

STUDENT FEEDBACK FORM-IAN BREWIS

This section details the *extent* to which the assessment criteria are demonstrated by you, which in turn determines your mark. The marks available for each category of skill are shown. Lecturers will use the space provided to comment on the achievement of the task(s), including those areas in which you have performed well and areas that would benefit from development/improvement.

Generic Assessment Criteria	Marks available	Marks awarded
1. Engagement with Literature Skills		
Your work has shown that you researched adequately into the topic and you demonstrated a good understanding of the subject. If you had used other sources such as books, papers, magazines etc, rather than simply relying on the internet would have gained the highest score. The quality of the references needs to be improved.	15	12
2. Knowledge and Understanding Skills		
The report shows good effort and good understanding of the topics covered. Good structure, detail and data presented in clear and appropriate format throughout.	20	20
3. Cognitive and Intellectual Skills		
Evidence of independent thought and ability to 'see beyond the question', suggesting a clear grasp of the broader field and wider concepts. Your work showed that you can summarise scientific facts into your own words without compromising academic style. There is a coherent flow in the writing and the reader can easily follow the experiment process and the analysis of the results. You have made it interesting to read and have also made attempts at drawing conclusions for the area you studied.	20	18
4. Practical Application Skills		
The report clearly demonstrates the student has a clear understanding of studies to date and also shows the student's ability to work as an individual. Good detail provided throughout the report with all aspects being suitably and sufficiently carried out. All tables and figures presented are thoroughly discussed in the assignment. All aspects have been covered in excellent detail.	20	18
5. Transferable Skills for Life and Professional Practice		
Mostly ordered presentation and structure in which relevant concepts are reasonably expressed. Demonstrates the basic skills for employment requiring the exercise of personal responsibility and decision-making, with some areas of minor weakness	25	13

Assessment Mark (Assessment marks are subject to ratification at the Exam Board. These comments and marks are to give feedback on module work and are for guidance only until they are confirmed.)

Late Submission Penalties (tick if appropriate)

Up to 1 week late (40% Max)

Over 1 week late (0%)

81%



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

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Name: Ian Richard Brewis

Student ID: BRE95040179

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Name of Lecturer: Dr Rabya Kadhim

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1. Introduction

This report will be looking at an overview of Fourier Analysis that is the study of how general functions can be decomposed to trigonometric and exponential functions with defined frequencies. The report will define the two types of Fourier expansions and explain the importance of Fourier Transforms in the field of engineering. The report will explain how a square wave is formed from a sum of odd harmonics. The report will show how a given function over a period can be graphically drawn and its Fourier trigonometric series representation written in terms of trigonometric or exponential functions for a certain number of harmonics. The function will be simulated with suitable software and graphical representation produced as well as obtaining values. The report will go on to critically analyse the simulated function against the calculated function.

2. Fourier Analysis has two extensions that are Fourier Series and Fourier Transform. Fourier Series is applied to periodic functions and Fourier Transforms apply to aperiodic/non-periodic functions.

2.1 Fourier series is a Mathematical way to represent a non-trigonometric periodic function as an infinite sum of trigonometric functions, by decomposing the function into a sum of Sines and Cosines that have different amplitudes and frequencies. (Weisstein, 2018)

A function $f(x)$ of period 2π can be represented by the Fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

2.2 Fourier Transforms is a generalisation of Fourier series that can be applied to aperiodic/nonperiodic functions by writing them as a continuous integral of trigonometric or exponential functions with continued possible frequencies. **Fourier Transform** method of transforming a function that expresses amplitude as a function of time into one that expresses amplitude as a function of frequency. Comprises contributions from all frequencies. (Weisstein, 2018)

$$f(x) = \int_{-\infty}^{\infty} C(k) e^{ikx} dk \quad \text{where} \quad C(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

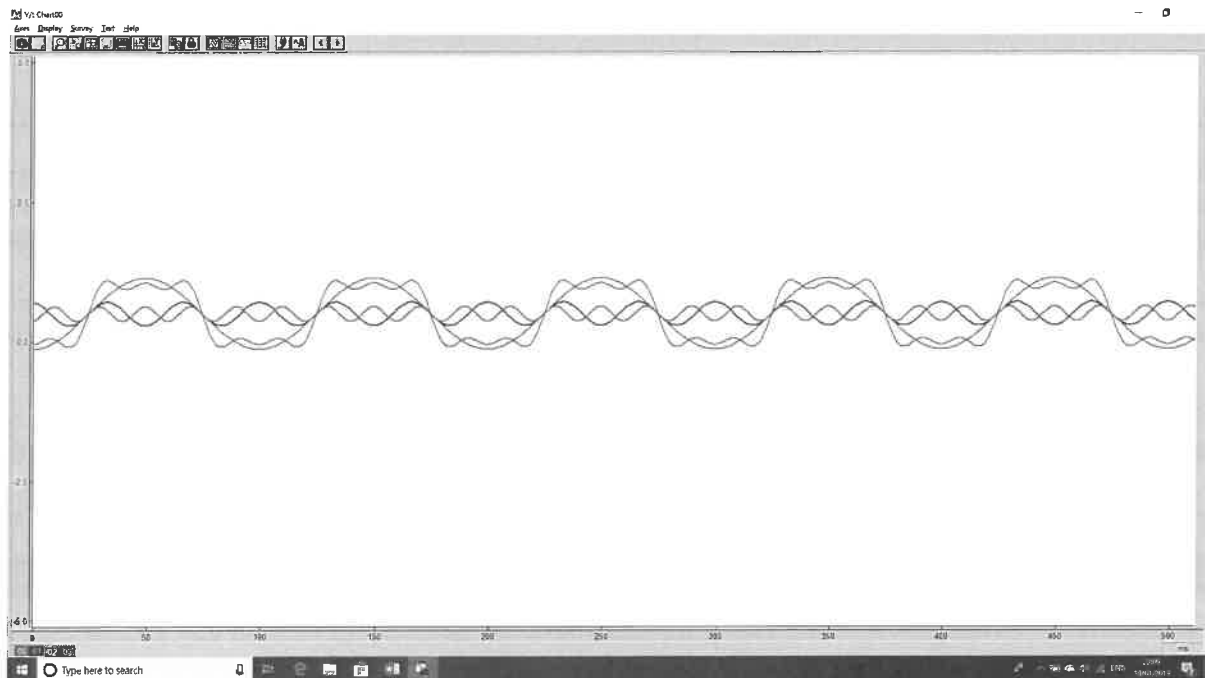
Where $C(k)$ is known as the Fourier transform of $f(x)$, and vice versa.

2.3 Fourier Transforms in field of engineering. In the real world most functions or signals that we come across are non-periodic. Fourier Series can only be useful for periodic functions or periodic signals. Therefore, Fourier Transform is extremely useful because it can handle non-periodic functions or signals. Fourier Transform is an extremely powerful mathematical tool that can be used to show different parts of a continuous signal and can convert a signal from time domain to frequency domain. By using Fourier Transform to analyse a signal it will show the frequencies that are present and their proportions. This is useful to see how much power is contained in a particular frequency. The Fourier Transforms has numerous applications in the field of engineering for analysis of vibration and acoustics, as well as electrically time varying signals.

