

EXAMINATION 2013 / 2014

AHOLIADAU 2013 / 2014

EXAM

FACULTY OF APPLIED DESIGN AND ENGINEERING

NAME OF COURSE/PROGRAMME: MSc in Non-destructive Testing & Evaluation

YEAR OR LEVEL: Level 7

NAME OF MODULE/UNIT/EXAM PAPER: Ultrasonic Methods

MODULE NO: M7X00731

DATE OF EXAMINATION:

EXAMINATION TIMES:

START TIME: 10.00am

FINISH TIME: 12.00 am

TIME ALLOWED: 2hrs

INSTRUCTIONS TO CANDIDATES:

Answer all questions.

Formulae and Material Properties on last two pages

Question 1. Total Marks - 10

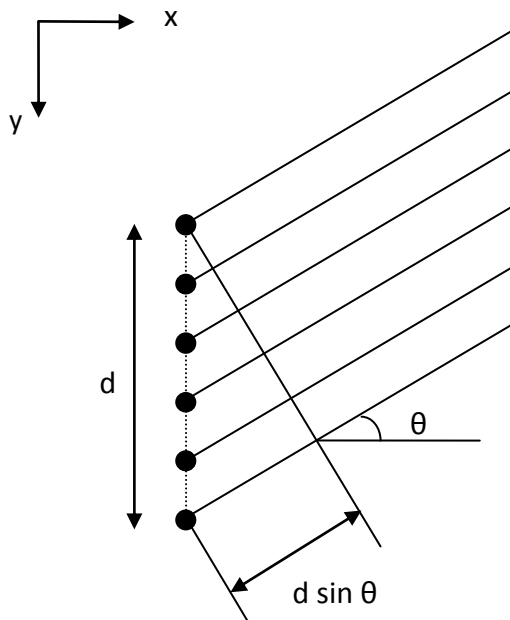
For a single element rectangular ultrasonic crystal, show that the field distribution in the far field can be given as:

$$A(\theta) = a_0 d \frac{\sin(\alpha)}{\alpha}$$

Where $\alpha = k \frac{d}{2} \sin(\theta)$ and k is the wave number ($2\pi/\lambda$) and d is the aperture width. **(10 marks)**

Answer:

Fraunhofer (far field) approximation



Assume all rays are parallel to a point far away. Probe is represented by N point sources. Extra path length as y increases $-y \cdot \sin(\theta)$

$$a(y) = a_0 e^{-jky \sin \theta}$$

For each point source a_0 is the same amplitude. Note: this form is only looking at a fixed angle from the centre of the probe to a point far away.

Now create the sum for all point sources (integrate over y)

$$a(\theta) = \int_{-d/2}^{d/2} a_0 e^{-jky \sin \theta} dy$$

$$a(\theta) = \left[\frac{a_0 e^{-jky \sin \theta}}{-jk \sin \theta} \right]_{-d/2}^{d/2}$$

$$a(\theta) = \frac{a_0}{-jk \sin \theta} \left[e^{-jk \frac{d}{2} \sin \theta} - e^{jk \frac{d}{2} \sin \theta} \right]$$

$$e^{-jk \frac{d}{2} \sin \theta} = \cos \left(k \frac{d}{2} \sin \theta \right) - j \sin \left(k \frac{d}{2} \sin \theta \right)$$

$$e^{jk \frac{d}{2} \sin \theta} = \cos \left(k \frac{d}{2} \sin \theta \right) + j \sin \left(k \frac{d}{2} \sin \theta \right)$$

Subtracting the exponential terms leaves:

$$a(\theta) = \frac{a_0}{-jk \sin \theta} \left(-2j \sin \left(k \frac{d}{2} \sin \theta \right) \right)$$

$$a(\theta) = a_0 2 \frac{\sin \left(k \frac{d}{2} \sin \theta \right)}{k \sin \theta}$$

Multiplying top and bottom by $\frac{d}{2}$

$$a(\theta) = a_0 d \frac{\sin \left(k \frac{d}{2} \sin \theta \right)}{k \frac{d}{2} \sin \theta}$$

