



University of Wales Trinity Saint David

School of Engineering

Determination for the optimum thermal insulation thickness for households and buildings roofs under different weather conditions in the Kingdom of Saudi Arabia

Name: Hamad Shehab ALsadoon

ID: P 137035

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Executive Summary

This project examines the use of roof insulation in the Kingdom of Saudi Arabia, and ways in which, in the future, it may be possible to reduce the growing pattern of electricity usage, either through improved household insulation or perhaps through the use of hybrid cooling units incorporated into the infrastructure. The project uses a Matlab® simulation to examine how quickly a traditional roof and insulation setting loses or gains heat through the transfer to the outside air.

The simulation uses explicit finite difference methods to calculate the relative rate of heat transfer, and defines the ways in which resources could be saved or adjusted to make more economical use of existing supplies.

Acknowledgement

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Terminology

A	<i>amplitude of periodic temperature variation</i>	$^{\circ}\text{C}$
a, b, c, d	<i>coefficients in the finite-volume equations</i>	
Bi	<i>Biot number</i>	
C_{cov}	<i>cloud-cover correction factor</i>	
c	<i>specific heat</i>	$(\text{J} / \text{kg.K})$
E	<i>rate of energy</i>	(W)
$ Fo$	<i>Fourier number</i>	
h	<i>heat transfer coefficient</i>	$(\text{W} / \text{m}^2 . \text{K})$
I_s	<i>solar radiation flux</i>	(W / m^2)
k	<i>thermal conductivity</i>	$(\text{W} / \text{m.K})$
L	<i>roof thickness</i>	(m)
N	<i>number of layers</i>	
n	<i>cloud-cover area or nodal point</i>	
q	<i>heat flux</i>	(W / m^2)
R	<i>thermal resistance per unit area</i>	$(\text{m}^2 . \text{K} / \text{W})$
T	<i>temperature</i>	$(^{\circ}\text{C})$
t	<i>Time</i>	(s)
U	<i>internal energy</i>	(J)
x	<i>coordinate direction</i>	(m)

Greek Letters

α	<i>thermal diffusivity</i>	(m^2 / s)
Δx	<i>intermodal distance</i>	(m)
ε	<i>surface emissivity</i>	
λ	<i>solar absorptivity of surface</i>	
ρ	<i>density</i>	(kg / m^3)
σ	<i>Stefan-Boltzmann constant</i>	$(\text{W} / \text{m}^2 . \text{K}^4)$
θ	<i>phase shift angle</i>	(rad)
ϕ	<i>relative humidity</i>	

Subscripts

c	<i>Convection</i>
f	<i>fluid (ambient)</i>
g	<i>ground</i>
i	<i>inside or nodal point</i>
j	<i>layer number</i>
N	<i>outside layer</i>
O	<i>Outside</i>
r	<i>Radiation</i>
0	<i>initial (e.g. t_0 means $t = 0$)</i>

Chapter 1 Introduction

1.1 Background

The Kingdom of Saudi Arabia is one of the fastest growing countries economically. The population is rapidly increasing which has resulted in constructions and buildings expanding in a noticeable way. Energy demand is high in the kingdom since the climate in the region is normally hot. The usage of Air-conditioning systems is very high and is a major concern for the Saudi Electricity Company. The reduction of the consumption of electricity is one of the most important items on the agenda in modern times (Saudi Electricity Company, 2015). Even from the first electricity distribution grid was implemented in cities and rural areas, it was necessary to control the amount that was consumed to manage the efficiency of the distribution system, but with the current emphasis on sustainability, households in cities and rural areas must be more efficient and functional. The optimisation of the appliances in each household or building is essential, since more than half of the electrical consumption in Saudi Arabia is by residential households (Algarni & Nutter, 2013), so an optimum choice for the insulation is a must for ensuring the efficiency of the home, and minimise electrical consumption.

Electricity consumption in Saudi Arabia increased during the 1990–2010 period due to rapid economic development (Saudi Electricity Company, 2015). Peak loads reached nearly 62.3GW in 2015 and expected to approach 99.4GW by 2024 (Worlddata, 2017) – the annual peak demand growth is 7.6%. A large amount of energy is wasted, and this can be reduced by the implementation of thermal insulation in building roofs. Insulation helps prevent a large amount of heat from escaping into the air. Therefore, thermal insulation is a very effective way to conserve energy.

Ranking the benefits of various energy projects done to save energy costs in a building, insulation is often at the top of the list. Usually determined by the cost-benefit trade off in sealing up very large air gaps in a home. Fortunately, many insulation projects can be done by a homeowner if they can and are willing to increase their capital expenditure only slightly.

1.2 Project Aims and Objectives

1.2.1 Aim

The aim of this project is to determine the optimum insulation thickness, and consequently optimum thermal resistance of building roofs under different climatic conditions in KSA. Climates of seven cities are considered; namely, Riyadh, Jeddah, Abha, Dhahran, Tabuk, Najran, and Jazan.

1.2.2 Objectives

- Confirming the project is of benefit to the society by looking at previous studies and literature review.
- Looking up and searching for related data for the project in Saudi Arabia.
- Choosing two or three types of material that are commonly used in Saudi Arabia.
- Taking note of different configurations used when applying thermal insulation on roof tops.
- Calculating the relative collected data and doing analysis on it using numerical methods.
- Using the finite-volume method for calculations of the data.
- Calculating the electricity loads under different materials and correlating them with the calculated data.

- Choosing the optimum thickness for the thermal insulation in regards with its economic value that was calculated using the data.

1.3 Problem Definition

Electric power is extensively used in buildings and in Saudi Arabia, most of the energy is consumed by air-conditioning (AC) equipment. Air-conditioning systems require a very high start-up current. A major part of the AC load is due to heat loss through roofs. To substantially reduce this loss, thermal insulation must be used and good design practices for implementing the insulation and thermal mass layers must be applied. The amount of insulation material to be used must be optimised, and under optimum conditions, the resulting roof thermal resistance (R-value) would be the recommended value to use. Several other economic parameters are involved in determining the optimum thickness of insulation, and these values will be discussed in the optimisation section – the recommended building roof R-value also depends on the type of insulation material to be used (particularly the thermal properties), roof geometric configuration, climatic conditions, and indoor design conditions.

1.4 Dissertation Structure

The aim, objective and problem definition of the study are explained in the first part of this chapter. This is followed by a literature review of previous works done in the same field of experimentation and investigations which will include roof configurations and insulation material. Chapter three is the experimental section. This part will clearly explain the calculations and methods used in conducting this experiment with previous examples of similar calculations. Matlab® simulation will also be shown and explained in brief followed by the final results from both calculation and the Matlab® simulation.

The results from Chapter 3 are then discussed in the following chapter. All the numbers reflected by the calculations that took place are discussed and explained in order to generate a clear conclusion from them. Application of the proposed experimental designs is also mentioned with all the criteria, constraints and specifications and alternative design possibilities are also presented before this project ends with the conclusion.

1.5 Summary

This project discusses three different types of roof insulations usage in a number of cities within the Kingdom of Saudi Arabia with a well-explained calculation methods and a Matlab® simulation. The three types are compared in many aspects such as optimum thickness, temperature change, orientation and material. Numbers generated are discussed in brief and explained thoroughly before concluding the outcome of this comparison.

Chapter 2 Review of Existing Literature

2.1 Previous Studies

The thermal performance of a composite roof under steady periodic conditions was first predicted by Zaki and Hassan (1986). They studied a two-layer roof, taking into consideration the effect of orientation and relative position of the insulating layer. Al-Nafeez and Rogers (1990) evaluated insulation materials using time lag, decrement factor, cost and R-value and found that polyurethane with an optimum thickness of one inch (25.4mm) appeared to be the most cost effective. They found that better thermal performance was instituted by locating the insulation within the outer side of the roof. Hasan (1999) worked out the optimised insulation thickness for roofs using the life cycle cost analysis, using the present value method. His study was limited to

heating only, without the losses when cooling the house with AC, and he estimated the transmission load using the degree-days idea. The pay-back periods were found to be between 1 year and 1.7 years for rock wool and 1.3 years to 2.3 years for polystyrene.

Al-Sanea (2000) presented a finite-volume procedure for the evaluation of the thermal characteristics of composite structures, accounting for solar radiation as well as convection. The dynamic R-values of roofs with different orientations were calculated using the climatic data for Riyadh. The dynamic R-value are then used to calculate the heat flow characteristics of a roof, making an allowance for the effects of heat storage, solar gains, and radiation exchange with surroundings as well as convection and conduction resistances. Using the same model, Al-Sanea and Zedan (2001a) investigated the effect of insulation location on heat transfer characteristics of building roofs under steady periodic conditions. The thermal performance of a roof with an insulation layer placed on the inside surface was compared to one where the insulation layer was placed on the outside. The results showed that insulation layer location had a minimal effect on daily mean heat transmission load, with a slight advantage for outside insulation in summer and inside insulation in winter. Outside insulation gave a smaller amplitude of load fluctuation with smaller peak loads in both summer and winter for all roof orientations.

However, under initial transient effects, Al-Sanea and Zedan (2001b) showed that the location of the insulation layer had a significant effect on both instantaneous and daily mean transmission loads. Under conditions of their study, they demonstrated that placing insulation on the inside presented a transmission load that was 80% less than the value for insulation placed on the outside, during first few hours of the transient process. Also, the duration of the transient process before settling to the steady

periodic state, and thus, the period of thermal discomfort due to radiation exchange was substantially reduced when using inside insulation. They found that, using internal insulation, the average heat transmission during the first 24 hours of AC operation was about one-third of that with external insulation. To reach this conclusion, dynamic R-values were averaged over the first 24 hours and compared for a range of roof orientations, seasons, and insulation locations. Their recommendation was that for buildings where the AC system is not running continuously, internal insulation should be utilised.

