

Decision Matrix for root finding methods and convergence comparison

Method	Robustness	Convergence rate	Ease of implementation	Derivative required	Best use case
Bisection	Very High	Linear	Easy	No	Initial interval guaranteed to contain a root, slow but reliable.
False Position	High	Superlinear	Moderate	No	Similar to bisection but faster for well-behaved functions.
Newton-Raphson	Moderate	Quadratic	Moderate	Yes	Fast convergence when the derivative is easy to compute.
Secant	Moderate	Superlinear	Moderate	No	Faster than bisection; good when the derivative is difficult to compute.
Fixed-point iteration	Low	Linear	Moderate	No - requires $g(x)$	Simple forms of $x=g(x)$ where convergence is ensured.
Brent's method	Very High	Superlinear	Complex	No	Highly robust and efficient for most general-purpose root-finding tasks.
Ridder's Method	High	Superlinear	Moderate	No	Reliable and faster than bisection; good alternative to Brent's.
Polynomial methods	Variable	Varies	Complex	No	Finding roots of polynomials (e.g., Durand-Kerner for higher degrees).

Comparison of Convergence Rates:

Linear Convergence:

$$e_{n+1} = C \cdot e_n, \quad 0 < C < 1$$

Error reduces by a constant factor C in each step

Superlinear Convergence:

$$e_{n+1} = o(e_n), \quad \text{as } n \rightarrow \infty$$

Error reduces faster than linear convergence but not exponentially

Quadratic Convergence:

$$e_{n+1} = C \cdot e_n^2, \quad C > 0$$

Error decreases proportionally to the square of the previous error

An iterative method is said to converge super linearly if:

$$\lim_{n \rightarrow \infty} \frac{e_{n+1}}{e_n} = 0$$