Let $\alpha = k \frac{d}{2} \sin \theta$

$$a(\theta) = a_0 d \frac{\sin \alpha}{\alpha}$$

$$a(\theta) = a_0 d \operatorname{sinc} \alpha$$

Question 2. Total Marks – 16

- a) What do we mean by the term ‘Q’ factor and how is it used with regards an ultrasonic probe specification. **(2 marks)**

Answer:

In practice the transducer does not generate a single pure frequency sound wave and will contain a band of frequencies. These are effected by:

the thickness of the crystal, its diameter or length and width,
the effects of the damping medium, and
the electrical characteristics of the transducer and associated circuits.

We refer to this spectrum as the ‘**Bandwidth**’ of the probe.

In a well-designed probe, the centre of this band should be the desired probe frequency and the lower and upper limits are usually defined as the frequencies at which the amplitude is reduced by a given factor. The upper and lower limits constitute the bandwidth of the probe, and can be based on 30% (-3 dB) and 50% (-6 dB) amplitudes.

One way of expressing bandwidth that is also common in other branches of electronics is the 'Quality factor' or 'Q' of the probe and is defined by the formula:

$$Q = \frac{f_c}{f_2 - f_1}$$

Where f_c = the centre frequency

f_1 = the upper -6dB frequency

f_2 = the lower -6dB frequency

- b) For a 10mm diameter PZT crystal of thickness 0.6mm, and given that the compression wave velocity for PZT is 3000m/s, determine the crystals near field distance when transmitting into Perspex. **(4 marks)**

Answer:

PZT Crystal: thickness = 0.6mm,
Velocity = 3000m/s,
Diameter = 10mm

To determine the near field distance in Perspex, we need the crystals resonant frequency:

$$f = \frac{v}{2 \times t} = \frac{3000 \times 10^3}{2 \times 0.6} = 2.5MHz$$

Using Longitudinal Velocity in Perspex = 2680m/s:

$$NF = \frac{D^2}{4\lambda} = \frac{10^2}{4\lambda}$$

Wavelength in Perspex at 2.5MHz:

$$\lambda = \frac{v}{f} = \frac{2680 \times 10^3}{2.5 \times 10^6} = 1.072mm$$

Substituting this into the Near Field Distance Equation:

$$NF = \frac{100}{4 \times 1.072} = 23.3mm \text{ to 1 decimal place.}$$

- c) If the crystal is in direct contact with a mild steel surface, calculate the theoretical absolute ultrasonic beam width for the crystal at a thickness in mild steel of 45mm. **(4 marks)**

Answer:

Using:

$$\sin \frac{\theta}{2} = \frac{1.22\lambda}{D}$$

Need to determine λ in mild steel.

$$\lambda = \frac{5960 \times 10^3}{2.5 \times 10^6} = 2.384 \text{ mm}$$

Therefore:

$$\frac{\theta}{2} = \sin^{-1} \left[\frac{1.22 \times 2.384}{10} \right] = 16.9^\circ$$

Substituting into:

$$\tan \frac{\theta}{2} = \frac{0.5 \times BW}{45}$$

$$BW = 2 \times 45 \times \tan 16.9 = 27.3 \text{ mm to 1 decimal place.}$$

- d) If a suitable Perspex delay line were to be used determine the change in -6dB half beam spread angle at a Perspex/mild steel interface. **(4 marks)**

Answer:

Determine the half spread angle for -6dB in Perspex.

$$\sin \frac{\theta}{2} = \left[\frac{0.56\lambda}{D} \right]$$

λ in Perspex: 1.07mm determined earlier.

$$\frac{\theta}{2} = \sin^{-1} \left[\frac{0.56 \times 1.07}{10} \right] = 3.43^\circ$$

Then use Snell's law:

$$\frac{\sin 3.43}{2680} = \frac{\sin i_r}{5960}$$

$$i_r = \sin^{-1} \left[\frac{5960}{2680} \times \sin 3.43 \right] = 7.64^\circ$$

