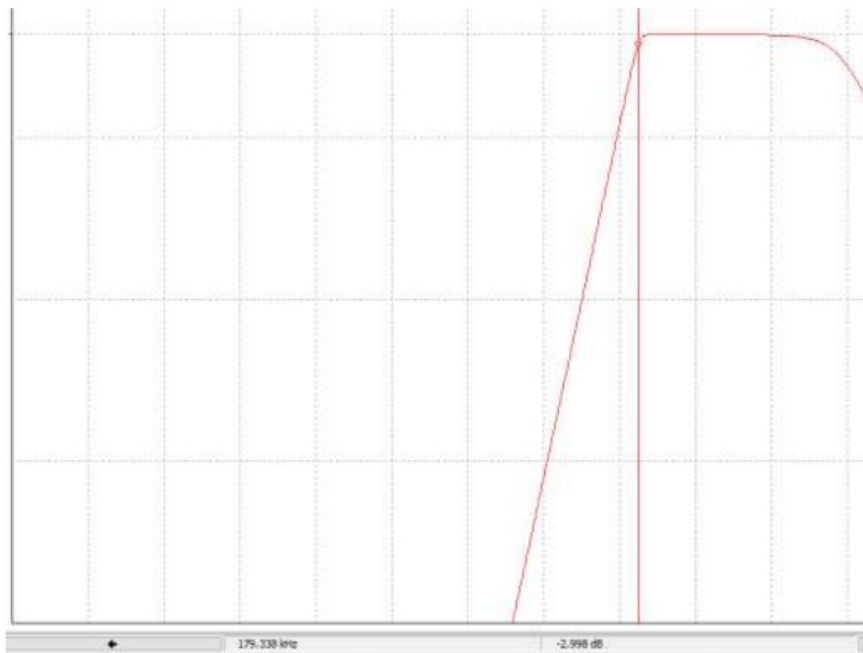


Figure 26 - Butterworth 6th Order High-pass Filter Roll Off

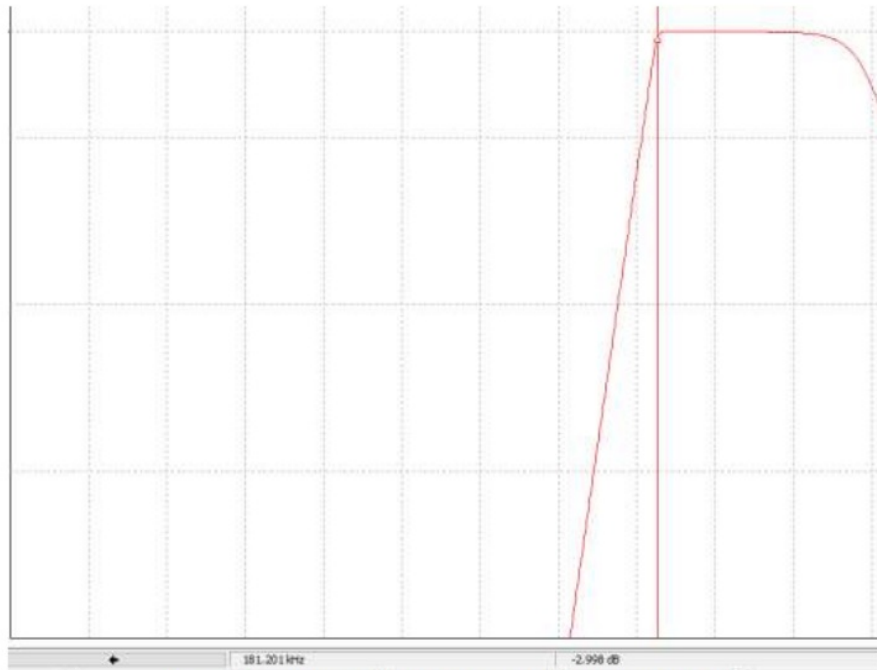


Figure 27 - Butterworth 9th Order High-pass Filter Roll Off

Butterworth High-pass Filter Comparison		
Filter Stage	DB Roll Off	Measured Frequency at -3dB Roll Off
3 rd Order	-2.999 Db	178.637 Khz
6 th Order	-2.998 Db	179.338 Khz
9 th order	-2.998 Db	181.201 Khz