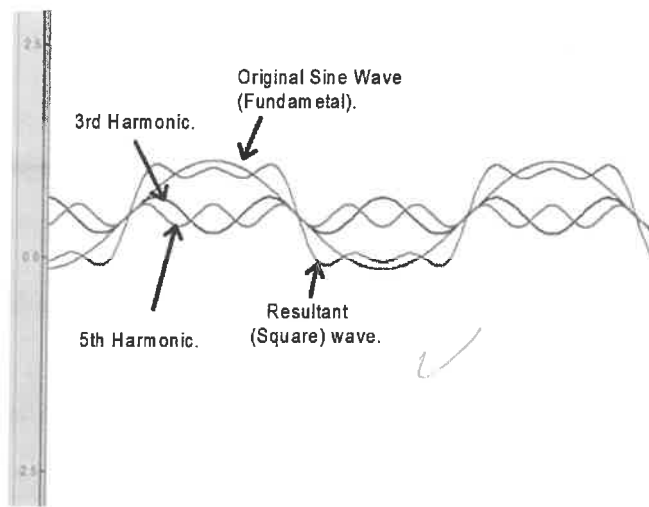
It is used extensively in Electrical Engineering in the analysis of circuits, control systems and signal processing. It is also used in mechanical engineering for vibration analysis, the characteristics of a vibration, such as phase amplitude to diagnose vibration problems. Civil Engineering uses Fourier Transforms to analyse attenuation and amplitude of vibration in structural dynamics.

3. How square wave is made up from summing odd harmonics.

A square wave is made up from summing odd harmonics of the square wave frequency. I have used computer software Daisy Lab (see Annex 1) to illustrate this. Below shows the fundamental wave in red and its resultant wave form in green after its 3rd in blue and 5th purple harmonics have all been summed together.



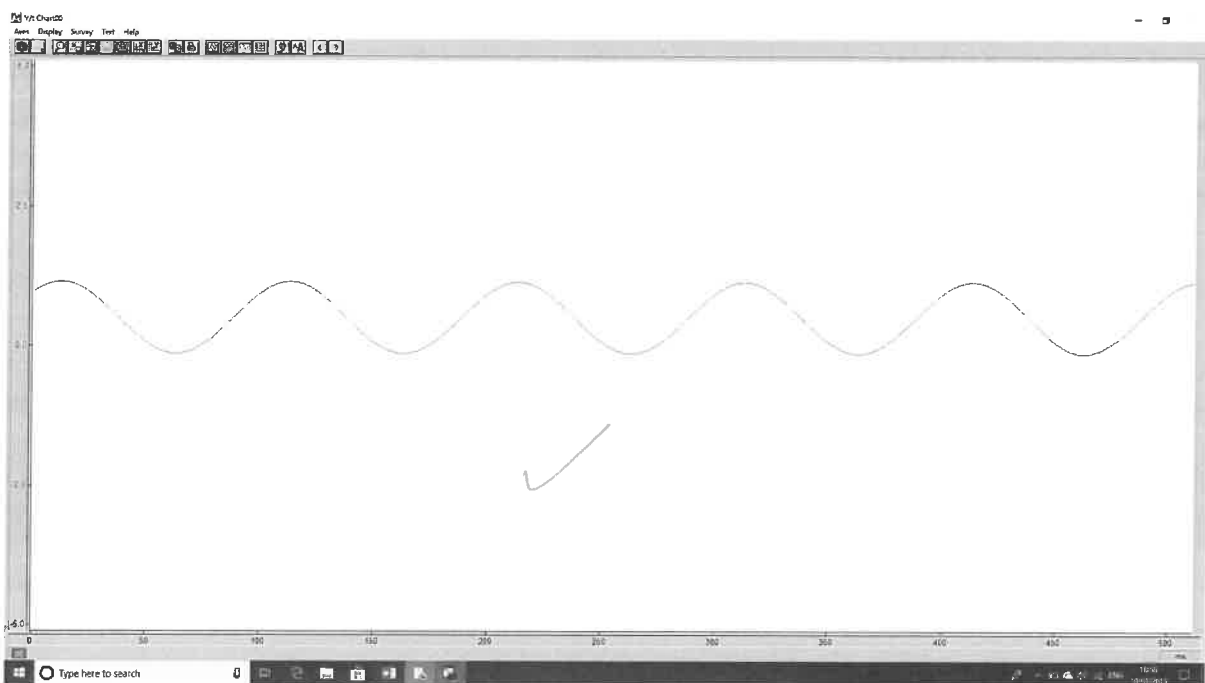
Zoomed in view taken from graph above.



Further continuation of this theory is demonstrated below again using the same computer software.

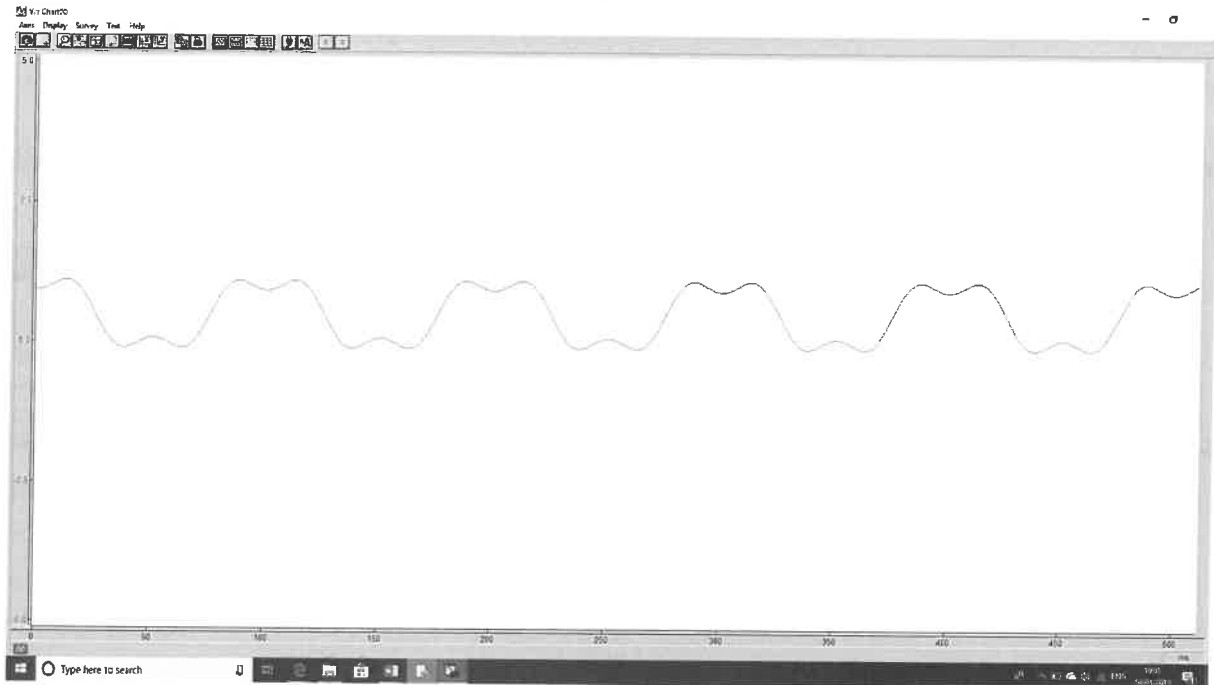
By starting with just a 10Hz sine wave, odd harmonics will be added to it until a square wave is produced.

This is a pure sine wave with a frequency of 10Hz. The Fundamental wave.