Therefore:

$$\Delta \text{Beam Angle} = 7.64 - 3.43 = 4.21^\circ \text{ increase.}$$

- e) For an ultrasonic probe with a 'Q' factor of 1.5 and wavelength of 0.987mm in water, estimate its centre frequency and spectral bandwidth. State the upper and lower frequency values that would have been used in its initial determination. **(2 marks)**

Answer:

$$Q = 1.5, \lambda_{\text{water}} = 0.987 \text{ mm}.$$

Need to calculate centre frequency:

$$\lambda_w = \frac{v_w}{f};$$
$$f = \frac{1480}{0.987} = 1.5 \text{ MHz to 1 decimal place}.$$

Using:

$$Q = \frac{f_c}{f_2 - f_1};$$

$$f_2 - f_1 = 1$$

Therefore, Bandwidth of Probe = 1MHz, with $f_2 = 2.0 \text{ MHz}$ and $f_1 = 1.0 \text{ MHz}$.

To determine defect sensitivity in mild steel, the rule of thumb is $>0.5 \times \lambda$.

We could use the probes centre frequency thus:

$$\lambda = \frac{5960 \times 10^3}{1.5 \times 10^6} = 3.97 \text{ mm}$$

Which indicates that defects $>1.985 \text{ mm}$ are likely to be detectable. However, using the upper frequency, $f_2 = 2.0 \text{ MHz}$ from the probes spectral response, this would indicate:

$$\lambda = \frac{5960 \times 10^3}{2.0 \times 10^6} = 2.98 \text{ mm}$$

Which would indicate that defects $>1.49 \text{ mm}$ are also likely to be detectable. This highlights the importance of knowing the spectral resolution of the probe to be used, and also requires consideration of material grain boundaries.

Question 3. Total Marks – 14

- a) A 32 element 2.5MHz array probe with an element pitch of 1mm is used to inspect a stainless steel pipe carrying a hydrocarbon (oil) product. The 'static' B-Scan image created by the probe uses an aperture size of 4 elements. Using the fraunhofer 'far-field' approximation, produce a normalised directivity graph for the specified aperture. (6 marks)

Answer:

Using:

$$A(\theta) = a_0 d \frac{\sin(\alpha)}{\alpha}$$

Where $\alpha = k \frac{d}{2} \sin(\theta)$ and k is the wave number ($2\pi/\lambda$) and d is the aperture width = 4mm.

Allowing $a_0 d = 1$ we calculate the following table of values and plot the graph accordingly:

Example calculations:

$$f = 2.5 \text{ MHz}; \quad V_L = 5740; \quad \text{Aperture} = 4 \text{ mm}; \quad \underline{a_0 d = 1.}$$

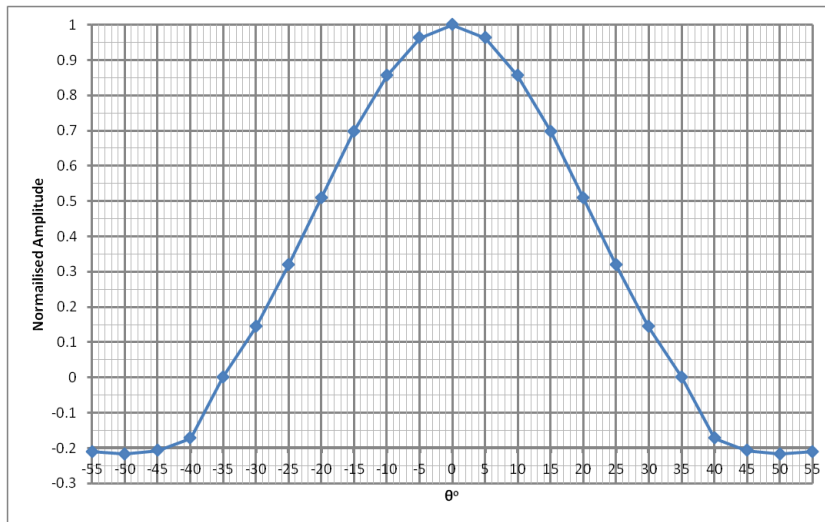
$$\text{when } \theta = 0^\circ \quad \underline{a(\theta) = 1} \quad *$$

$$\theta = 5^\circ = \frac{\pi}{36} \text{ radians}; \quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi \times 2.5 \times 10^6}{5740} \\ = \underline{2736.58},$$