Table 16 - Butterworth High-pass Filter Frequency Comparison

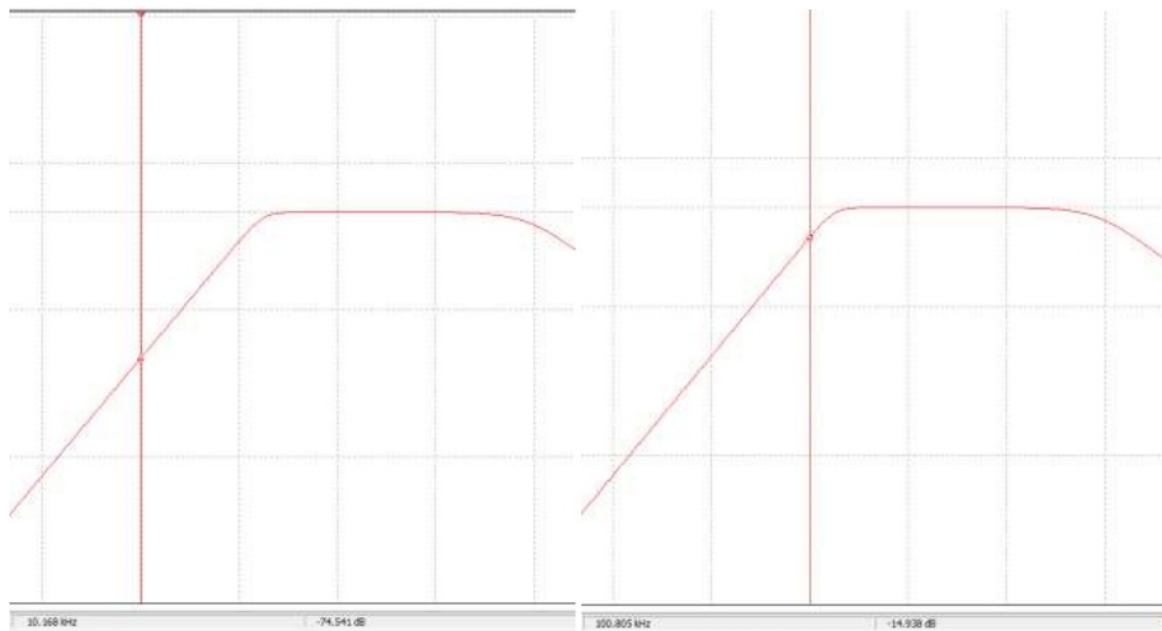


Figure 28 – Butterworth 3rd Order High-pass Filter Logarithmic Frequency

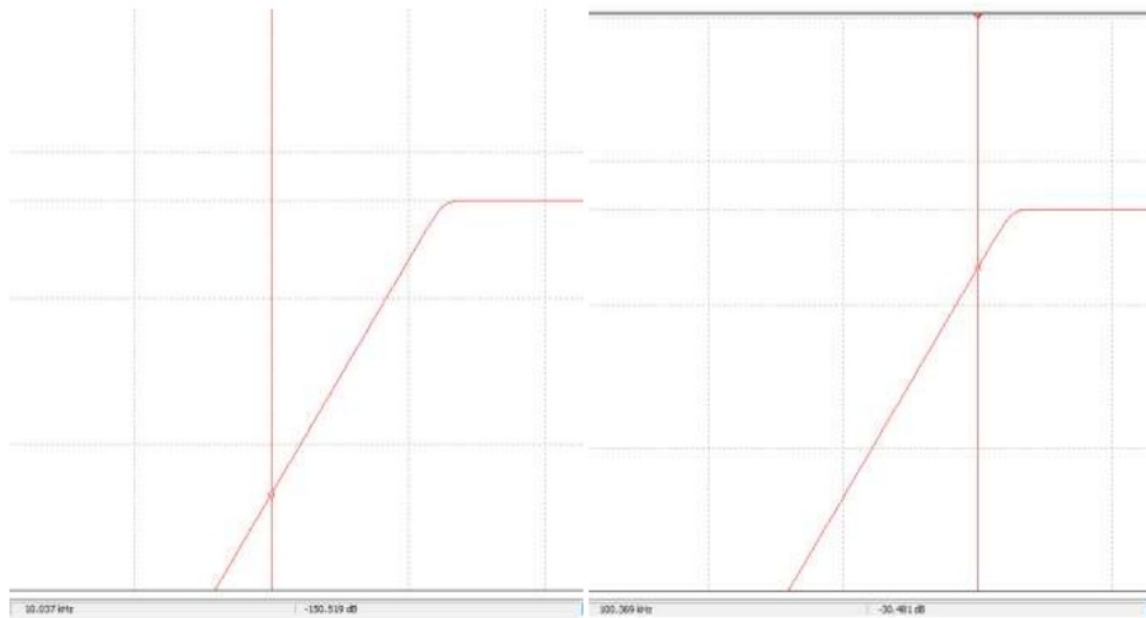


Figure 29 - Butterworth 6th Order High-pass Filter Logarithmic Frequency

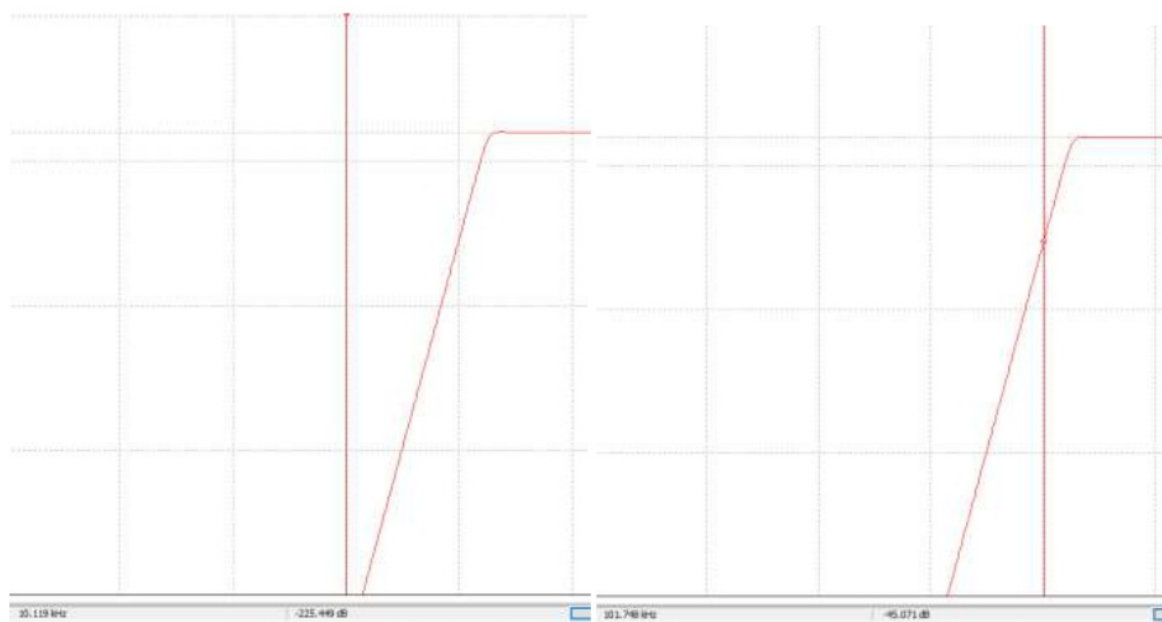


Figure 30 - Butterworth 9th Order High-pass Filter Logarithmic Frequency

Butterworth High-pass Filter Comparison					
Stage	Logarithmic Frequency low		Logarithmic Frequency High		Rate of Roll off
3 rd Order	10.168 Khz	-74.541 dB	100.805 Khz	-14.938 dB	-59.603 dB
6 th Order	10.037 Khz	-150.037 dB	100.369 Khz	-30.481 dB	-119.556 dB
9 th order	10.119 Khz	-225.449 dB	101.748 Khz	-45.071 dB	-180.378 dB

Table 17 - Butterworth High-pass Filter Logarithmic Frequency Comparison

As illustrated in the images and tables above a number of notable differences between each stage of the Butterworth filters can be seen. Measuring the frequency of each filter at the -3.0dB roll-off, each stage is within an acceptable value of the required 180Khz.

Measuring the rate of roll-on for each filter stage, the different stages of the Butterworth filter all measure within $\pm 0.5\text{dB}$ of the required value of 20dB per decade. The expected roll-off rate for 3rd, 6th and 9th order filters are -60dB, -120dB and -180dB respectively.

The image below succinctly illustrates the information gathered from the images and tables above, showing the variation in the Butterworth filter's passbands and rate of roll-on for each stage. While each stage has a flat passband, the filter's passband is varied in duration/curvature. Each stage has a different roll-on, and as the stages increase from 3 to 6 to 9, the transition from stopband to passband decreases, making the transition steeper for each stage added to the filter.