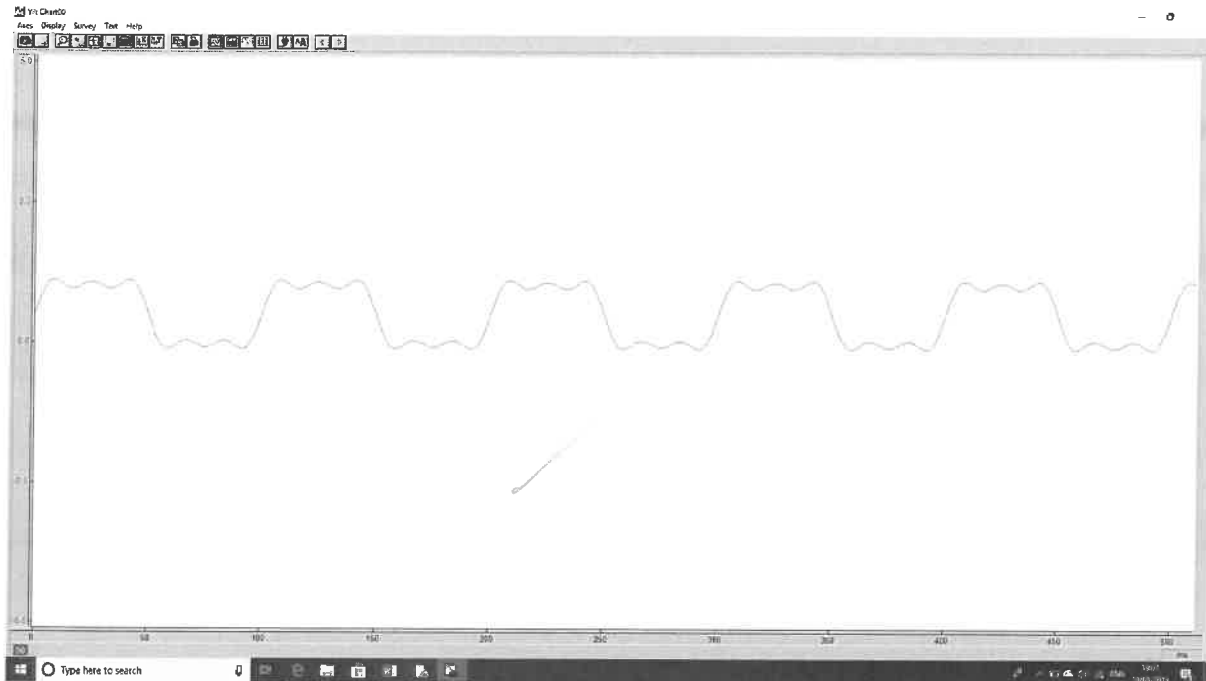


Now a first odd harmonic of 30Hz is added to the fundamental. This new sine wave will be 3 times the fundamental frequency.

As you can see in the graph below the wave has become the sum of the two and has dramatically changed shape.



Now the second odd harmonic of 50Hz is added to the fundamental wave, which will be 5 times the fundamental frequency.



Now the third 70Hz odd harmonic wave is added to the fundamental, this is 7 times the fundamental frequency.

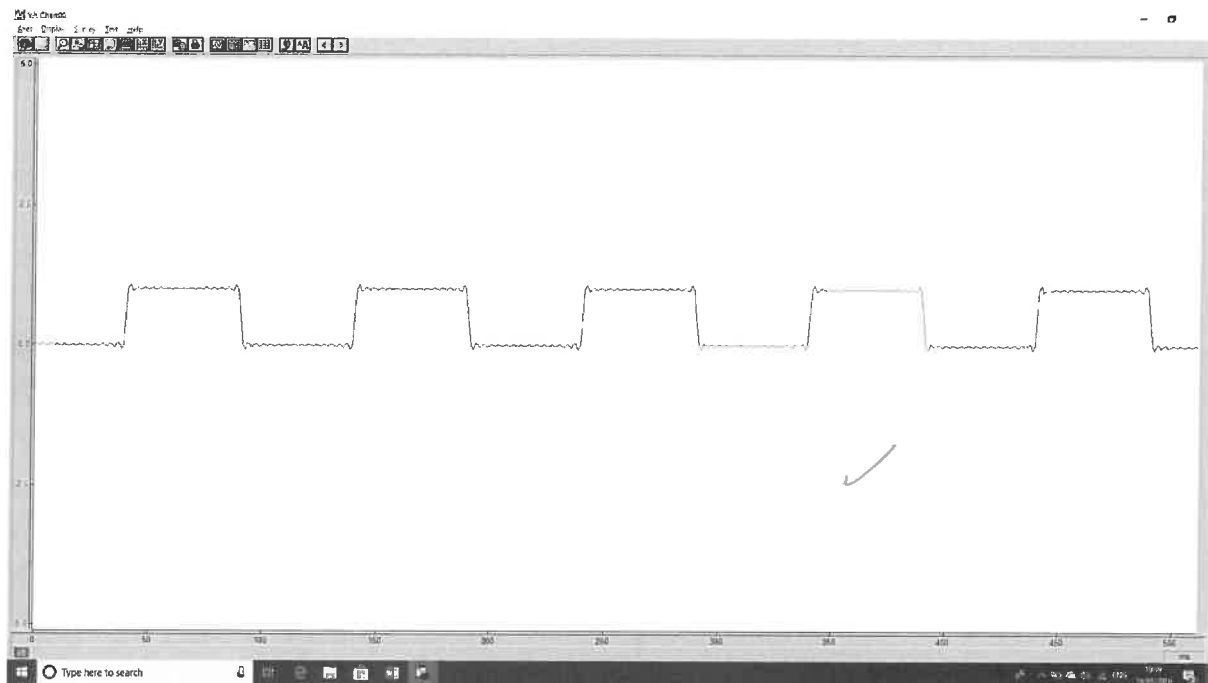
By now the wave form is looking more like a square wave than the original sine wave, its peaks and troughs are gone and its sides are getting steeper.



The wave will continue to change shape if more harmonics are added and become a more defined square wave.

Below is the wave after **15** odd harmonics (up to the 29th) have been added to the fundamental, this is the maximum that can be generated with the computer software.

It is now a non-sinusoidal but still a periodic waveform.



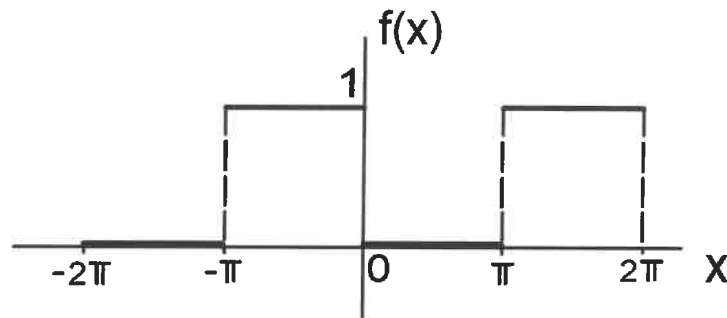
The wave form will never be perfectly square, a perfect square would have straight edges that would give instantaneous shift from its high and low values. It will only become a better defined square wave the more odd harmonics are added to it and would take an infinite amount of odd harmonics before it is perfectly square. This is unachievable.

4. Example Fourier trigonometric representation of a function.

Function:

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$$

Below the function is graphically drawn to show $f(x)$ over 2 periods (-2π to $+2\pi$).