The finite-volume model used by Al-Sanea and Zedan (2001b) was refined by Al-Sanea, et al. (2003) To determine the actual thermal performance and optimum insulation thickness required for cavity roofs. They investigated different insulation materials and tried these at different locations in the cavity. Annual cooling and heating transmission loads were calculated for each insulation thickness and then different economic models were tried, to minimise the total cost of insulation and energy consumption over the lifetime of the building. The investigation compared air spaces of different sizes, insulation layers with different surface conditions, and insulation with different thicknesses, with control cases with no air space or with no insulation.

Al-Sanea and Zedan (2002) used annual transmission loads, annually averaged dynamic R-value, time lag, and decrement factor to determine the optimum thickness of insulation in non-cavity roofs with different orientations. A parametric study was performed to establish the sensitivity of the optimum insulation thickness to changes in economic parameters. It was found that the optimum insulation thickness increased with the cost of electricity, length of building lifetime, and the inflation rate, and decreased with the increasing cost of insulation material, coefficient of performance of AC equipment, and discount rate. Their results also showed that roof orientation had

a significant effect on thermal behaviour, but a smaller effect on optimum insulation thickness. A south-facing roof was found to be best, since it gave a 12% lower annual transmission load and a 5% lower total cost, when compared to the least favourable west-facing orientation. In this study, moulded polystyrene was found to be the most economical type of insulation material.

In summary, there is an increasing level of interest and research into the performance and optimum thickness of building envelopes with specific emphasis on the thermal insulation of the roof. In this study, performing a thermal analysis of building roofs and comparing results under different climatic conditions in Saudi Arabia, and using different insulation materials and roof configurations is intended to add to the data needed to calculate the optimum level and position of roof insulation.

This Chapter discusses some of the basic concepts one should investigate when considering an insulation project.

2.1.1 Insulation Thickness

Insulation is typically appraised as having a specific R-value per inch or some set thickness, for example, $\frac{3}{4}$ inch (Frazier, 2014). The R-value increments in a direct, or consistent, rate as the thickness increments. For instance, on the off chance that one inch of fiberglass batt insulation has an R-value of 3.2, then two crawls of a similar fiberglass batt will have an R-value of 6.4 and have double the insulation capacity of the one-inch thickness (Frazier, 2014). Diverse encasings will have distinctive R-values, and these distinctive resistances can include in the event that they are layered together. A normal divider may include block confronting, vapour retardant, molecule board, fiberglass insulation, a gypsum board and even a layer of stucco/paint. There would be at least six R-values including to compute the aggregate R-value of this

divider (Frazier, 2014). The insulation thickness is one of the main considerations in the material's capacity to oppose warm development.

2.1.2 Insulation Material

The other prime component of a structure's ability to hold warm in or out is the honest to goodness material of the insulation (Frazier, 2014). Strikingly, there is a parallel between warmth conduction and electrical conduction. Metals, for instance, copper are excellent electrical and warmth transmitters. Most plastics make extraordinary electrical and warmth separators. Dead air is a not too bad warmth cover like a vacuum (thermos bottle). Common gasses, for instance, air or argon make poor warmth conductors (good insulators) and are used as a piece of a couple of windows. All building insulation uses this property to some degree. Fiberglass insulation is genuinely about the air trapped between the fibres. Plastic foams, both open and close cells, have little air pockets or ways that trap air or gasses and this opposes warmth development (Frazier, 2014). Numerous materials have been contemplated, and their insulation properties are notable. Diverse materials can have altogether different insulation properties. On the off chance that the R-value is known, the normal temperatures and the range of a divider or roof, the measure of warmth that will go through in a specific time can be computed, utilizing the accompanying condition: $Q = U A \Delta T$ or $Q = (A \Delta T) / R$, Where: Q=heat development, A=wall or roof region, ΔT =Temperature contrast from inside and outside the structure (Frazier, 2014). The present warmth development through a divider is figured, and the insulation R-value is changed to a higher number (thicker or diverse material) and recalculated. The difference in warmth going through is the measure of energy saved from a radiator or ventilation system keeping up a similar temperature. This is fundamentally how energy examining programming predicts the energy and related cost reserve funds from

including insulation in a building. This is likewise how a contractual worker ought to decide the span of warming and cooling gear for a building.

2.1.3 Water Vapour Movement

Notwithstanding controlling temperature, inside solace of a structure depends to some degree on the capacity to control the development of dampness and moistness through the structure. Structures encounter inward water loads from showers, cooking, painting by individuals (QTR, 2015). This water vapour (alongside different vapours and exhaust) needs to escape the space at some rate. The outside condition might be higher in water vapour focus. For this situation, the vapour will attempt to move into the building. High stickiness in the inside of structures can prompt solace and medical problems (QTR, 2015). This can likewise encourage the development of mould in the building inside.

A portion of the insulation materials portrayed here can go about as vapour obstructions that stop the development of water (Frazier, 2014). Others essentially back off the water development (semi-permeability) and some enable dampness to go straightforwardly through the insulation. Appropriate arrangement of vapour hindrances relies on upon neighbourhood atmospheres and building structure utilization. Continuously counsel an expert when introducing vapour boundaries. It is not prudent to introduce insulation with a vapour hindrance in a building that as of now has a vapour boundary since this can trap dampness between building materials layers, where it can obliterate dividers and roofs (QTR, 2015).

2.2 Roof Configurations

Roof insulation should be able to play two major roles in a roof:

- 1- It is the primary thermal barrier for the top surface of the building envelope, and
- 2- It serves as a base for the roofing system (QTR, 2015).

For these deliverables to be generated successfully, all roof insulation should possess many characteristics (Frazier, 2014). Design should have an acceptable thermal resistance to meet the design criteria. The design should also have high resistance to indentation, compression and crushing during installation of the roof membrane and during periodic maintenance, once the roof is completed (QTR, 2015). Cover boards are always recommended by designers and specialists. Moreover, dimensional stability and resistance to moisture absorption are also important for long life span assurance. The more the design absorbs moisture, the weaker it becomes. It must also be rigid enough to support the roof membrane and to span the rib openings of metal roof decks. Also, the design must be compatible with all used techniques. Resistance to wind speed and power to avoid roof lifting is also of great importance. Finally, the design must be acceptable by local laboratories and quality control units.

The easiest part of a home exterior components to insulate is the Attic over flat ceilings due to their accessibility and enough room for ventilation (Frazier, 2014). However, most homes use ceilings that provide small space for insulation. It is completely important to assure proper insulation for any type of roof used. In addition, designers are suggesting builders should begin to insulate the roof deck to create a conditioned attic space. This can reduce duct usage and loss because the HVAC system will be installed into a conditioned space (QTR, 2015).

In winter, properly designed roof vents expel moisture that could otherwise accumulate and deteriorate insulation or other building materials. In summer, ventilation reduces roof and ceiling temperatures, thus saving on cooling costs and lengthening the roof's life. This makes it essential that a proper ventilation system must be present.

Construction and building experts are researching attic ventilation. For years, researchers have believed that the cooling benefits of ventilating a well-insulated attic are negligible. However, some experts are now debating whether ventilation is even effective at moisture removal and this remains a debating subject.

A large number of factors affect the optimum thickness of insulation in building roofs, and this then affects the recommended roof R-values. One of these factors is the roof physical conformation or design. In addition to the different types of thermal insulation materials used (see following section) in roofs, roofs also differ in respect of different building materials. This could be the type (properties), amount of masonry or tiles and the location of the masonry or timber. Also, the same roofing materials could be used in different ways such as solid slab and Hord block with or without insulation. This means there are a lot of possible roof configurations, theoretically, but the most commonly used structures have been examined here (Al-Sanea & Zedan, 2002). The three most common configurations in roof structures in Saudi Arabia are designated R1, R2, and R3 in the Figures below.

Figure 1: Roof Type R1: Solid slab roof.

