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Analytical Methods: ACHA4000

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Question One

$$(i) \begin{matrix} & A & B \\ AB & \begin{bmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix} & \begin{bmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{bmatrix} \end{matrix}$$

$$\begin{bmatrix} (8 \times -2) + (-1 \times 2) + (7 \times 0) & (8 \times 8) + (-1 \times 5) + (7 \times -9) & (8 \times 0.5) + (-1 \times 2) + (7 \times 1) \\ (-5 \times -2) + (2 \times 2) + (5 \times 0) & (-5 \times 8) + (2 \times 5) + (5 \times -9) & (-5 \times 0.5) + (2 \times 2) + (5 \times 1) \\ (3 \times -2) + (0 \times 2) + (2 \times 0) & (3 \times 8) + (0 \times 5) + (2 \times -9) & (3 \times 0.5) + (0 \times 2) + (2 \times 1) \end{bmatrix}$$

$$AB = \begin{bmatrix} -18 & -4 & 9 \\ 14 & -75 & -0.5 \\ -6 & 6 & 3.5 \end{bmatrix}$$

$$(ii) \begin{matrix} & B & A \\ BA & \begin{bmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{bmatrix} & \begin{bmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix} \end{matrix}$$

$$\begin{bmatrix} (-2 \times 8) + (8 \times -5) + (0.5 \times 3) & (-2 \times -1) + (8 \times 2) + (0.5 \times 0) & (-2 \times 7) + (8 \times 5) + (0.5 \times 2) \\ (2 \times 8) + (5 \times 5) + (2 \times 3) & (2 \times -1) + (5 \times 2) + (2 \times 0) & (2 \times 7) + (5 \times 5) + (2 \times 2) \\ (0 \times 8) + (-9 \times 5) + (1 \times 3) & (0 \times -1) + (-9 \times 2) + (1 \times 0) & (0 \times 7) + (-9 \times 5) + (1 \times 2) \end{bmatrix}$$

$$BA = \begin{bmatrix} -54.5 & 18 & 27 \\ -3 & 8 & 43 \\ 48 & -18 & -43 \end{bmatrix}$$

(iii) AC

A	C
$\begin{bmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{bmatrix}$

$$\begin{bmatrix} (8 \times 2) + (-1 \times -6) + (7 \times 1) & (8 \times 5) + (-1 \times 0) + (7 \times 12) \\ (-5 \times 2) + (2 \times -6) + (5 \times 1) & (-5 \times 5) + (2 \times 0) + (5 \times 12) \\ (3 \times 2) + (0 \times -6) + (2 \times 1) & (3 \times 5) + (0 \times 0) + (2 \times 12) \end{bmatrix}$$

$$AC = \begin{bmatrix} 29 & 124 \\ -17 & 35 \\ 8 & 39 \end{bmatrix}$$

(iv) CA

C	A
$\begin{bmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{bmatrix}$	$\begin{bmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix}$

$$\begin{bmatrix} (2 \times 8) + (5 \times -5) & (2 \times -1) + (5 \times 2) \\ (-6 \times 8) + (0 \times -5) & (-6 \times -1) + (0 \times 2) \\ (1 \times 8) + (12 \times -5) & (1 \times -1) + (12 \times 2) \end{bmatrix}$$

$$CA = \begin{bmatrix} -9 & 8 \\ -48 & 0 \\ -52 & 23 \end{bmatrix}$$

(vii) $A+B$

$(8+2)$	$(-1+8)$	$(7+0.5)$
$(-5+2)$	$(2+5)$	$(5+2)$
$(3+0)$	$(0+(-9))$	$(2+1)$

$$A+B = \begin{bmatrix} 6 & 7 & 7.5 \\ -3 & 7 & 7 \\ 3 & -9 & 3 \end{bmatrix}$$

$$\begin{aligned}
 d) \quad & 2x + 3y + 2z = 14 \\
 & x + 2y + 5z = 4 \\
 & 3x + y + 2z = 19
 \end{aligned}$$

$$\begin{array}{c}
 + \quad - \quad + \\
 \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 5 \\ 3 & 1 & 2 \end{bmatrix}
 \end{array}$$

$$2 + 5 + 1 + 2 = 10$$

$$\Rightarrow D = 2 \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 2(10) - 3(11) + 2(7)$$

$$= 2(10) - 3(11) + 2(7) = 1$$

Question Two

a) (i) $Z_1 + Z_3 = 1 + 6j$ (ii) $Z_1 \cdot Z_3 =$ (iii) $\frac{3+7j}{2-10j} \times \frac{2+10j}{2+10j} = \frac{-64 + 44j}{104}$

$$\frac{3+7j}{2-10j} = \frac{3+7j}{2-10j} \times \frac{2+10j}{2+10j} = \frac{-64 + 44j}{104}$$