While the Butterworth filters can be used in applications such as signal processing, stopping low frequencies while permitting higher frequencies to pass, the variation in the stages allow for the filter to be more selective in filtering out lower frequency signals as the stages increase. (AspenCore, 2020)

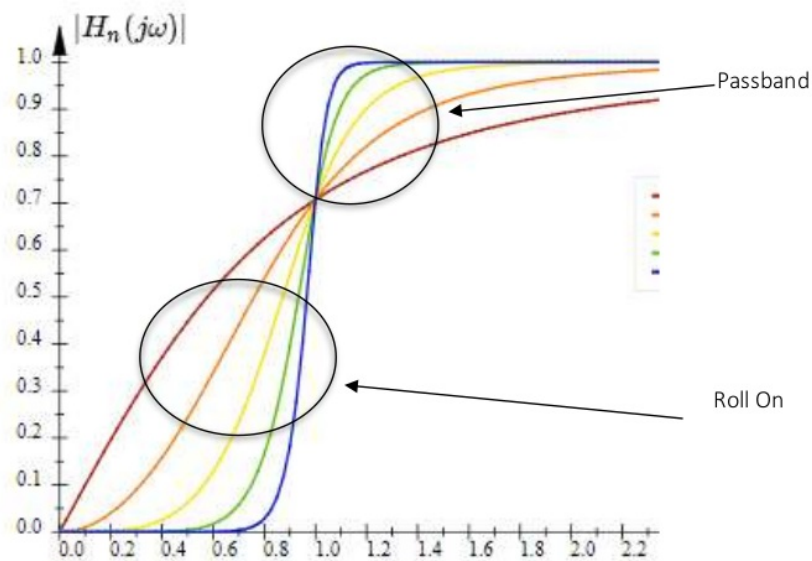


Figure 31 - Butterworth High-pass Filter Comparison

(Pieter.P, 2020)

5. Conclusions

Calculation, design and simulation of the three 5th order filter types, along with the 3 stages of the Butterworth filter was successfully completed, each filter circuit functioning as expected. Using the simulation, a successful comparison of characteristics of each of the circuits revealed features such as roll-off rates and passband magnitude comparable with that stated in the literature.

Whilst there is no ideal filter, the use of such design and simulation technology can be used to test filter designs for suitability to the intended application, and adjustments to components such as capacitor values, can be made and simulation carried out, to test the effect of such alterations on the characteristics.

Recommendation

One recommendation would be to alter the design stage of the filters. Initially the capacitor value, C1, selected is 1nF, the value of C1 remaining the same throughout the filter designs. By altering the value of C1 through the filter stages, the filter responses can then be analysed and compared to see what the difference in capacitor values will make to the filters' other components, passband, rate of roll off and stopband. By altering the component values within the filter, each stage of the filter could be made to be more effective and possibly display more accurate values when testing and analysing the filters.

6. Reference List & Bibliography

References

Administrator, 2019. *www.electronicshub.org*. [Online]

Available at: <https://www.electronicshub.org/butterworth-filter/>
[Accessed 11 05 2020].

Anysilicon, 2016. *anysilicon.com*. [Online]

Available at: <https://anysilicon.com/fpga-vs-asic-choose/>
[Accessed 23 01 2020].

AspenCore, 2020. *www.electronics-tutorials.ws*. [Online]

Available at: https://www.electronics-tutorials.ws/filter/filter_6.html
[Accessed 11 05 2020].

Barkolov, A. & Titarenko, 2009. *www.researchgate.net*. [Online]

Available at: https://www.researchgate.net/publication/220696440_Logic_Synthesis_for_FSM-Based_Control_Units
[Accessed 26 01 2020].

Circuitstoday, 2020. *circuitstoday.com*. [Online]

Available at: <http://www.circuitstoday.com/active-filter-types>
[Accessed 13 05 2020].

Davies, N., 2017. *www.allaboutcircuits.com*. [Online]

Available at: <https://www.allaboutcircuits.com/technical-articles/an-introduction-to-filters/>
[Accessed 14 05 2020].

Electronics-note, 2020. *www.electronics-notes.com*. [Online]

Available at: <https://www.electronics-notes.com/articles/radio/rf-filters/what-is-bessel-filter-basics.php>
[Accessed 13 05 2020].