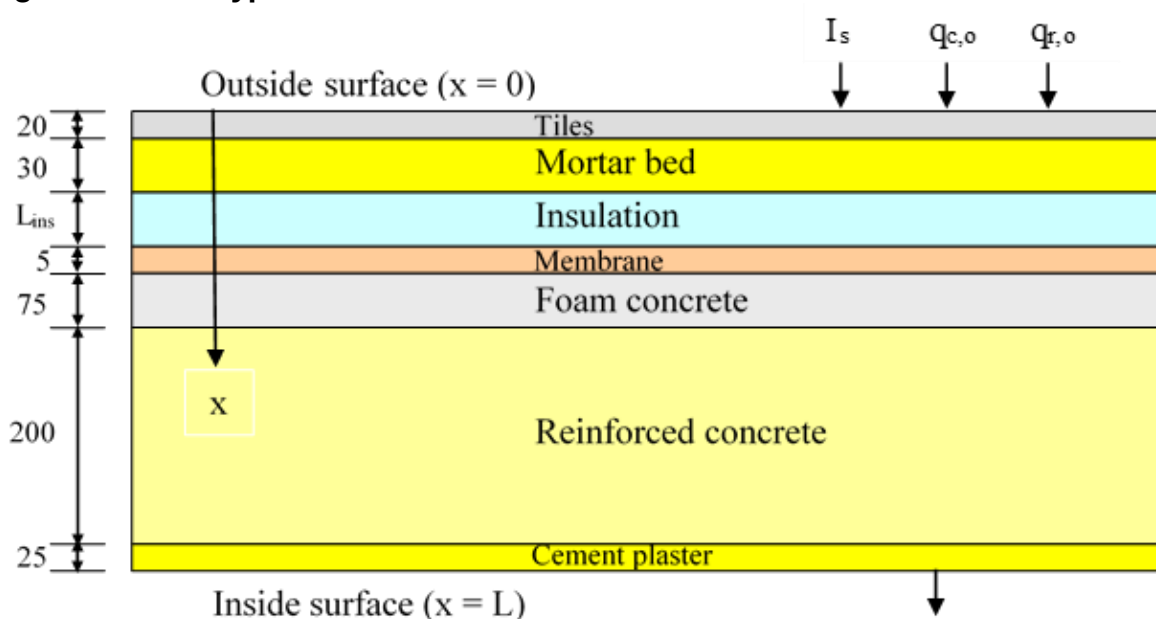
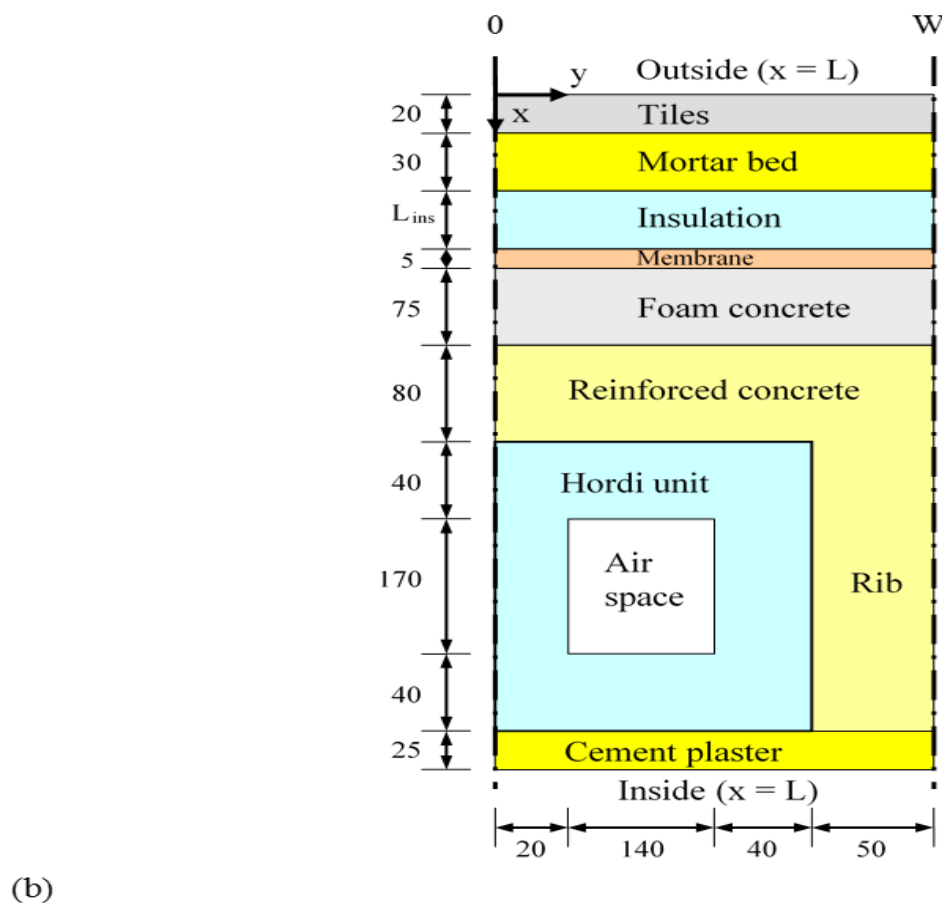
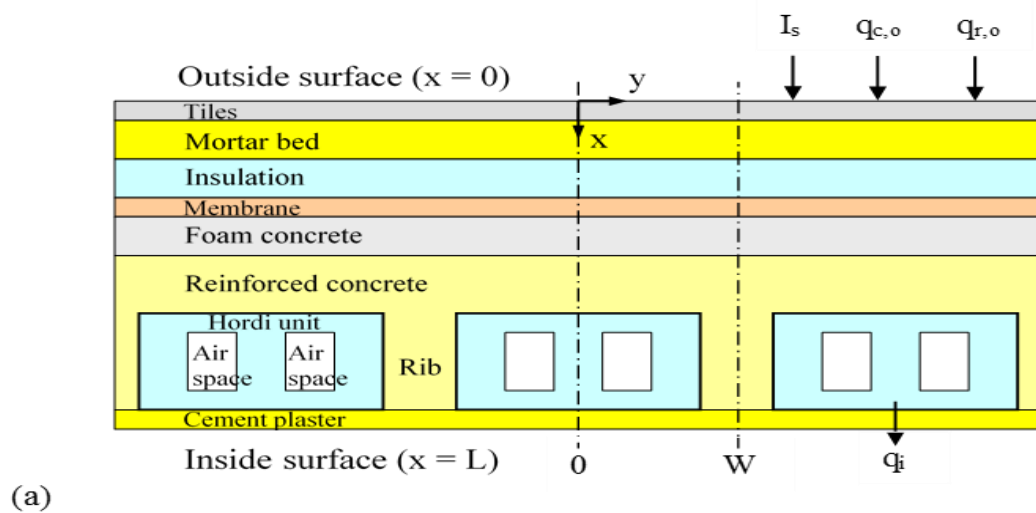


Figure 2: Schematic of a front-view section in a typical rib-slab (Hordi) roof showing (a) symmetric region of interest and (b) enlarged symmetric region with dimensions in mm (not to scale); Hordi unit material is concrete for roof R2 and is polystyrene for roof R3.



2.3 Roof Insulation Materials

There are several materials which are used for roof insulation in Saudi Arabia; rock wool, polyurethane, moulded polystyrene, polystyrene (Hasan, 1999), glass fibre, and mineral wool (Jackson & Day, 2007, p. 261). In addition to these, there are also variations and products used in other countries which may be utilised by contractors. There are also several different ways of applying the insulation (Jackson & Day, 2007);

- **Blanket insulation.** This is typically made of glass fibre, mineral wool, or rock wool and is available in rolls or as batting – it is non-flammable, and can be supplied with or without an attached vapour barrier. The standard thicknesses available are 100mm, 150mm, 170mm, and 200mm (see Figure 3).

Figure 3: Blanket insulation. Source: (OVO Energy, 2017)



- **Fiberglass Batts.** This material has been around for quite a while. It is generally reasonable and the householders can introduce this themselves. The issue is that it must be cut into exceptionally exact pieces and shapes to cover the range it is attempting to protect. This can turn out to be a significant task for

odd formed spaces and corners around wood shafts. The batts must fit firmly into all spaces – no breaks. Introducing fiberglass insulation is not troublesome, but rather it can be a bothersome, hot occupation. Eventually, fiberglass can settle and free its underlying R-value of roughly 3.5 for every inch of thickness.

- **Rigid sheet insulation.** This is ideal for the inside of sloping roofs, and is also available in the same range of thicknesses (see Figure 4). Also, sometimes known as;
- **Foam Boards:** These premade sheets can have high insulation (R) values. They are genuinely simple to deal with and install, and are certainly worth considering for the householder willing to attempt to introduce insulation themselves. Be that as it may, their cost tends to be higher than free cellulose, fiberglass batts or splash foams.

Figure 4: Rigid Sheet Insulation. Source: (Ecohome, 2014)



- **Foiled quilts.** These are multi-layer quilted sheets of plastic sheets interleaved with reflective foils. It is relatively expensive, but a 25mm foiled quilt has the

same U value as a 200mm blanket. The highest quality quilts also have a layer of very high quality 'bubble wrap' (see Figure 5).

Figure 5: Foiled Quilt insulation. Source: (Thinsulex, 2016)

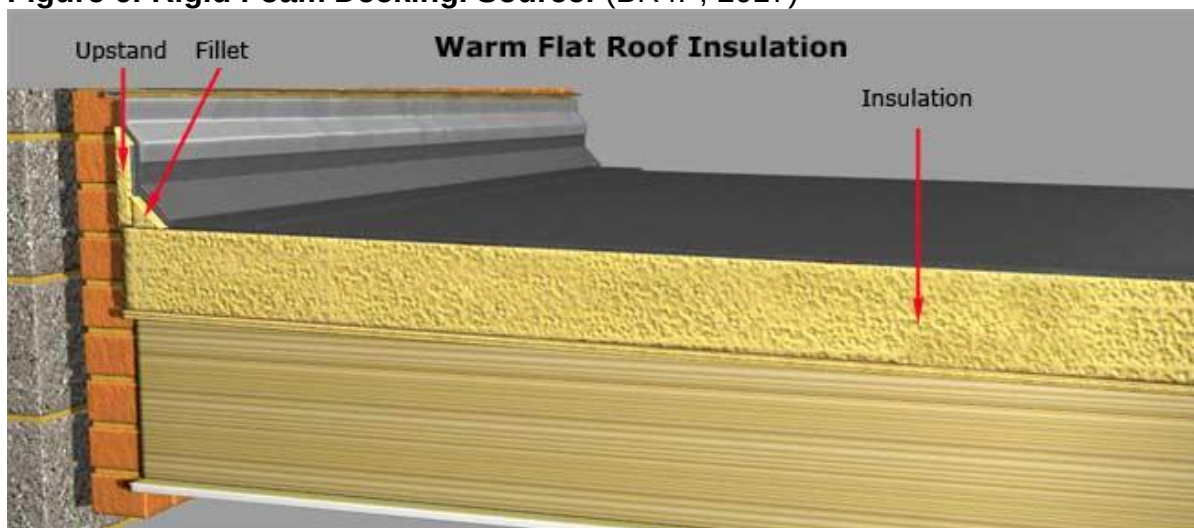


- **Shut Cell Foam.** In recent years, shower on insulation foam has become more popular in the USA. The foam is showered onto a surface, extends and solidifies, then is trimmed and completed with different covers (or left uncovered). If it is connected well, the foam will totally seal any surface and can easily be applied to the undersides of surfaces. The R-value of shut cell foam is very high per inch of insulation at around 6.5 for each inch. These advantages are extremely appealing, as air development from the outside is adequately stopped. This implies that dampness, or water vapour, development into the space is additionally halted. Application is fast and easy once setup. The inconveniences incorporate a high starting expense around three times higher than fiberglass insulation. A part of the less important issues includes the fact

that anything underneath the foam is totally solidified and stuck together. For instance, wiring covered by foam is impossible to repair, and the foam can likewise conceal water harm from view that may otherwise be generally noticed.