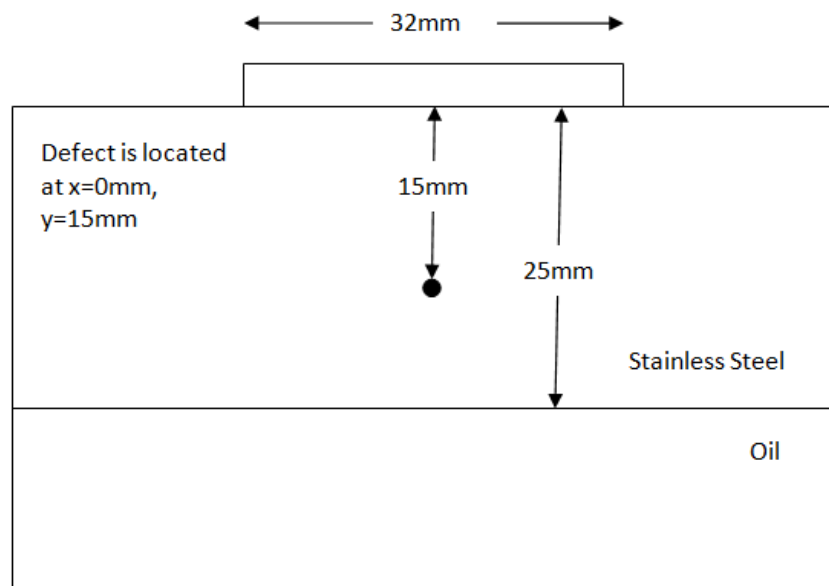
$$K = \frac{k d}{2} \sin \theta = 2736.58 \times \frac{4 \times 10^{-3}}{2} \times \sin\left(\frac{\pi}{36}\right) \\ = 0.477$$

$$a(5^\circ) = \frac{\sin(0.477)}{0.477} = \underline{0.9625}$$

θ°	A
0	1
± 5	0.963
± 10	0.856
± 15	0.698
± 20	0.510
± 25	0.319
± 30	0.144
± 35	0.001
± 40	-0.172
± 45	-0.207
± 50	-0.217
± 55	-0.211



- b) Using the directivity function as determined in part a) along with relevant calculations, provide a scaled illustration of the B-scan image that would be observed using this probe in the presence of the defect as indicated in the following figure. Assume an aperture width of 4 elements and a scan resolution of 1 element. **(6 marks)**



Answer:

-6dB Half Angle from directivity plot: $\pm 20^\circ$

-20dB Half Angle from directivity plot: $\pm 32^\circ$

Calculate the lateral distances and indicated depths from defect centre:

-6dB Lateral Distance:

$$\begin{aligned}
 ld_{-6dB} &= \text{defect depth} * \tan \theta_{-6dB} \\
 &= 15\text{mm} \times \tan 20^\circ = 5.46\text{mm}
 \end{aligned}$$

-6dB Indicated Depth:

$$id_{-6dB} = \frac{\text{defect depth}}{\cos\theta_{-6dB}}$$

$$= \frac{15\text{mm}}{\cos 20^\circ} = 15.96\text{mm}$$

-20dB Lateral Distance:

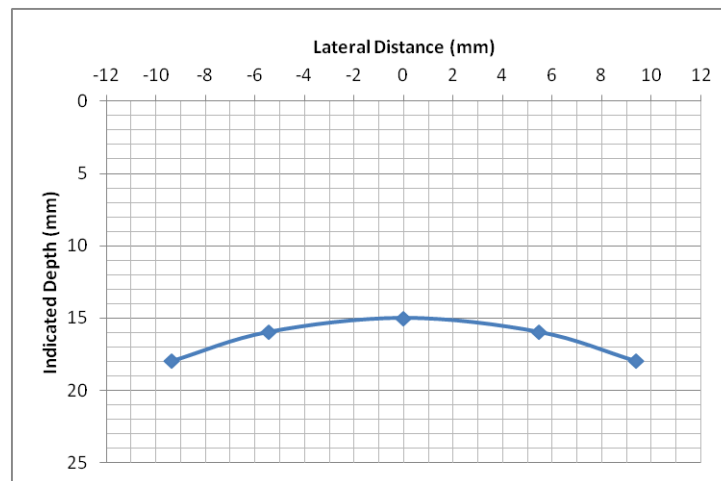
$$ld_{-20dB} = \text{defect depth} * \tan\theta_{-20dB}$$

$$= 15\text{mm} \times \tan 32^\circ = 9.37\text{mm}$$

-20dB Indicated Depth:

$$id_{-20dB} = \frac{\text{defect depth}}{\cos\theta_{-20dB}}$$

$$= \frac{15\text{mm}}{\cos 32^\circ} = 17.69\text{mm}$$



- c) Estimate the reflection coefficient for the stainless steel/product interface and determine the percentage energy transmitted into the product. **(2 marks)**

Answer:

$$RE = \left(\frac{Z_{st.steel} - Z_{oil}}{Z_{st.steel} + Z_{oil}} \right) \times 100\%$$

$$= \left(\frac{44.8 - 1.3}{44.8 + 1.3} \right) \times 100\% = 89.04\%$$

$$\text{Therefore } TE = 100 - 89.04 = 10.96\%$$

Question 4. Total Marks - 20

A TOFD inspection system is to be used to inspect a mild steel weld with a nominal wall thickness of 75mm. It has already been decided that 60° compression probes with a centre frequency of 2.5MHz are to be utilised along with a probe separation distance of 100mm.

- a) Disregarding wedge delays and using probe index points only, determine the approximate arrival times for the lateral and back wall signals. **(4 marks)**

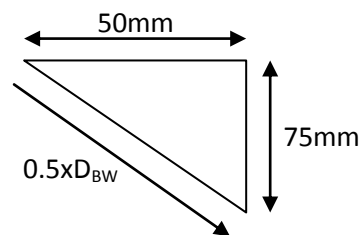
Answer:

Lateral Wave Travel Time:

Lateral Distance = 100mm at longitudinal Velocity in Mild Steel = 5960 m/s

$$t_l = \frac{d}{v_L} = \frac{100}{5960 \times 10^3} = 16.78 \mu s \text{ to 2 decimal places.}$$

Back wall Signal Travel Time:



With reference to the above figure, back wall travel distance can be calculated as follows:

$$D_{BW} = 2 \times \sqrt{50^2 + 75^2} = 180.28 \text{ mm to 2 decimal place.}$$

Therefore, using the longitudinal velocity of sound, as before:

$$t_{bw} = \frac{180.28}{5960 \times 10^3} = 30.25 \mu s$$

- b) If the transmit pulse length is assumed to be 2 cycles at the probe centre frequency, what would be the minimum theoretical depth from the weld surface that near surface indications could be resolved. **(4 marks)**

Answer:

Pulse width = 2 cycles at the longitudinal wavelength (2.5MHz) in mild steel:

$$\lambda_L = \frac{v_L}{f} = \frac{5960 \times 10^3}{2.5 \times 10^6} = 2.384 \text{ mm}$$