Electronics-notes, 2020. *www.electronics-notes.com*. [Online]

Available at: <https://www.electronics-notes.com/articles/radio/rf-filters/what-is-bessel-filter-basics.php>
[Accessed 11 05 2020].

Electronics-notes, 2020. *www.electronics-notes.com*. [Online]

Available at: <https://www.electronics-notes.com/articles/radio/rf-filters/what-is-chebychev-filter-basics.php>
[Accessed 11 05 2020].

Elprocus, 2020. *www.elprocus.com*. [Online]

Available at: <https://www.elprocus.com/types-active-filters-and-applications/>
[Accessed 14 05 2020].

Engineering_360, 2020. *www.globalspec.com*. [Online]

Available at:
https://www.globalspec.com/learnmore/semiconductors/analog_mixed_signals/amplifier_linear_devices/analog_active_filters
[Accessed 14 05 2020].

Instruments, T., 2019. *www.ti.com*. [Online]

Available at: <http://www.ti.com/lit/an/sboa319a/sboa319a.pdf?ts=1589527768486>
[Accessed 14 05 2020].

Island, U. o. R., 2010. *Classic Filters*, Rhode Island: University of Rhode Island, College of Engineering.
Kugelstadt, T., 2008. *Active Filter Design Techniques*. Texas, Texas Instruments.

Lai, E., 2003. *www.sciencedirect.com*. [Online]
Available at: <https://www.sciencedirect.com/topics/engineering/antialiasing-filter>
[Accessed 13 05 2020].

Lankshear, I., 2019. *www.electronicdesign.com*. [Online]
Available at: <https://www.electronicdesign.com/technologies/embedded-revolution/article/21808278/the-economics-of-asics-at-what-point-does-a-custom-soc-become-viable>
[Accessed 26 01 2020].

Matej Machalec, J. Š. (., 2010. *www.elektrorevue.cz*. [Online]
Available at: <http://www.elektrorevue.cz/en/articles/analogue-technics/0/synchronous-fsm-design-methodology-for-low-power-smart-sensors-and-rfid-devices/>
[Accessed 26 01 2020].

Pieter.P, 2020. <https://tttpa.github.io>. [Online]
Available at: <https://tttpa.github.io/Pages/Mathematics/Systems-and-Control-Theory/Analog-Filters/Butterworth-Filters.html>
[Accessed 11 05 2020].

Rashid, M., 2011. *www.sciencedirect.com*. [Online]
Available at: <https://www.sciencedirect.com/science/article/pii/B9780123820365000410?via%3Dihub>
[Accessed 14 05 2020].

Sciencedirect, 2020. *www.sciencedirect.com*. [Online]
Available at: <https://www.sciencedirect.com/topics/engineering/low-pass-filters>
[Accessed 11 05 2020].

Smuits, j., 2016. *www.blog.opticontrols.com*. [Online]
Available at: <https://blog.opticontrols.com/archives/1319>
[Accessed 13 05 2020].

Unknown, 2020. *www.tutorialspoint.com*. [Online]
Available at:
https://www.tutorialspoint.com/linear_integrated_circuits_applications/linear_integrated_circuits_applications_active_filters.htm
[Accessed 14 05 2020].

Watford, 2014. *www.ems12lead.com*. [Online]
Available at: <http://ems12lead.com/2014/03/10/understanding-ecg-filtering/>
[Accessed 12 05 2020].

Wentz, Z., 2018. *www.arrow.com*. [Online]
Available at: <https://www.arrow.com/en/research-and-events/articles/using-capacitors-to-filter-electrical-noise>
[Accessed 13 05 2020].

Yerzley, D., 2019. *www.quora.com*. [Online]
Available at: <https://www.quora.com/What-is-roll-off-rate>
[Accessed 12 05 2020].