- **Open Cell Foam.** This is fundamentally the same as shut cell foam, however it varies in that the little upsurges in the foam interfere with each other. This diminishes the R-value to around 4.2 for every inch and enables some dampness to go through the material. All in all, open cell foam costs about half as much as shut cell foam. Application is fundamentally the same as shut cell as are the advantages and disadvantages. Foams are a good decision if no-one needs to change a storage room space into a living space. The odd states of surfaces in the loft are moderately simple to seal with the shower application.
- **Rigid foam decking.** This is used in cases where the roof space is used for storage, as it forms a solid layer on which a person may walk, but the panel makers recommend that it is used in conjunction with blankets underneath between the ceiling joists (see Figure 6).

Figure 6: Rigid Foam Decking. Source: (BR4P, 2017)



- **Blown-fibre insulation.** This is rarely used in Saudi Arabia, but essentially it is blown through a large hose into the spaces between the joists until an even layer 270mm thick is achieved. It then dries into a thick blanket (see Figure 7).

Figure 7: Blown Fibre Insulation. Source: (Mammoth, 2017)



- **Cellulous:** This material is an alternative to the foam shown in Figure 7, and is frequently recently finely shredded newspaper. The material is just blown free into the space where it is required (upper room or dividers). This is regularly not a householder project since exceptional hardware is expected to convey the material through a blower and hoses. The cost is on the low side for insulation, contingent upon the seller (roughly \$0.04 per R-value per square foot introduced). One of cellulose's points of interest is that the little particles tend to seal up air spills in oddly moulded ranges. Establishment time can be minutes once everything is set up, essentially blow the material into the storage room to the required depth. A few of the weaknesses of free cellulose include: Careless application can have the shredded material covering required vents in the upper

room (soffit, and so on.). High winds can move the insulation around and leave uninsulated roof regions (fences around vents reduce this). The material is subject to settling. An extensive part of the R-value of insulation is because of its thickness and the air trapped inside. As insulation settles and turns into a thinner layer with time, its R-value drops. Free cellulose is exceptionally unprotected from this settling and should be reviewed at regular intervals.

- **Polyisocyanurate Board (ISO).** This is a premade unbending foam board, usually with an aluminium paper backing. The ISO or PIR board has a high R-value of around 6.2 to 7.2 for every inch thickness. The cost is moderately high and in accordance with the range of splash foams for dollar per R-value. An advantage of PIR board are that it is generally minimal per measure of insulation. Being grooved, ISO is easy to deal with and join to vertical surfaces. ISO does not require much tidying and finishing, and is excellent for use in living areas. One of the drawbacks, like fiberglass batts, is that exact geometric pieces must be sliced to fill in voids and make spaces impervious to the wind stream. Properly introduced, ISO material can also act as a vapour boundary.
- **Expanded Polystyrene Board (EPS).** This is an inflexible foam board that requires no support. This board is regularly used as a part of insulated solid structures. The material has great insulation properties (R-value breaks even with 4 for each inch of thickness), however the sheets are expensive – although EPS is the least expensive of the premade foam boards. The sheets break effortlessly if misused. As a rule, the EPS sheets both retain and permit water vapour to go through. In this manner, they are not considered as vapour seals.
- **Extruded Polystyrene Board (XPS).** Is more grounded than EPS and is likewise a foam board. The R-value of 5 for each inch of insulation puts it

amongst EPS and ISO sheets in its warm resistance capability. This material does not, however, prevent water vapour from passing, so it is not a vapour obstruction, but rather a vapour retardant. The XPS board is one of the more expensive board products.

- **Other Materials.** There is an assortment of other insulation materials that are sometimes use. Cotton, sheep's fleece, mineral fleece and different plastics can be utilised as building insulation. These are not standard and the property holder would have trouble finding a contractual worker to introduce these materials.

From the types of insulation materials available locally, three insulation materials are chosen and used in the present investigation. These are: moulded polystyrene, polyurethane board, and rock wool; designated in the present study as insulations MP, PB, and RW, respectively. Remember that the optimum thickness of insulation in roofs depends upon: thermal properties of the insulation material used, cost of insulation, roof configuration, indoor design conditions and outdoor climatic conditions. It is not the objective of the present study to classify insulation materials into categories of good insulation and bad insulation. The type of application may dictate which insulation material to use with required specific characteristics other than thermal properties.

2.4 Insulation and environment

The overall use of insulation is to reduce power consumption by HVAC systems. The higher the quality of insulation, the less the heat loss values (Frazier, 2014). This results in a lower energy loss from the HVAC system by reducing the power used by it. Using less power results in using less fuel which meets the criteria of being more environmental friendly. Although insulation has a positive impact on environment since

it decreases heat emission and power consumption, materials used in insulation have always generated several studies and debates to investigate their negative impact on environment and health. Concerns were regarding the health of the installer and individuals who will be living close to the installations and to the ozone layer that might be affected by materials used. The installer breathes in mineral wool fibres which causes breathing problems both in the short and long term. As we biologically know, lung's alveoli do not get rid of particles that enter them easily, therefore, accumulated particles with time could cause serious breathing problems and might also be a cause in triggering cancer problems but not yet proven to have a similar impact.

Cellulose usage in insulation materials also cause flammability issues. This issue was solved by adding fire retardant chemicals which along with its greater density provides great fire safety. Foam products used to contain Chlorofluorocarbon (CFCs), blowing agent, which has been replaced by pentane, Hydrochlorofluorocarbon (HCFC) and Carbon Dioxide because CFCs have a direct impact on the ozone layer unlike the replacements which have minor effects. However, the usage of pentane is not completely harmless because pentane increases smog formation. The insulation materials of made of polystyrene, Polyisocyanurate and polyurethane use primarily HCFCs because they are a lot less harmful to the environment. It must be assured that the insulation system used is least harmful to environment and health of whoever is in contact with the insulation system.

Chapter 3 Experimental Section

3.1 Method of Calculation

The simulation is completed using Matlab®, using the finite difference model, which is developed from the work of the English mathematician, Brook Taylor (1685-1731) and the Scottish mathematician Colin MacLaurin (1698-1746). This method of differentiation begins with the Taylor Series.

3.1.1 Taylor Series

The definition of the Taylor Series is that, in some circumstances, a function $f(x)$ can be written in terms of an infinite series sum of other functions, for example;

$$f(x) = f(x_0) + f'(x_0) \cdot \frac{(x - x_0)^1}{1!} + f''(x_0) \cdot \frac{(x - x_0)^2}{2!} + f'''(x_0) \cdot \frac{(x - x_0)^3}{3!} + f^{(4)}(x_0) \cdot \frac{(x - x_0)^4}{4!} + \dots$$

Which can be written more neatly in a compact form using the summation notation;

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(x_0) \cdot \frac{(x - x_0)^n}{n!}$$

Equation 1: Taylor series expansion of f(x)

N.B. if $x_0 = 0$, this is known as the Maclaurin series.

3.1.2 Example

The function $f(x) = e^{5x}$ is expanded as below;

Therefore,

$$e^{5x} = 1 + 5 \cdot \frac{(x - 0)^1}{1!} + 5^2 \cdot \frac{(x - 0)^2}{2!} + 5^3 \cdot \frac{(x - 0)^3}{3!} + 5^4 \cdot \frac{(x - 0)^4}{4!} + \dots$$

Hence,

$$e^{5x} = \sum_{n=0}^{\infty} 5^n \cdot \frac{x^n}{n!}$$

this is the Maclaurin series expansion for the function $f(x) = e^{5x}$

Equation 2: The Maclaurin series expansion for $f(x) = e^{5x}$

3.1.3 Numerical Differentiation

This section looks at ways to numerically approximate the first and second derivative of a function.

3.1.3.1 Forward, Backward and Central Difference:

The Taylor series relates the value of a function at a point x_0 to the value of the function at a nearby point $(x_0 + h)$, that is

$$f(x_0 + h) = f(x_0) + h \cdot f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f^{(3)}(x_0) + \dots \quad (3)$$

Equation 3: Forward, Backward and Central Difference Example i

This is rearranged to get

$$\frac{f(x_0 + h) - f(x_0)}{h} = f'(x_0) + \frac{h}{2!} f''(x_0) + \frac{h^2}{3!} f^{(3)}(x_0) + \dots \quad (4)$$

Equation 4: Forward, Backward and Central Difference Example ii

and then used to make the forward difference approximation to $f'(x_0)$ thus

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h} \quad (5)$$

Equation 5: Forward Difference Approximation.