Calculating the coefficients of the frequency domain expansion.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^0 f(x) dx + \frac{1}{\pi} \int_0^{\pi} f(x) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^0 1 dx + \frac{1}{\pi} \int_0^{\pi} 0 dx$$

$$= \frac{1}{\pi} [x]_{-\pi}^0$$

$$= \frac{1}{\pi} (\pi)$$

$$= 1$$

$$\therefore a_0 = 1$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 f(x) \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 1 \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} 0 \cos nx \, dx$$

$$= \frac{1}{\pi} \left(\left[\frac{\sin nx}{n} \right]_{-\pi}^0 \right)$$

$$= \frac{1}{n\pi} \left([\sin nx]_{-\pi}^0 \right)$$

$$= \frac{1}{n\pi} [\sin 0 - \sin(-n\pi)]$$

$$= \frac{1}{n\pi} [0 + \sin(n\pi)]$$

$$= \frac{1}{n\pi} [0 + 0]$$

$$a_n = 0 \quad \checkmark$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^0 f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 1 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 0 \sin nx \, dx$$

$$= \frac{1}{\pi} \left(\left[-\frac{\cos nx}{n} \right]_{-\pi}^0 \right)$$

$$= -\frac{1}{\pi} \left([\cos nx]_{-\pi}^0 \right)$$

$$= -\frac{1}{\pi} (1 - \cos(-n\pi))$$

$$b_n = \frac{-1}{n\pi} (1 - (-1)^n)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = 1$$

$$a_n = 0$$

$$b_n = \frac{1}{n\pi} (1 - (-1)^n)$$

$$\text{i.e. } b_n = \begin{cases} 0, n \text{ even} \\ -\frac{2}{n\pi}, n \text{ odd} \end{cases}, \text{ since } (-1)^n = \begin{cases} 1, n \text{ even} \\ -1, n \text{ odd} \end{cases}$$

The Fourier series representation of the first 7 harmonics:

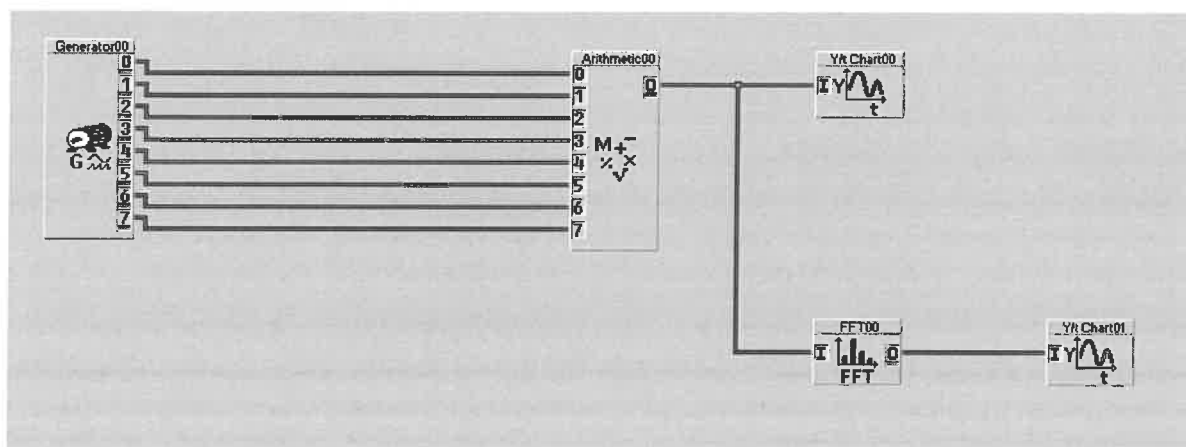
$$f(x) = \left(\frac{1}{2}\right) - \frac{2}{\pi} (\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x + \frac{1}{9} \sin 9x + \frac{1}{11} \sin 11x + \frac{1}{13} \sin 13x)$$

5. Validity of the transformation by reversing the process using waveform computer software (1111DaisyLab).

Wave No	0	1	2	3	4	5	6	7
Wave type	Constant	Sin	Sin	Sin	Sin	Sin	Sin	Sin
Harmonic No	-----	1 st	3 rd	5 th	7 th	9 th	11 th	13 th
Frequency (Hz)	-----	10	30	50	70	90	110	130
Calculated Amplitude	0.5= 0.5	$2/\pi =$ 0.6366	$2/3 \pi =$ 0.2122	$2/5 \pi =$ 0.1273	$2/7 \pi =$ 0.0909	$2/9 \pi =$ 0.0707	$2/11 \pi =$ 0.0578	$2/13 \pi =$ 0.0489

Note: based on a 10Hz square wave.

Circuit Diagram of 1111DaisyLab

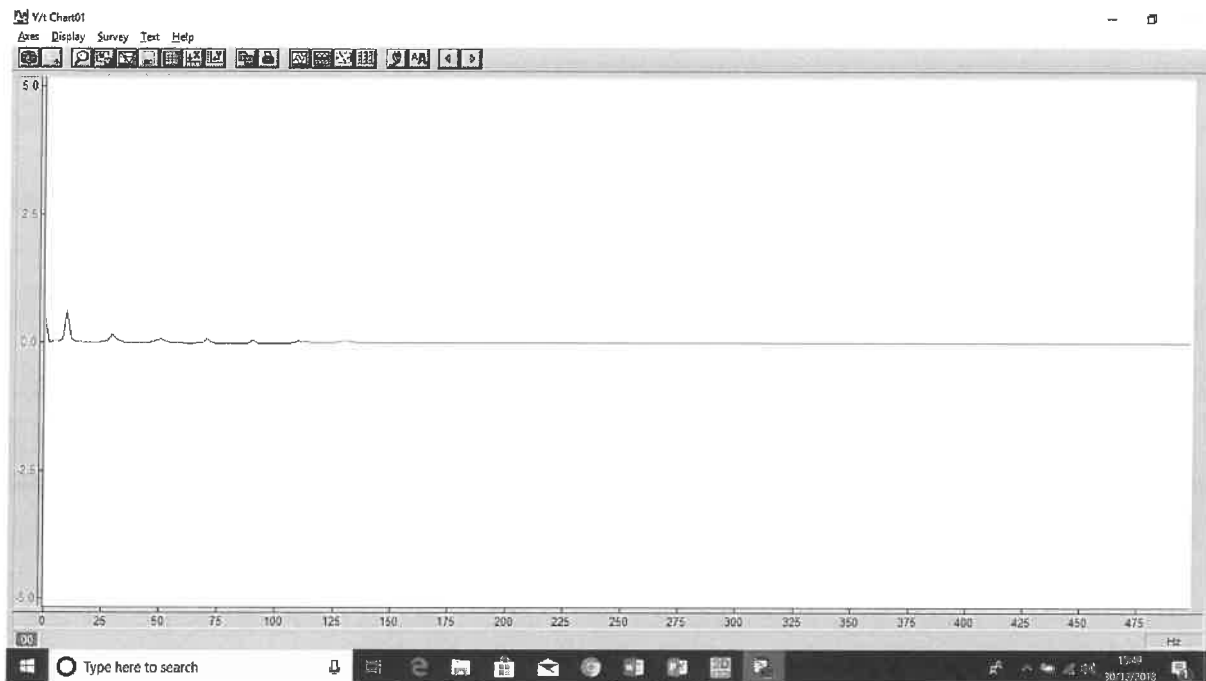


Graph to show sum of 7 harmonics in the time domain. Y/X Chart00



Note: the square wave amplitude is 1, as required.