7. Appendix

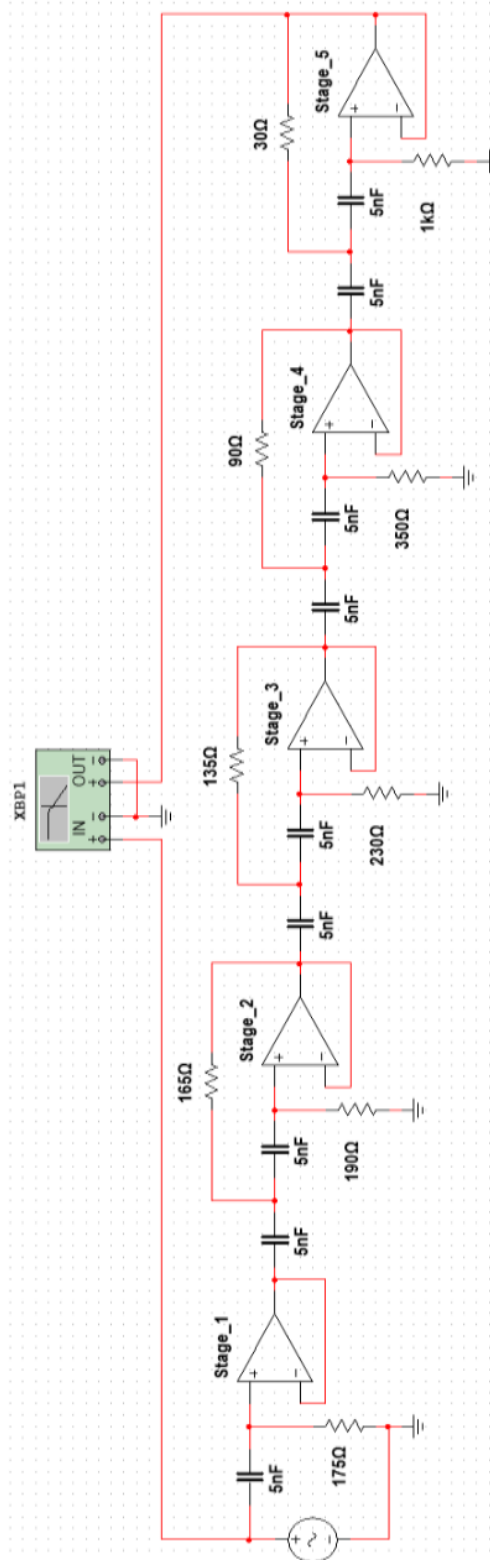


Figure 32 - Butterworth 9th Order High-pass Filter

1809810 - SGEE5000 Assignment II

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

83/100

PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9

PAGE 10

PAGE 11

PAGE 12

PAGE 13

PAGE 14

PAGE 15

PAGE 16

PAGE 17

PAGE 18

PAGE 19

PAGE 20

PAGE 21

PAGE 22

PAGE 23

PAGE 24

PAGE 25

PAGE 26

PAGE 27

PAGE 28

PAGE 29

PAGE 30

PAGE 31

PAGE 32

PAGE 33

PAGE 34

PAGE 35

PAGE 36

PAGE 37

PAGE 38

PAGE 39

GRADING FORM: CSG-RUBIC

DAVID
THOMAS

0

RESEARCH-INFORMED LITERATURE



There was very good evidence of further reading and research from credible and reliable sources. Your interpretation and understanding of the additional research was good and demonstrated a very good underlying appreciation of the subject. Your references were appropriate to the task and applied with few errors or inconsistencies. Your reference list was of an appropriate length for this assignment.

7/10

KNOWLEDGE AND UNDERSTANDING



The knowledge and understanding of the subject demonstrated by this assignment was generally excellent. There were very few gaps in your knowledge of the basic principles and your understanding of the higher level concepts was demonstrated to a high degree. You have demonstrated a highly competent critical understanding of the main concepts and expanded your knowledge well beyond that contained in the course notes.

24/30

ANALYSIS



Your analysis of the task was excellent, both in class and as shown in this assignment. You have demonstrated highly competent analytical skills and your approach to the task was highly logical. You carried out a detailed evaluation of the results and included an excellent conclusion relating your practical results to accepted theory. You have demonstrated a capacity for independent thought and your interpretation of the quantitative and qualitative data was excellent with sound judgements based on wholly valid arguments.

17/20

PRACTICAL APPLICATION AND DEP.



Your application of practical skills for this task was generally excellent. You used both the basic and advanced features of the resources appropriately when designing the circuit and you used appropriate advanced options for testing and evaluating the circuit performance. You managed to design an effective circuit that functioned well within the specification and achieved this by rarely asking for assistance thus demonstrating some advanced problem solving skills. Your results were comprehensive and allowed for a detailed conclusion to be written.

26/30

SKILLS FOR PROFESSIONAL PRAC.



In this assignment you have demonstrated skills in working individually and also collaboratively. Your written report was exceptionally well structured and written in a highly coherent and logical way. The

9/10

report was presented in a professional manner using appropriate language and formatting, good diagrams in appropriate places, good sentence/paragraph structure and clear calculations.