This is only an approximation, although it is one that is more accurate as h gets smaller (indeed in the limit as $h \rightarrow 0$ this is exact).

But, how and why precisely does the accuracy depend on h ?

If we let ϵ_f denote the error incurred using this forward difference approximation, then

$$\begin{aligned} \epsilon_f &\equiv f'(x_0) - \frac{f(x_0 + h) - f(x_0)}{h} \\ &= -\frac{h}{2!} f''(x_0) - \frac{h^2}{3!} f^{(3)}(x_0) + \dots \end{aligned} \quad (6)$$

Equation 6: Forward, Backward and Central Difference Example iii

Thus, this approximation is of the order $O(h)$. So, that as ' h ' gets smaller the error of the forward difference approximation is also reduced in a linear way. This can be improved by examining how the function varies with the decrease, *i.e.* create the backward differences approximation. Using the Taylor series, we can write

$$f(x_0 - h) = f(x_0) - h \cdot f'(x_0) + \frac{h^2}{2!} f''(x_0) - \frac{h^3}{3!} f^{(3)}(x_0) + \dots \quad (7)$$

Equation 7: Forward, Backward and Central Difference Example iv

This is then rearranged

$$\frac{f(x_0 - h) - f(x_0)}{h} = -f'(x_0) + \frac{h}{2!} f''(x_0) - \frac{h^2}{3!} f^{(3)}(x_0) + \dots \quad (8)$$

Equation 8: Forward, Backward and Central Difference Example v

From this the following **backward difference approximation**, can be found

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h} \quad (9)$$

Equation 9: Backward Difference Approximation.

Again, this approximation is more accurate as h gets smaller, but to quantify how the error depends on h requires inspection of the equations, which yields

$$\begin{aligned} \epsilon_b &\equiv f'(x_0) - \frac{f(x_0) - f(x_0 - h)}{h} \\ &= \frac{h}{2!} f''(x_0) - \frac{h^2}{3!} f^{(3)}(x_0) + \dots \end{aligned} \quad (10)$$

Equation 10: Forward, Backward and Central Difference Example vi

This approximation is also of the order $O(h)$. It has been shown that as ' h ' gets smaller the error is reduced in a linear way, therefore the forward difference approximation has not been improved. However, these two approximations can now be combined to get an improved version – by taking the later equation from the early equation it can be seen that

$$\begin{aligned}
f(x_0 + h) - f(x_0 - h) &= \left[f(x_0) + h.f'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f^{(3)}(x_0) + \dots \right] \\
&\quad - \left[f(x_0) - h.f'(x_0) + \frac{h^2}{2!}f''(x_0) - \frac{h^3}{3!}f^{(3)}(x_0) + \dots \right] \\
&= 2. \left[h.f'(x_0) + \frac{h^3}{3!}f^{(3)}(x_0) + \dots \right] \quad (11)
\end{aligned}$$

Equation 11: Forward, Backward and Central Difference Example viii

From this, follows

$$\frac{f(x_0 + h) - f(x_0 - h)}{2.h} = f'(x_0) + \frac{h^2}{3!}f^{(3)}(x_0) + \dots \quad (12)$$

Equation 12: Forward, Backward and Central Difference Example ix

And from this the **central difference approximation** can be constructed

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0 - h)}{2.h} \quad (13)$$

Equation 13: Central Difference Approximation

To find the error of this approximation the following is used

$$\begin{aligned}
\epsilon_c &\equiv f'(x_0) - \frac{f(x_0 + h) - f(x_0 - h)}{2.h} \\
&= -\frac{h^2}{3!}f^{(3)}(x_0) + \dots \quad (14)
\end{aligned}$$

Equation 14: Forward, Backward and Central Difference Example x

The central difference approximation is thus accurate to order $O(h^2)$ and so, for small values of h , will in general be better than both the forward and backward approximations.

3.1.3.2 Second Order Derivatives

A similar procedure can be followed to derive approximations to higher order derivatives. If we add equation (3) and equation (8) we get

$$f(x_0 + h) + f(x_0 - h) = \left[f(x_0) + h.f'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f^{(3)}(x_0) + \dots \right]$$

$$\begin{aligned}
& + \left[f(x_0) - h \cdot f'(x_0) + \frac{h^2}{2!} f''(x_0) - \frac{h^3}{3!} f^{(3)}(x_0) + \dots \right] \\
& = 2 \cdot \left[f(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^4}{4!} f^{(4)}(x_0) + \dots \right]
\end{aligned} \tag{15}$$

Equation 15: Second Order Derivative Example i

Therefore, from this

$$\frac{f(x_0 + h) - 2 \cdot f(x_0) + f(x_0 - h)}{h^2} = f''(x_0) + \frac{h^2}{12} f^{(4)}(x_0) + \dots \tag{16}$$

Equation 16: Second Order Derivative Example ii

From this we can construct the **central difference approximation** to the 2nd derivative

$$\boxed{f''(x_0) \approx \frac{f(x_0 + h) - 2 \cdot f(x_0) + f(x_0 - h)}{h^2}} \tag{17}$$

Equation 17: Central Difference Approximation

As in previous examples, the error is of order $O(h^2)$ and is given by

$$\begin{aligned}
\epsilon_c & \equiv f''(x_0) - \frac{f(x_0 + h) - 2 \cdot f(x_0) + f(x_0 - h)}{h^2} \\
& = -\frac{h^2}{12} f^{(4)}(x_0) + \dots
\end{aligned} \tag{18}$$

Equation 18: Second Order Derivative Example iii

3.1.4 Heat Transfer Calculations Using Finite Difference Equations

3.1.4.1 Time-Dependent Thermal Diffusion Model using the Finite Difference Method

Croft and Lilley (1977) provide the required information for this section. Conduction is the heat transfer which takes place in solids or fluids at rest (*i.e.* no convective motion resulting from the displacement of the main portion of the medium) due to an internal temperature gradient. Direct physical contact between interacting systems is required for conduction to happen. The transfer of thermal energy occurs at the molecular and atomic levels. In solids, conduction of heat proceeds by both lattice vibration and migration of free electrons.

Conduction in solids may be described by the following thermal diffusion equation

$$\nabla^2 T = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

(19)

Equation 19: Thermal Diffusion - Conduction in Solids.

where ∇^2 is the Laplacian defined by $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in three dimensions; and where t is time, T is the temperature, $\alpha = k/\rho \cdot c$ is the thermal diffusivity [$\text{m}^2 \cdot \text{s}^{-1}$], k is the thermal conductivity [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$], ρ the density [$\text{kg} \cdot \text{m}^{-3}$], and c the specific heat [$\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$].

If there is also an internal heat source Q to be considered, the thermal diffusion equation (19) becomes;

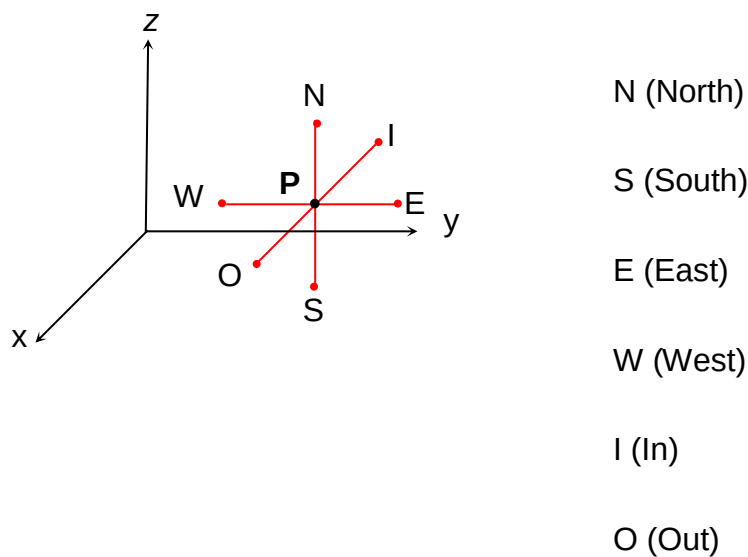
$$k \cdot \nabla^2 T + Q = \rho \cdot c \cdot \frac{\partial T}{\partial t}$$

(20)

Equation 20: Modified Thermal Diffusion Equation.

The region of interest is assumed to be covered by a uniform grid, as shown in the following figure, where the **grid spacing** is $h = \Delta x = \Delta y = \Delta z$.

Figure 8: Typical point P with its six neighbours



The idea behind the finite difference technique is the substitution of the derivatives in the differential equation by forward, backward and central differences in the Taylor series expansion. This transforms the differential equation into a set of algebraic equations solvable using the initial and boundary conditions. There are then several alternative methods depending on the stability, convergence, accuracy and speed required. A detailed analysis of the finite difference approximation is given by Croft and Lilley in the text book "*Heat Transfer Calculation Using Finite Difference Equations*" (Croft & Lilley, 1977).

3.1.4.2 Explicit Method

The equation (20) is shown in finite-difference form by using central differences to discretise $\nabla^2 T$ and forward differences to discretise $\partial T / \partial t$.

So: At the point P

$$\begin{cases} \frac{\partial^2 T}{\partial x^2} = \frac{T_O^n - 2.T_P^n + T_I^n}{h^2} \\ \frac{\partial^2 T}{\partial y^2} = \frac{T_E^n - 2.T_P^n + T_W^n}{h^2} \\ \frac{\partial^2 T}{\partial z^2} = \frac{T_N^n - 2.T_P^n + T_S^n}{h^2} \end{cases} \quad (21)$$

Equation 21: First Step for Finite Difference.

and

$$\frac{\partial T}{\partial t} = \frac{T_P^{n+1} - T_P^n}{\Delta t} \quad (22)$$

Equation 22: Second Step for Finite Difference

with the notation $T_P^n = T_P(t)$ temperature evaluated at time-level t and location P , and $T_P^{n+1} = T_P(t + \Delta t)$ temperature evaluated at time-level $t + \Delta t$ and location P .