Therefore the Pulse Width:

$$PW = 2 \times 2.384 \text{ mm} = 4.768 \text{ mm}$$

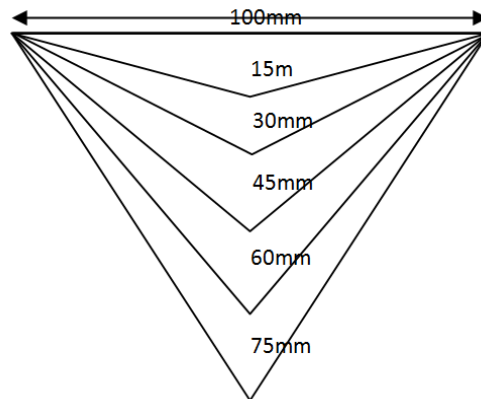
This equates to a travel distance of 104.768mm. Using Pythagoras theorem:

$$\begin{aligned} \text{Minimum Theoretical Depth} &= \sqrt{(0.5 \times 104.768)^2 - 50^2} \\ &= 15.62 \text{ mm to 2 decimal places.} \end{aligned}$$

- c) Develop and sketch a TOFD depth/arrival time correlation curve for the TOFD configuration described previously. Use the basic TOFD assumption that defect signals are assumed to originate centrally between the probe pair, and use at least 5 equally spaced depth values. Mark on the illustration the theoretical minimum depth value calculated previously, and comment on the depth resolution characteristics of the technique. **(12 marks)**

Answer:

Using depths of 0, 15, 30, 45, 60 and 75mm:



Note: We know the travel times for 0mm and 75mm from part a) of the question.

$$t_l = 16.78\mu S$$

$$t_{bw} = 30.25\mu S$$

15mm Travel Time:

$$D_{15mm} = 2 \times \sqrt{50^2 + 15^2} = 104.4mm$$

$$t_{15mm} = \frac{104.4}{5960 \times 10^3} = 17.52\mu S$$

30mm Travel Time.

$$D_{30mm} = 2 \times \sqrt{50^2 + 30^2} = 116.62mm$$

$$t_{30mm} = \frac{116.62}{5960 \times 10^3} = 19.57\mu S$$

45mm Travel Time.

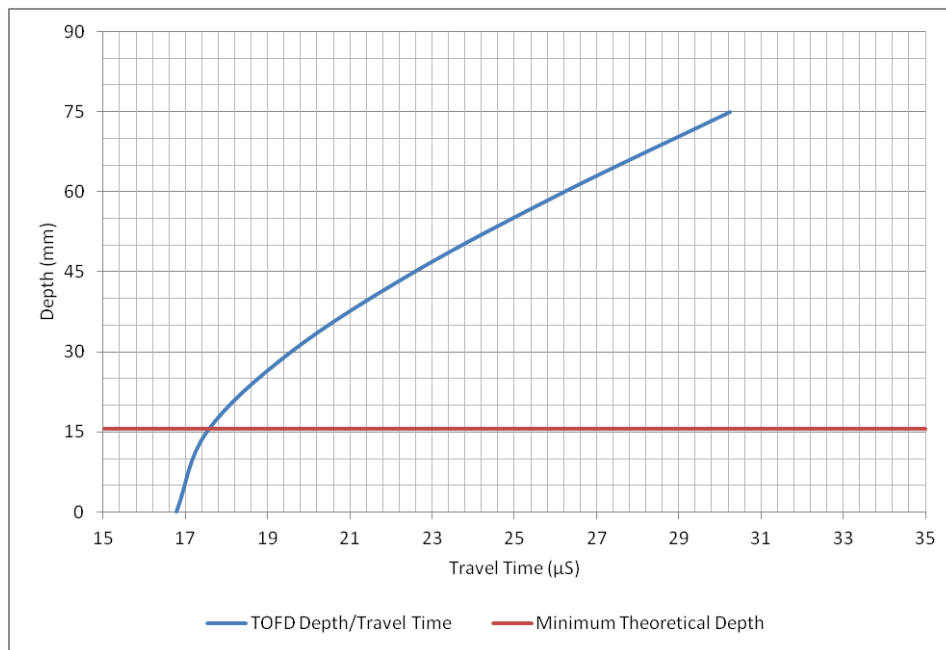
$$D_{45mm} = 2 \times \sqrt{50^2 + 45^2} = 134.54mm$$

$$t_{45mm} = \frac{134.54}{5960 \times 10^3} = 22.57\mu S$$

60mm Travel Time.

$$D_{60mm} = 2 \times \sqrt{50^2 + 60^2} = 156.20mm$$

$$t_{60mm} = \frac{156.20}{5960 \times 10^3} = 26.21\mu S$$



From the characteristic, it can be stated that the resolution of the technique increases with increasing depths. Also, from the parameters given, it is improbable that defects less than 15.62mm could be determined accurately.