Fast Fourier Transform graph showing what values are present at each frequency in the frequency domain. Y/X Chart01



Zoomed in graph showing what values are present at each frequency in the frequency domain. Taken from chart01

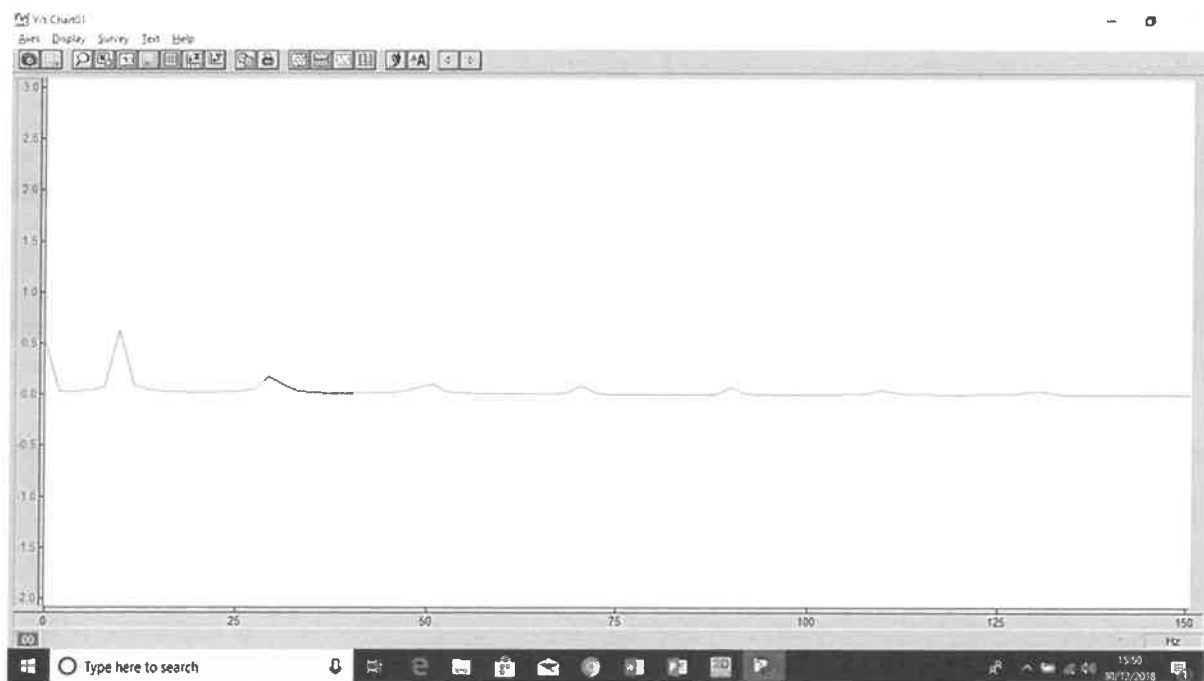


Table to show Comparison of harmonic amplitudes

Harmonic No	1	2	3	4	5	6	7
Frequencies	10	30	50	70	90	110	130
Simulation Frequency measured	9.77	29.30	19.53	70.31	89.84	109.38	130.86
Amplitude							
Calculated	0.6366	0.2122	0.1273	0.0909	0.0707	0.0578	0.0489
Simulation Measured	0.62	0.17	0.09	0.08	0.07	0.05	0.03

Note: All the measured values screen shots are in **Annex Two**.

6. Conclusion

Calculation

For a square wave of period $(-\pi, \pi)$, it can be seen that a_n is always 0 for all values of n and b_n is 0 for n even.

$$i.e. b_n = \begin{cases} 0, n \text{ even} \\ -\frac{2}{n\pi}, n \text{ odd} \end{cases}, \text{ since } (-1)^n = \begin{cases} 1, n \text{ even} \\ -1, n \text{ odd} \end{cases}$$

Therefore, the contributions for the Fourier series approximation for a square wave of this type are from the a_0 term and the summation of only the odd harmonic terms.

$$f(x) = \left(\frac{1}{2}\right) - \frac{2}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x + \frac{1}{9} \sin 9x + \frac{1}{11} \sin 11x + \frac{1}{13} \sin 13x \right)$$

Simulation

The Daisy Lab validation shows that the Fourier Series simulation is a good and controllable representation of the original. The square wave has an amplitude of 1 as required.

7. Recommendations

Sources of inaccuracy like the limitation to 7 harmonics and rounding errors are selectable, meaning that the degree of accuracy of the simulation is adjustable to the degree required by circumstance - rounding errors such as decimal places will account for accuracy. Also, the more harmonics summed together (**see Annex 3**) with a minimization of rounding errors will produce a better representation of the original function.

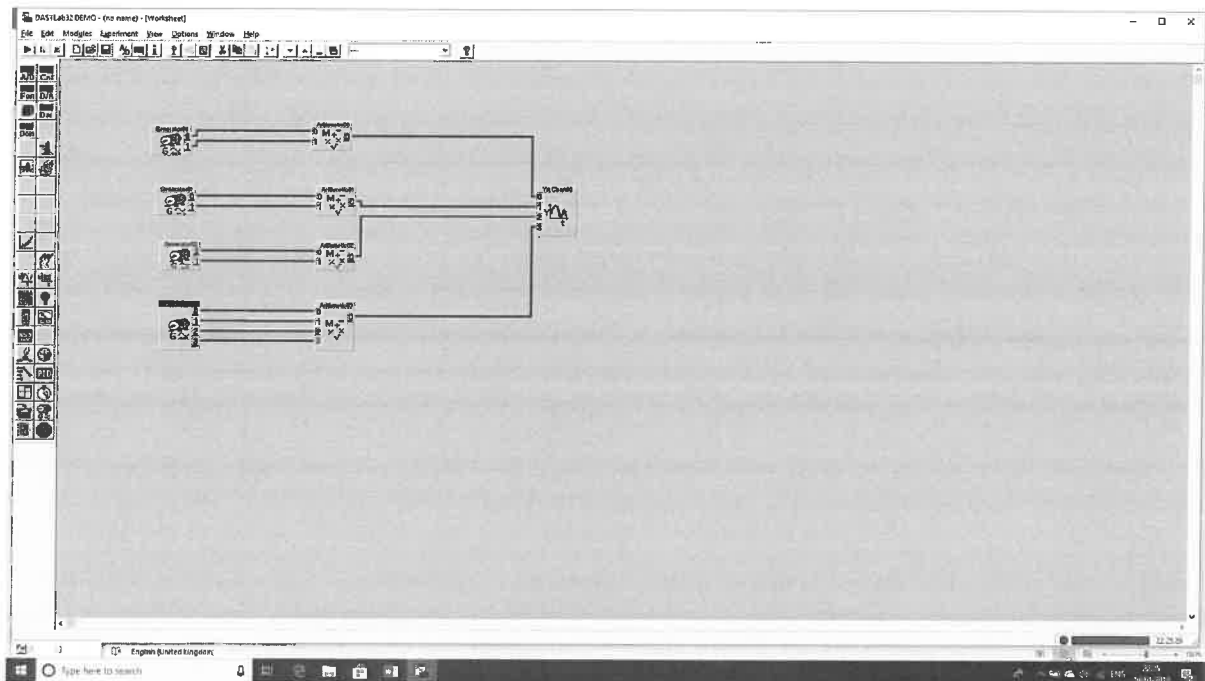
8. References

Weisstein, E. W., 2018. "Fourier Series.". [Online]
Available at: <http://mathworld.wolfram.com/FourierSeries.html>
[Accessed 14/12/2018 December 2018].