Substituting equations (21) and (22) into the thermal diffusion equation (20) gives

$$\frac{1}{F_0} T_P^{n+1} = T_O^n + T_I^n + T_E^n + T_W^n + T_N^n + T_S^n - \left(6 - \frac{1}{F_0}\right) \cdot T_P^n + \frac{h^2 \cdot Q}{k} \quad (23)$$

Equation 23: Substituting (21) and (22) into Thermal Diffusion (20)

where F_0 is a dimensionless expression known as the **Fourier number** i.e.;

$$F_0 = \frac{\alpha \cdot \Delta t}{h^2} \quad (24)$$

Equation 24: Fourier Number for the Equation

Thus, the Finite Difference representation given by equation (23) is called the explicit form because the unknown temperature T_P' can be explicitly determined knowing the temperatures at the previous time step. The main disadvantage of this method is that once α and h are fixed, there is a maximum permissible time-step size Δt which should not exceed the value imposed by the following **stability criterion**

$$0 < F_0 \leq \frac{1}{6} \quad \text{in 3D} \quad (25)$$

Equation 25: Stability Criterion for the Time Step

So, for given values of α and h , if the time step Δt exceeds the limit imposed by the above criterion, the numerical calculations become unstable because of the amplification of errors.

The derivation can also be calculated for a 2D conducting region which yields the analogous expression;

$$\frac{1}{F_0} T_P^{n+1} = T_O^n + T_I^n + T_E^n + T_W^n - \left(4 - \frac{1}{F_0}\right) \cdot T_P^n + \frac{h^2 \cdot Q}{k} \quad (26)$$

Equation 26: Analogous Expression for a 2D Region.

with the following stability criterion

$$0 < F_0 \leq \frac{1}{4} \quad \text{in 2D} \quad (27)$$

Equation 27: Stability Criterion for 2D Calculation

3.1.4.3 Implicit Method

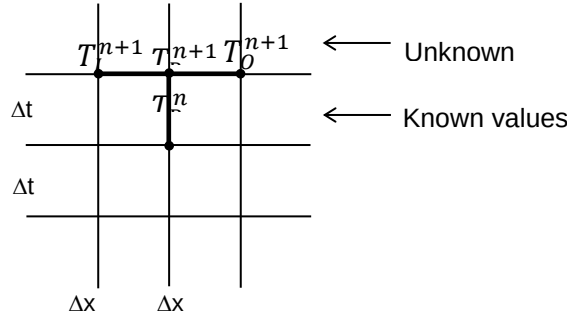
The computation of the explicit method, above, is relatively simple, but the stability criterion becomes an issue when calculations are performed over large time periods. In this case, as the restriction on F_0 tends to lead to small Δt , the number of calculations needed may become prohibitively large. The implicit method has been developed to overcome this limitation.

The Finite Difference approximation of equation (20) with the implicit scheme is given by;

$$-\frac{1}{F_0}T_P^n = T_O^{n+1} + T_I^{n+1} + T_E^{n+1} + T_W^{n+1} + T_N^{n+1} + T_S^{n+1} - \left(6 + \frac{1}{F_0}\right) \cdot T_P^{n+1} + \frac{h^2 \cdot Q'}{k} \quad (28)$$

Equation 28: Finite Difference Approximation of Thermal Diffusion Equation (20)

This is unconditionally stable. This is an implicit scheme because, at each time level, the nodal temperatures at the future time step are required for the neighbouring points around the point P, in contrast to the explicit method (23) where the neighbouring nodal temperatures at only the old time step are required. For the entire three-dimensional grid, a series of equations are to be solved simultaneously to determine the nodal temperatures at the next time level.

Figure 9: Simple implicit scheme implementation.

The Figure 9 shows the implementation of the simple implicit scheme in one dimension. The temperatures at the new time level $T_p^{n+1} = T_p(t + \Delta t)$ cannot be directly computed. If the problem involves N unknown node temperatures, a simultaneous solution of N equations is then required at each time step. Such a solution procedure is more involved computationally than the procedure of the explicit method, but benefits from the fact that no stability restrictions limit the size of the time step Δt .

3.1.4.4 Combined Method

A combination of the explicit method given by equation (23) and the implicit method given by equation (28) is written as

$$\begin{aligned} \frac{T_p^{n+1} - T_p^n}{\alpha \cdot \Delta t} = & \frac{\theta}{h^2} \cdot \left(T_O^{n+1} + T_I^{n+1} + T_E^{n+1} + T_W^{n+1} + T_N^{n+1} + T_S^{n+1} - 6 \cdot T_P^{n+1} + \frac{h^2 \cdot Q'}{k} \right) \\ & + \frac{(1 - \theta)}{h^2} \cdot \left(T_O^n + T_I^n + T_E^n + T_W^n + T_N^n + T_S^n - 6 \cdot T_P^n + \frac{h^2 \cdot Q}{k} \right) \end{aligned} \quad (29)$$

Equation 29: The Combined Equation, the Explicit (23) and the Implicit (24) incorporated

where the constant θ ($0 \leq \theta \leq 1$) is the weight factor which represents the degree of implicitness. Depending on the value chosen for θ , the simple explicit, the simple implicit, and the Crank-Nicolson methods are obtained as special cases:

$\theta = 0$: The explicit scheme. The stability constraint on the time step Δt is $0 < F_0 \leq \frac{1}{6}$.

$\theta = 1/2$: The Crank-Nicolson scheme. The finite difference equations are unconditionally stable.

$\theta = 1$: The implicit scheme. The finite difference equations are unconditionally stable.

For values of $0.5 \leq \theta \leq 1$, the scheme is unconditionally stable.

3.1.4.5 Alternating Direction Implicit (ADI) Method

Another possibility is the ADI method, which consists of splitting the time-level integration into equal time steps for X, Y and Z directions in three dimensions. The time-step restriction for stability is removed by the implicit algorithm, although this requires a matrix calculation at every time step. The technique subsequently requires one direction of the second-order space derivatives of equation (20) to be represented fully implicitly and the other two explicitly for each third-step level. This approach has the advantage of generating a set of equations expressible in the matrix form $A.X = Y$, with X and Y being vectors and A being a tridiagonal matrix. Therefore, an efficient inversion algorithm specific to the tridiagonal expression of A can be used to significantly reduce the computation time and solve for all the rows of vector X at each time level. Thus, when all the rows have been solved sequentially at time $(n+1/3)$ – supposing the computations are to be advanced from time level (n) to $(n+1)$ –, it is possible to advance to the next time level $(n+2/3)$ by changing the direction of explicit and implicit representation. Once vector X has been solved at that latter level, progression to time level $(n+1)$ is achieved by expressing the last space derivative explicitly and the remaining two implicitly and solving for X.

3.1.4.6 Boundary Conditions

Assume that the problem has a grid system covering the solution domain with boundaries coinciding with mesh lines. To be soluble, a Finite Difference Equation

(FDE) must be provided for each one of the entire set of node points, which in general consists of all internal points and all boundary points. The FDE for each internal point can be deduced from the material presented so far; the objective here is to supply an equation for each boundary point.

In heat transfer problems two cases often arise for each boundary point:

3.1.4.6.1 If Boundary Surface Temperature is Known

This is the Dirichlet known boundary condition, and so;

$$T_B = T_G \quad (30)$$

Equation 30: Dirichlet Known Boundary Equation

This applies where T_G is the known temperature given in the problem specification.

Thus, equation (30) provides a simple FDE for the point B ; without it, T_B would occur in the set of unknowns and the problem would not be solvable.

3.1.4.6.2 If Boundary Surface Temperature is Not Known

This is the Neumann derivative boundary condition, so the additional problem specification is in the form of knowing the outward normal gradient of temperature $\partial T / \partial n$. This may be:

$$\frac{\partial T}{\partial n} = K \quad (\text{given}) \quad (31)$$

Equation 31: Derivative Boundary Condition where K is Given

where K is simply a given value;

$$\frac{\partial T}{\partial n} = -\frac{h}{k}(T_B - T_A) \quad (\text{convective}) \quad (32)$$

Equation 32: Derivative Boundary Condition - Convective

where h is the surface heat transfer coefficient (which is usually assumed to be known), k is conductivity, and T_B and T_A are boundary and ambient temperatures, respectively. Equation (31) may also take the form;

$$\frac{\partial T}{\partial n} = -\frac{FE\sigma}{k}(T_B^4 - T_A^4) \quad (\text{radiative}) \quad (33)$$

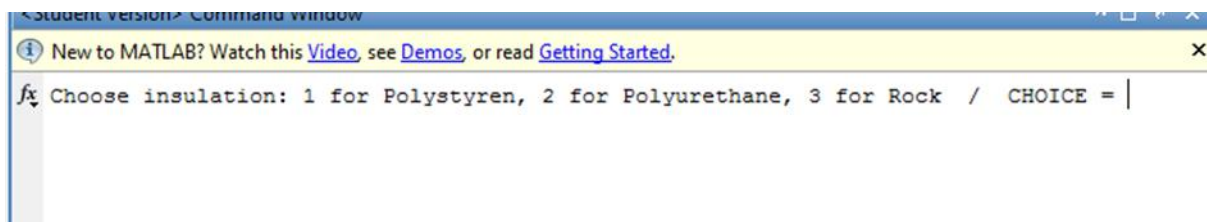
Equation 33: Derivative Boundary Condition - Radiative

where F is a geometric factor, E is emissivity and σ is the Stefan-Boltzmann constant.