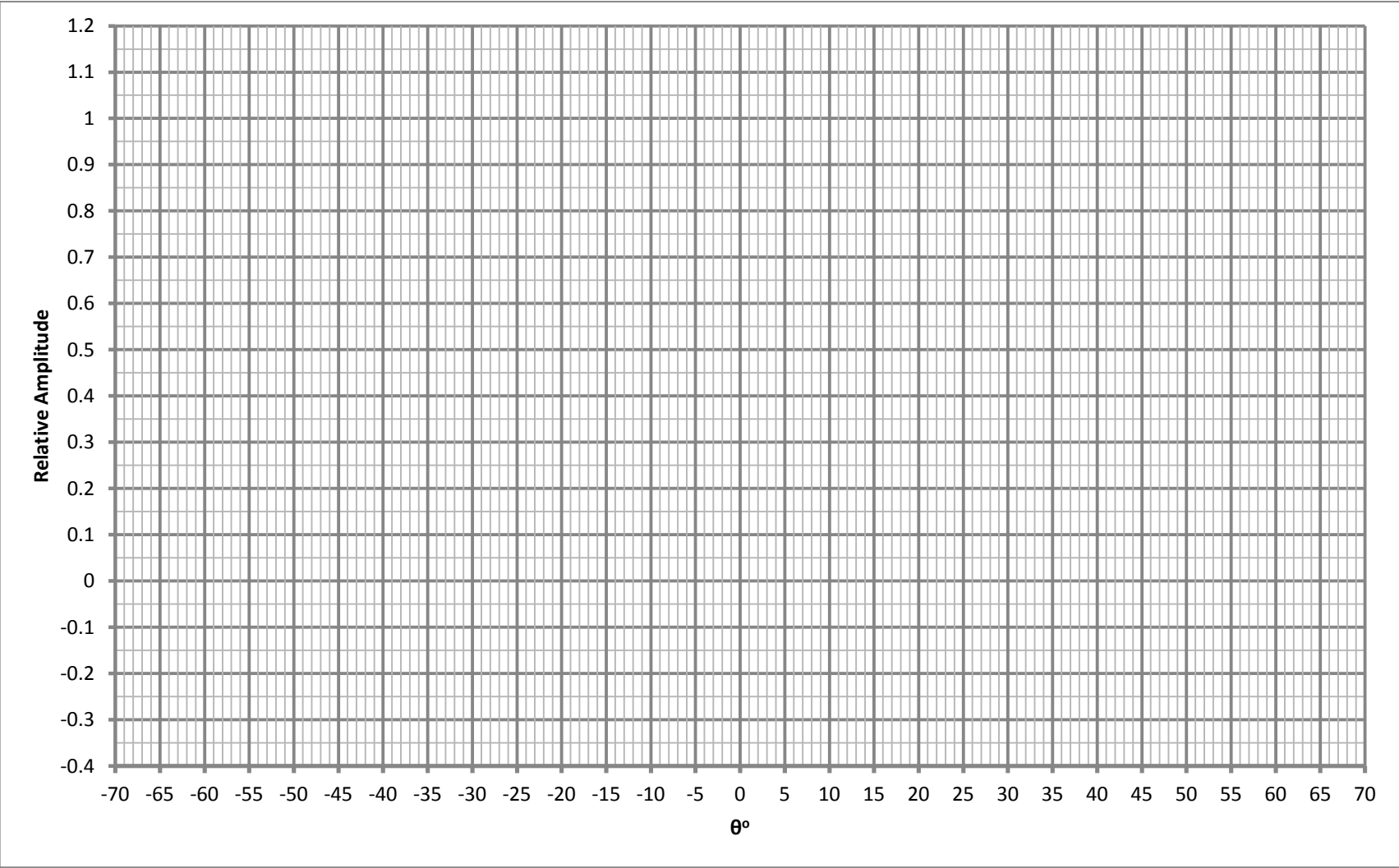
Material Properties.