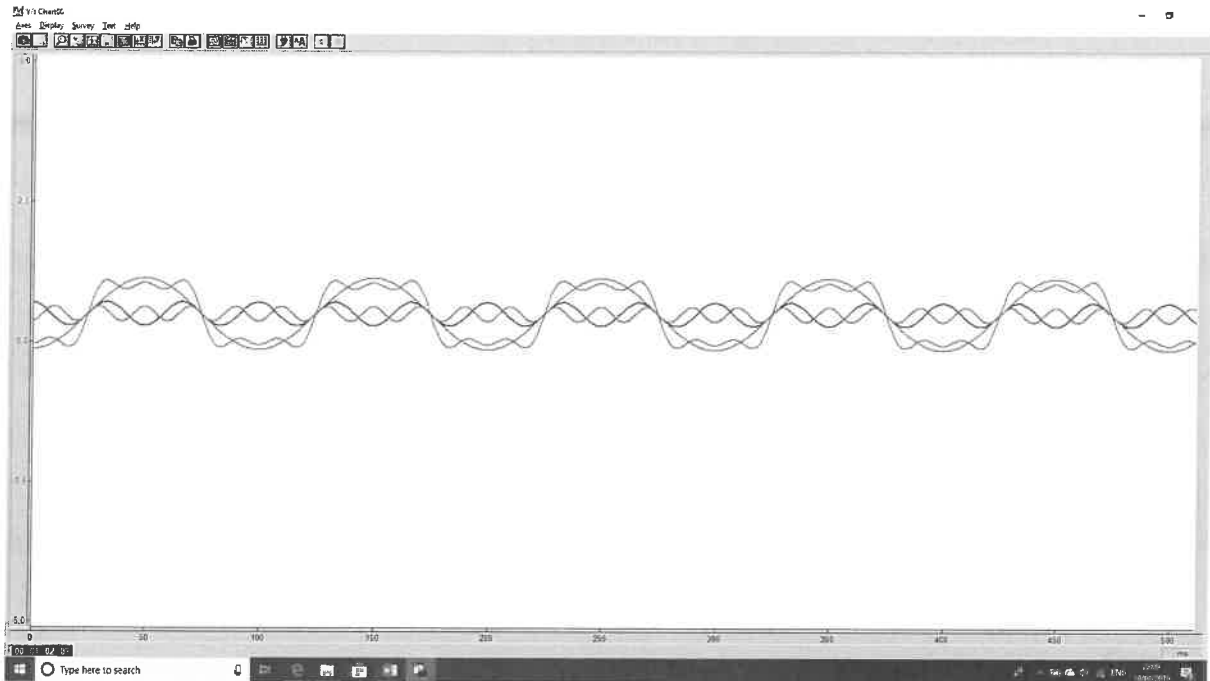
Weisstein, E. W., 2018. *Fourier Transform*. [Online]
Available at: <http://mathworld.wolfram.com/FourierTransform.html>
[Accessed 20th December 2018].

Annex One.

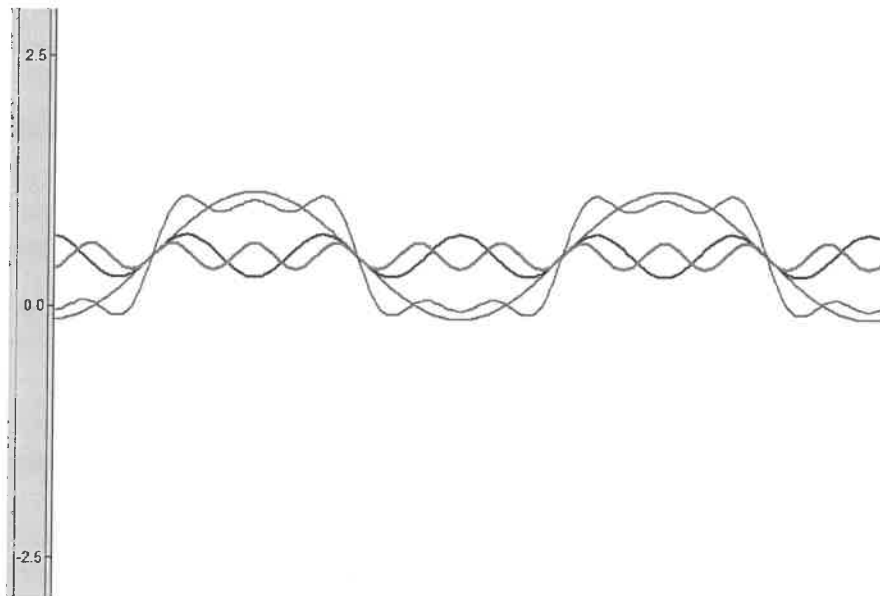
Daisy Lab software was used to produce 4 different waves to graphically illustrate how a square wave is produced.



Resultant of 4 generator channels going into a summer above then run/viewed on an X/Y graph shown below.



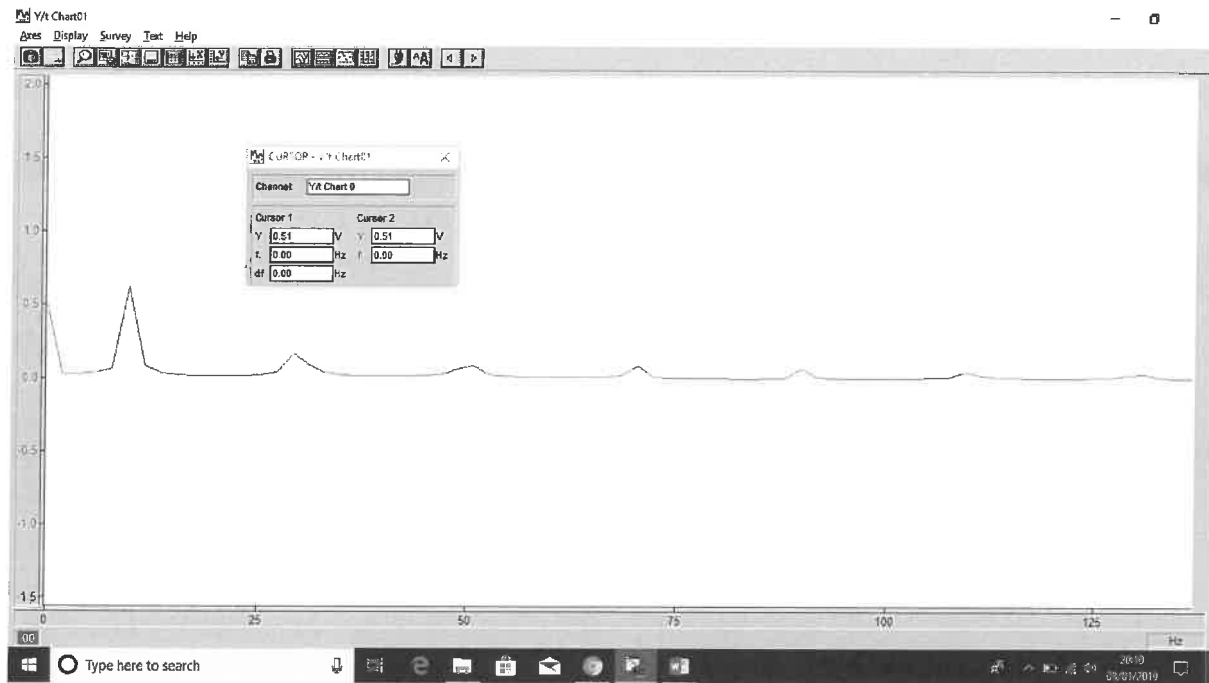
Zoomed in view taken from graph above.