3.2 Matlab® Simulation

The method of calculation described in the earlier part of this chapter is very complex and time-consuming if carried out manually, but in the Matlab® simulation, the data for the materials, roof, weather, direction the roof faces, and all the essential information is entered, and the actual calculations are carried out by the programme.

Figure 10: The Opening Screen for the Simulation.



The operator makes the selection as prompted in Figure 10, and the selects the East-West boundary conditions, and finally summer or winter. The Matlab® then uses the finite differences method to calculate how long it would take to stabilise the temperature, a measure of how efficient the insulation is.

3.3 Matlab® Assumptions

For the Matlab® simulation, the roof is a solid slab roof (Roof Type R1), and the programmed structure consists of the layers listed in Table 1;

Material Thickness (m)	
Tiles	0.020
Mortar	0.030
Insulation	0.045
Membrane	0.005
Foam Concrete	0.075
Concrete	0.200
Plaster	0.025

Table 1: The construction of the solid slab roof (R1) for the simulation.

The programme solves the heat diffusion equation (Equation 23), so that for each of the layers of the roof the thermal diffusivity (α) needs to be calculated

$$\alpha = \text{Thermal Conductivity} / (\text{density} \cdot \text{specific heat})$$

$$\alpha = k / (d \times c)$$

Equation 34: Thermal Diffusivity Calculation Used

The values k , d , and c for each material are programmed into the Matlab® simulation, and the Fourier number (Equation 24) is $(\alpha \cdot dt) / h^2$.

3.3.1 Governing Equation

The governing equation for the Matlab® code is Equation 23 derived from Equation 20, Equation 21, and Equation 22, which are repeated here for clarity. This uses the explicit finite difference for the numerical calculations;

$$k \cdot \nabla^2 T + Q = \rho \cdot c \cdot \frac{\partial T}{\partial t} \quad (20)$$

$$\begin{cases} \frac{\partial^2 T}{\partial x^2} = \frac{T_O^n - 2 \cdot T_P^n + T_I^n}{h^2} \\ \frac{\partial^2 T}{\partial y^2} = \frac{T_E^n - 2 \cdot T_P^n + T_W^n}{h^2} \\ \frac{\partial^2 T}{\partial z^2} = \frac{T_N^n - 2 \cdot T_P^n + T_S^n}{h^2} \end{cases} \quad (21)$$

$$\frac{\partial T}{\partial t} = \frac{T_P^{n+1} - T_P^n}{\Delta t} \quad (22)$$

$$\frac{1}{F_0} T_P^{n+1} = T_O^n + T_I^n + T_E^n + T_W^n + T_N^n + T_S^n - \left(6 - \frac{1}{F_0}\right) \cdot T_P^n + \frac{h^2 \cdot Q}{k} \quad (23)$$

This is the ‘thermal diffusion’ equation already discussed, but the ‘thermal diffusion’ itself is defined below.

3.3.2 Thermal Diffusion

When conducting a heat transfer analysis, thermal diffusion is defined as being the thermal conductivity divided by density multiplied by specific heat capacity at constant pressure. It measures the rate of transfer of heat of a material from the warm side to the cool side. It has the SI derived unit of m²/s. The simulation calculates the time take to reach thermal equilibrium – the point where internal and external temperatures are the same.

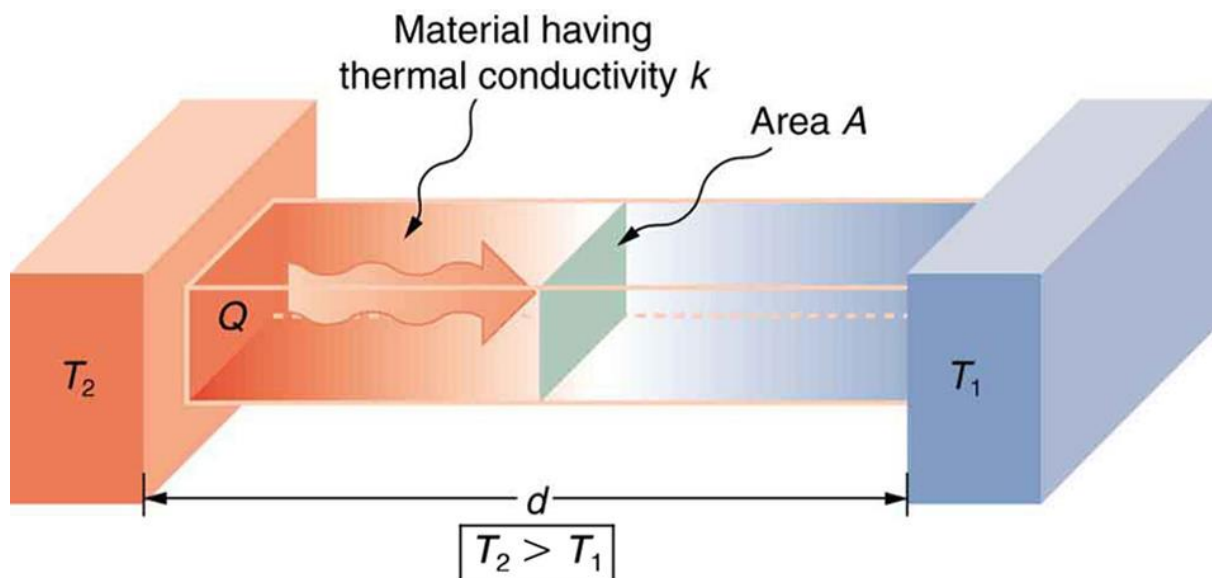


Figure 11: The Principal of Thermal Diffusion. Source: (Khan Academy, 2017)

3.3.3 Initial and Boundary Conditions for the Simulation

The user can input two alternative boundary conditions 'linear' and 'insulated', and the initial temperature differential is based on a standard internal temperature and a typical or average external temperature for the chosen season (Winter or Summer).

These assumptions and conditions represent the necessary information which Matlab® needs to make the complex calculations, and on which the results are therefore based – In the Tables in the following section, the Matlab® results are given. These tables allow direct comparison, but the figures are discussed in depth in the following Chapter 4 - Discussion.

3.4 Results

Time to Thermal Equilibrium	Polystyrene, Linear, Summer	Polystyrene, Linear, Winter	Polystyrene, Insulated, Summer	Polystyrene, Insulated, Winter
Seconds	15,201.40	10,793.20	14,803.80	8,509.40
Minutes	253.36	179.89	246.73	141.82
Hours	4.22	2.99	4.11	2.36

Table 2: Matlab® Results for Material 1 – Polystyrene.

Time to Thermal Equilibrium	Polyurethane, Linear, Summer	Polyurethane, Linear, Winter	Polyurethane, Insulated, Summer	Polyurethane, Insulated, Winter
Seconds	15,763.80	10,796.40	15,541.20	8,512.80
Minutes	262.73	179.94	259.02	141.88
Hours	4.38	3.00	4.32	2.36

Table 3: Matlab® Results for Material 2 – Polyurethane.

Time to Thermal Equilibrium	Rock Wool, Linear, Summer	Rock Wool, Linear, Winter	Rock Wool, Insulated, Summer	Rock Wool, Insulated, Winter
Seconds	15,911.00	10,797.2	15,690.80	8,513.40
Minutes	265.18	179.95	261.51	141.89
Hours	4.42	3.00	4.36	2.36

Table 4: Matlab® Results for Material 3 – Rock Wool

As noted, the figures are based on a roof of type R1, using the optimised grid spacing – it was important to judge this correctly, as if the grid spacing is very small, the time taken to completed the calculation will be very long. Although this is a more accurate

application the additional time makes it impractical. Conversely, if the grid is spaced too widely accuracy is lost. Since accuracy is important, the smallest practical grid spacing was used, with run-times of around 10-12 seconds.

In the simulation, from the initial conditions with a differential of temperature from outside to inside, there is a repetitive loop on a time step at the end of which the temperature differential is considered again. This is reiterated until the inside and outside temperatures have equalised – the condition referred to as thermal equilibrium. The tolerance values are, by necessity, quite coarse for the simulation – the figures for conductivity of the materials are based on statistical information, so are only approximations. However, the figures give a clear optimisation of the time taken for each material type and as such give guidance to builders and householders.

Chapter 4 Discussion

4.1 Introduction

The figures obtained from Matlab® used fixed values and predetermined thicknesses for each type of insulation. The value of R was not calculated, because the simulation was measuring the diffusion rate for a type R1 roof. Since R increases linearly with thickness, the simulation could be used to estimate the thickness required in a specific case.

4.2 Comparative Values

There is not a large difference in performance between the three materials; if the summer values for the linear loss boundary are considered, the least effective is polystyrene, where the temperature equalises after 4.22 hours. Next is polyurethane which equalises after 4.38 hours, and the best is rock wool, which equalises after 4.42 hours – an overall difference between the three of around 12 minutes. In the winter, the difference between the three is barely perceptible – 4 seconds separates the worst figure from the best figure, and the overall order is maintained.

When the results from the ‘insulated’ boundary conditions are examined, in ‘summer’ conditions, the three materials perform only slightly less well – polyurethane equalises more quickly by 6.6 minutes, and the others each by 3.2 minutes. In the ‘winter’ comparisons, however, the difference between materials virtually disappears, with a difference range of only 6 seconds. Although the ‘winter’ performance with the insulated boundary condition is far worse – all materials reach the equilibrium point 38.0 minutes sooner than the equivalent linear condition, with a range of 0.01 minutes (0.6 seconds), effectively all identical in comparison.