Material Units	Velocity (C) m/s x 10 ³	Velocity (S) m/s x 10 ³	Density kg/m ³	Acoustic Impedance
Air	0.33	-	1.2	0.0004
Aluminium	6.40	3.13	2700	17.3
Barium Titanate A	5.26	-	5700	30.0
Barium Titanate B	5.53		5700	31.5
Beryllium	12.89	8.88	1800	23.2
Brass	4.37	2.10	8450	37.0
Cast Iron	3.5 to 5.6	2.2 to 3.2	7200	25 to 40
Copper	4.76	2.33	8930	42.5
Glass (Crown)	5.66	3.42	3000	17.4
Gold	3.24	1.20	19300	63.0
Iron	5.96	3.22	7850	46.8
Lead	2.40	0.79	11300	27.2
Lead Niobate	2.76	-	5800	16.0
Lead Zirconate Titanate A	3.00	-	7600	22.8
Lead Zirconate Titanate B	3.00	-	7500	22.5
Lithium Sulphae	5.45	-	2060	11.2
Magnesium	5.74	3.08	1720	9.90
Mercury	1.45	-	13550	19.6
Molybdenum	6.25	3.35	10200	63.7
Nickel	5.48	2.99	8850	48.5
Oil	1.44	-	900	1.3
Perspex	2.68	1.32	1200	3.2
Platinum	3.96	1.67	21400	85.0
Polystyrene	2.35	1.12	1060	2.5
Steel (Mild)	5.96	3.24	7850	46.7
Steel (Stainless)	5.74	3.13	7800	44.8
Silver	3.70	1.70	10500	36.9
Tin	3.38	1.61	7300	24.7
Titanium	5.99	3.12	4500	27.0
Tungsten	5.17	2.88	19300	100.0
Tungsten-Araldite	2.06	-	10500	21.65
Tungsten-Carbide	6.65	3.98	10000 to 15000	66.5 to 98.5
Uranium	3.37	2.02	18700	63
Water	1.48	-	1000	1.48
Zinc	4.17	2.48	7100	29.6
Zirconium	4.65	2.30	6400	29.8

Formulae

Snell's Law: $\frac{\sin i}{V_1} = \frac{\sin r}{V_1} = \frac{\sin R}{V_2}$	Reflected Energy Equation: $RE = \frac{Z_1 - Z_2}{Z_1 + Z_2} \times 100\%$
Ultrasonic Beam Formulae: $NF = \frac{d^2}{4\lambda}$ Where: NF = Near Field Distance d = Crystal diameter or aperture width λ = wavelength	Far Field Approximation: $A(\theta) = a_0 d \frac{\sin(\alpha)}{\alpha}$ Where $\alpha = k \frac{d}{2} \sin(\theta)$ K is the wave number ($2\pi/\lambda$) and d is the aperture width.
$\sin \frac{\theta}{2} = \frac{1.22\lambda}{D}$ Defines the absolute edge $\sin \frac{\theta}{2} = \frac{0.56\lambda}{D}$ Defines the 6 dB edge	$\sin \frac{\theta}{2} = \frac{1.08\lambda}{D}$ Defines the 20 dB edge
Q Factor $Q = \frac{f_0}{f_2 - f_1}$ Where: f₀ = the centre frequency f₂ = the upper -6dB frequency f₁ = the lower -6dB frequency	
Frequency thickness crystal constant: $f \times t = V/2$ Where, • f = desired frequency • t = crystal thickness, and • V = compression wave velocity in the crystal material	

Q3. a)



Q3. b)