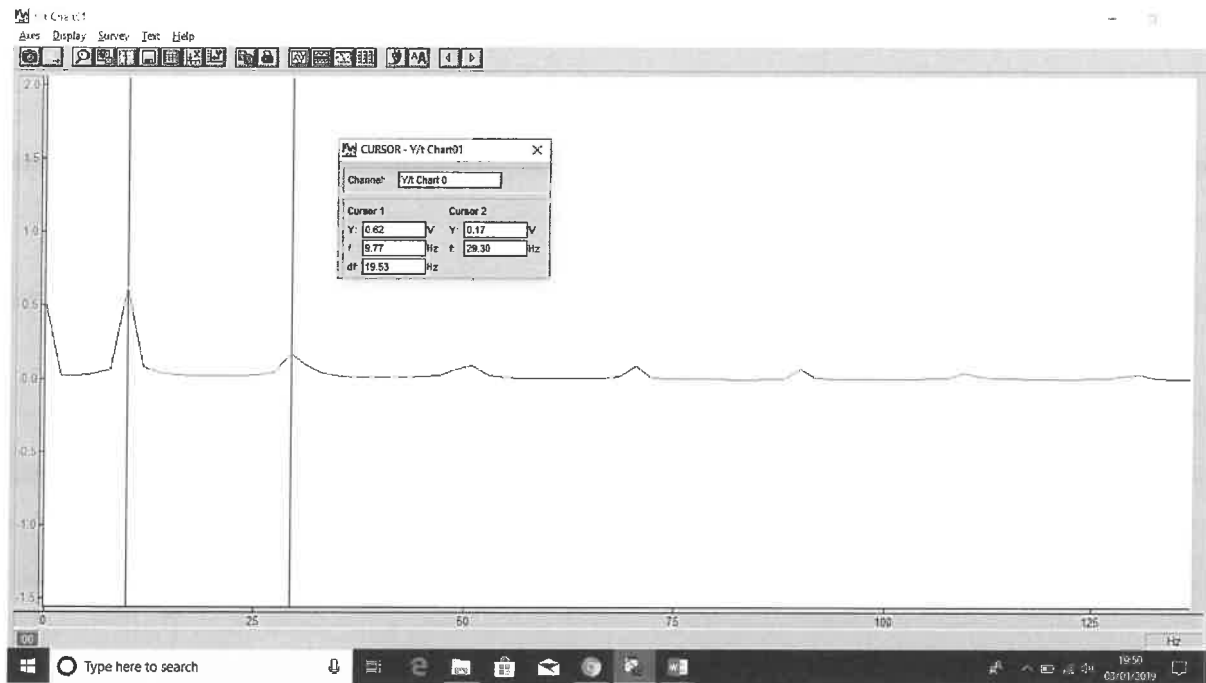
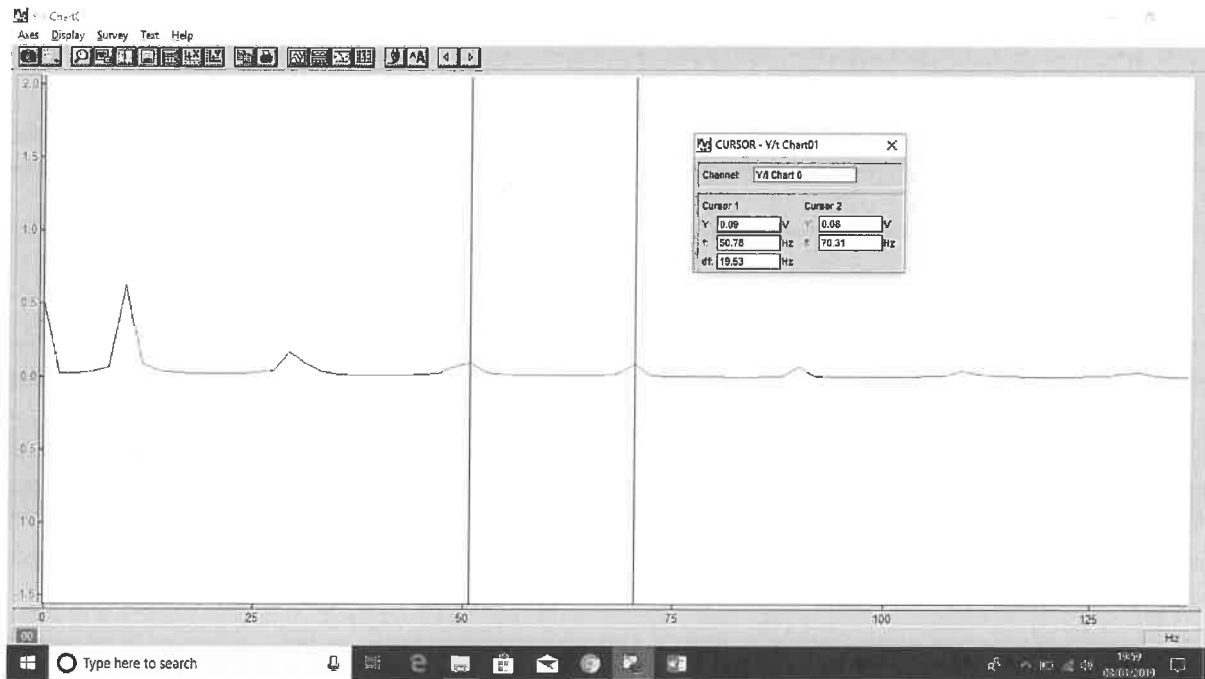