Finally, a comparison of the best of each category to the worst case for each gives the following results (Table 5) the range of ranges is $0.2h = 12$ minutes and the range of the mean values is $0.1h = 6$ minutes.

Material	Best case – Summer, Linear	Worst case – Winter, insulated	Range	Mean
Polystyrene	4.22h	2.36h	1.86h	3.29h
Polyurethane	4.38h	2.36h	2.02h	3.37h
Rock Wool	4.42h	2.36h	2.06h	3.39h

Table 5: Comparison of Best/Worst with Range and Mean

Chapter 5 Application

5.1 Design Concept

The original application of the project was to optimise or reduce the electricity usage for home AC plants in Saudi Arabia. The simulation discussed above is the first step in that process. The next step would be to create another Matlab® simulation in which the insulation thickness was a variable, and in which the cost of the materials was included in the overall comparison.

This would then be combined with a final simulation model which would estimate the hourly AC cooling/heating requirements with different thicknesses and materials – when the cost of adding more thickness is greater than the saving in energy, then the break-even or optimum thickness has been reached. This would then be the final design for implementation.

The design is intended to consider which of the materials used is most suitable for roofs in Saudi Arabia – although the difference between the materials is only slight, as the results show. Nevertheless, by manipulating the simulation slightly, it could be made to display the thickness required to take a long enough time to thermal equilibrium.

5.1.1 Design Constraints

- Low insulation thermal conductivity; the thermal conductivity of insulation should not exceed a value of about 0.05 kW/m.
- High roof R-value; roof R-value should not be lower than a value of about 1 m²/kW.
- Low cost of insulation material.
- Availability of insulation material in local market.

5.1.2 Design Criteria

- Cost – the additional cost of installation must be less than the energy saved to ensure that the project is viable.
- R-value – this is important, but only if the cost fits the cost criterion above.
- Durability – the insulation should last the length of the building, if possible.
- Good installation – good practice in any case but it must be considered as an essential part of the design.
- Moisture absorption – and vapour sealing are both important, but moisture absorption that prevents the householder from realising there is a leak would be very bad.
- Fireproof – most of the modern materials are fire retardant or fireproof, and in a concrete slab roof the actual structure itself is a fire barrier of a kind.

5.2 Product Design Specification

The logical thought progression from the Matlab® model is that the ‘ideally’ insulated roof would, in the worst case, exceed eight hours to reach thermal equilibrium, since this would mean that the load on the AC system was reduced at the times when it would previously peaked – in the hottest part of the day and the coldest part of the night. In real tests in remote desert locations (Zoubeidi, et al., 2012) it is clear that the peak load is less than 8hrs per day, which coincides well with the design proposal (see Figure 12). Although Zoubeidi, et al., (2012) were working in UAE, conditions there are similar to the Kingdom of Saudi Arabia, and their conclusion was that, because of this relatively short demand period, hybrid coolers (which have lower running costs) were suited to this kind of use – despite Salem’s (2014) Psychrometric Chart (Figure 13) suggesting that they are of limited use here.

Daily Profile

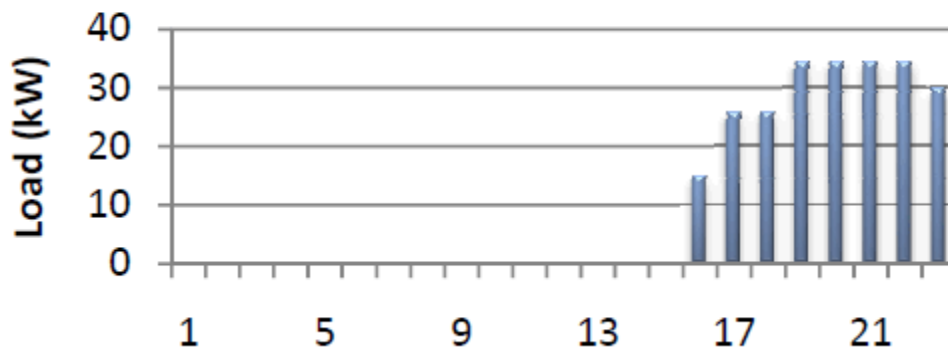


Figure 12: Daily Cooling in UAE Desert Location (Al-Ain) is c. 8 hrs per day. Source: (Zoubeidi, et al., 2012)

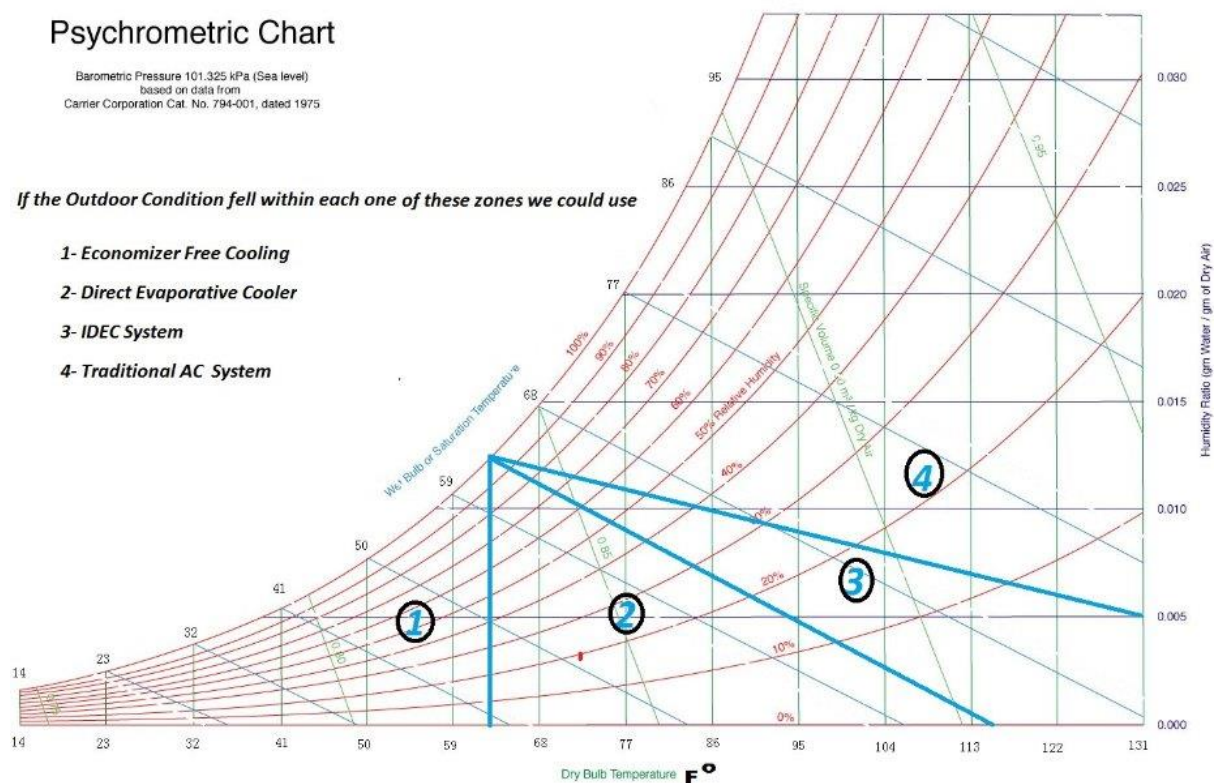


Figure 13: Decision Chart for Hybrid AC Units Indicates Need for Traditional Systems in Saudi Arabia. Source: (Salem, 2014)

5.2.1 General Overview

It appears probable that a further Matlab® simulation is required, but Zoubeidi, et al.'s, (2012) suggestion that Hybrid cooling may also save energy should be incorporated in the new code if possible – if the temperature normalisation period can be made to be the same length as the demand period, substantial saving should result.

5.3 Initial Design

The initial design is a Matlab® simulation to increase the effectiveness of insulation to cover the duration of peak energy demand, combined with an alternative or hybrid cooling system to substantially reduce the AC electricity usage in the KSA. The programme will be a simple development of the existing simulation, but with additional variables, material costs, and electricity usage throughout the twenty-four hours of one day.

5.4 Alternative Design Possibilities

It may be possible to design a variation on this design, but this would be forced to involve the hybrid designs discussed by Zoubeidi, et al., (2012) and by Salem (2014), as this seems to be an ideal complement to existing systems.

5.5 Conclusion

The study used a Matlab® simulation to compare three different roof insulation materials. The rate of heat transfer – or the time taken to reach thermal equilibrium, to be more precise – was then calculated from the known properties of the material in the specific configurations in the simulation. The initial comparison of the figures showed very little variation between the three, so that the final decision may come down to comparative cost of the three materials tested. In the 'worst case' scenario, the materials were, to all intents and purposes, identical – the variation in the time to

equilibrium having a range of less than six seconds. In the 'best case', RW had a clear advantage, but even in these circumstances the difference in time was only 12 minutes (0.2h) against MP and 2.4 minutes (0.04h) against PB.

When 'best' and 'worst' cases were averaged, the difference between the three materials was closer still (see Table 5), although overall, RW probably represents the best choice on these figures, because it is easier to apply correctly, and is less likely to be damaged by mishandling – a problem suffered by both the alternative materials.

By examining the theoretical diffusion rates for different materials making up a Type R1 roof, the most commonly found roof type in Saudi Arabia, the simulation has provided a realistic view of the effectiveness of the insulation in summer and winter conditions, with a mean value of around three hours.

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