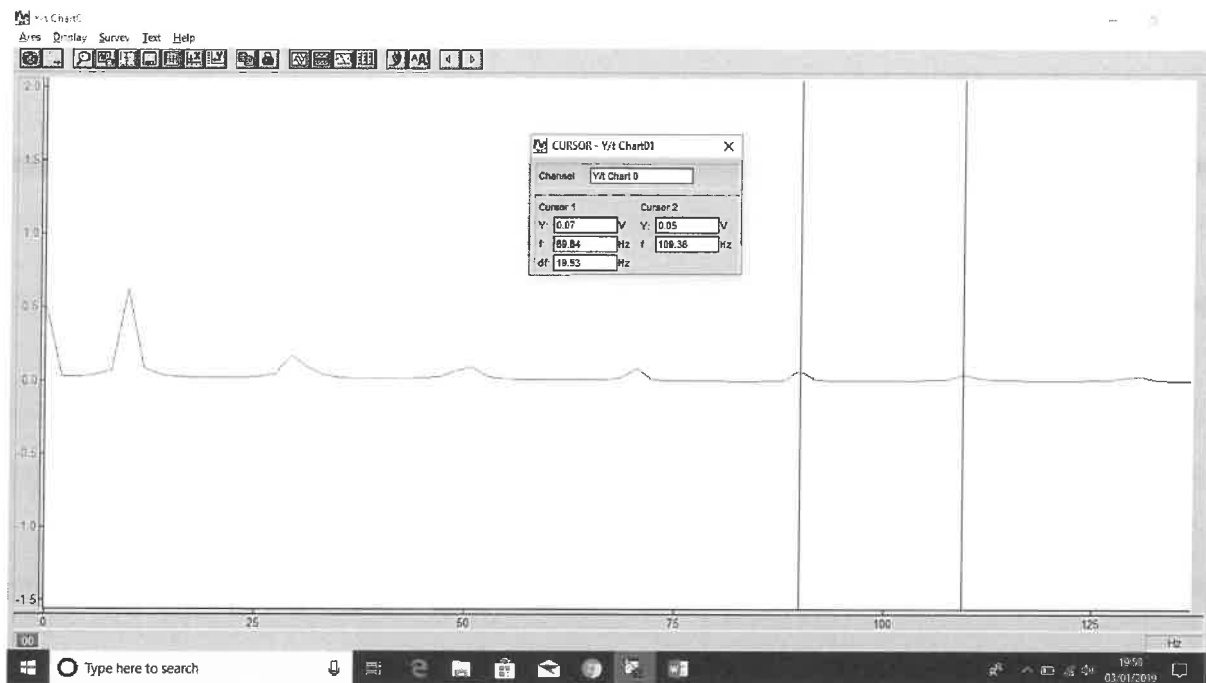
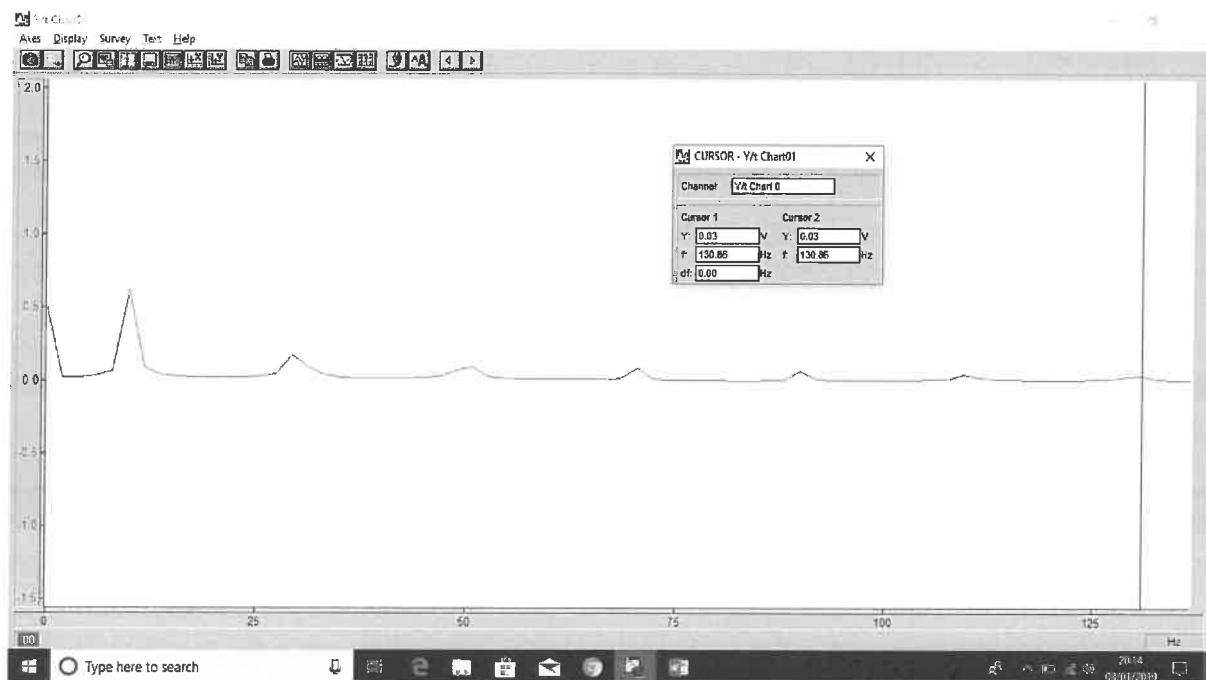
Annex Two.

Amplitude and frequency measurements taken from simulation.

Measuring the Constant value.

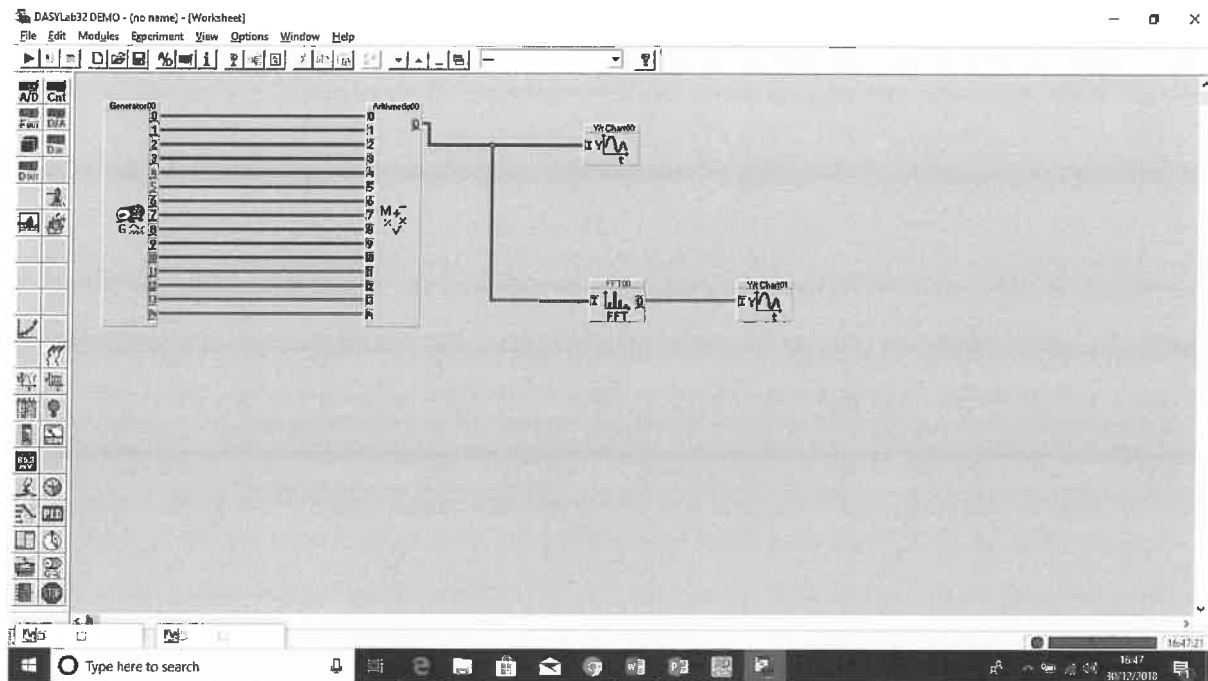


Measuring Harmonic 1 and 2.**Measuring Harmonic 3 and 4.**

Measuring Harmonic 5 and 6.**Measuring Harmonic 7.**

Annex Three.

Verification of function with 14 Harmonics.



Wave No	8	9	10	11	12	13	14
Wave type	Sin	Sin	Sin	Sin	Sin	Sin	Sin
Harmonic	15 th	17 th	19 th	21 st	23 rd	25 th	27 th
Frequency	150	170	190	210	230	250	270
Calculation	2/15Π	2/17Π	2/19Π	2/21Π	2/23Π	2/25Π	2/27Π
Amplitude	0.0424	0.0374	0.0335	0.0303	0.0276	0.0254	0.0235

Shown below are two wave forms, one with 7 harmonics and the other with 14.

Waveform with 7 Harmonics



Waveform with 14 Harmonics