$$\frac{3+7j}{2-10j} = \frac{3+7j}{2-10j} \times \frac{2+10j}{2+10j} = \frac{-64 + 44j}{104}$$

$$\frac{6+44j+70j^2}{4-100j^2} = \frac{6+44j-70}{4+100}$$

Simplify = $\frac{-8 + 11j}{13 + 26}$

(iv) $Z_1 = 3+7j$
 $Z_2 = 2-10j$
 $Z_2 = 2+10j$

$$Z_1 - Z_2 = 3+7j - (2-10j) = 1+17j$$

(v) $Z_3 = -4j$ $Z_1 = 3+7j$ $Z_1 = 3-7j$

$$(Z_3)^3 = (-4j)(-4j)(-4j)$$

$$-16+4j+4j^2(-4-j)$$

$$-16+4j+4j^2(-4-j)$$

$$(-15+8j)(-4-j)$$

$$= (2j)^3 - Z_1 =$$

$$\frac{68-17j}{-3-7j}$$

$$= 65-10j$$

b) $8x^2 = -3x - 18$
 $8x^2 + 3x + 18 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(8)(18)}}{2(8)}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(8)(18)}}{2(8)}$$

$$x = \frac{-3 \pm 23.81j}{16}$$

$$x = \frac{-3 \pm \sqrt{-567}}{16}$$

$$x = \frac{-3}{16} + \frac{23.81j}{16}$$

$$x = \frac{-3}{16} - \frac{23.81j}{16}$$

$$x = -0.19 + 1.49j$$

$$x = -0.19 - 1.49j$$

c)

$$d) = j^8$$

$$e) (9+4j)(-7-5j)(2-5j)$$

$$(18-45j+7j-5j)(2-5j)$$

$$(16-38j)(2-5j)$$

$$32-26-38j+38j^2$$

Question Three

a)

$$(i) \int (x^5 - 2x)(5x^3 + 6x) dx$$

$$\int 5x^8 + 6x^6 - 10x^4 - 12x^2 dx$$

$$\int 5x^8 dx + \int 6x^6 dx - \int 10x^4 dx - \int 12x^2 dx$$

$$\frac{5x^9}{9} + \frac{6x^7}{7} - 2x^5 - 4x^3$$

$$\left\{ \frac{5x^9}{9} + \frac{6x^7}{7} - 2x^5 - 4x^3 + C \right.$$

$$(ii) (e^{5x} + 1)(e^{3x} + 2) dx$$

$$(dx e^{5x} + dx) \times (e^{3x} + 2)$$

$$dx e^{8x} + 2dx e^{5x} + dx e^{3x} + 2dx$$

$$dx e^{8x} + 2dx e^{5x} + dx e^{3x} + 2dx$$

$$(iii) \int_0^2 (3x^3 + 2x^3 + 4x + 3) dx$$

$$\int (3x^3 + 2x^3 + 4x + 3) dx$$

$$\int 7x^3 + 54x^2 + 36x^5 + 8x^3 + 4x + 3 dx$$

$$\left(\frac{7x^{10}}{10} + \frac{27x^8}{4} + 6x^6 + 2x^4 + 2x^2 + 3x \right) \Big|_0^2$$

$$\frac{7 \times 2^{10}}{10} + \frac{27 \times 2^8}{4} + 6 \times 2^6 + 2 \times 2^4 + 2 \times 2^2 + 3 \times 2 - \left(\frac{7 \times 0^{10}}{10} + \frac{27 \times 0^8}{4} + 6 \times 0^6 + 2 \times 0^4 + 2 \times 0^2 + 3 \times 0 \right)$$

$$= \frac{24614}{5}$$

$$(iv) \int_2^4 (2x+3)(x+5) dx$$

$$\int (2x+3) \times (x+5) dx$$

$$\int 2x^2 + 13x + 15 dx$$

$$\int 2x^2 dx + \int 13x dx + \int 15 dx$$

$$\frac{2x^3}{3} + \frac{13x^2}{2} + 15x$$

$$\left(\frac{2x^3}{3} + \frac{13x^2}{2} + 15x \right) \Big|_2^4$$

$$\frac{2 \times 4^3}{3} + \frac{13 \times 4^2}{2} + 15 \times 4 - \left(\frac{2 \times 2^3}{3} + \frac{13 \times 2^2}{2} + 15 \times 2 \right)$$

$$= \frac{436}{3}$$

$$b) \int 4x \cos 3x dx$$

$$4 \times \int x \times \cos(3x) dx$$

$$4 \left(\frac{x \times \sin(3x)}{3} - \frac{1}{3} \times \int \sin(3x) dx \right)$$

$$4 \left(\frac{x \times \sin(3x)}{3} - \frac{1}{3} \times \int \frac{\sin(t)}{3} dt \right)$$

$$4 \left(\frac{x \times \sin(3x)}{3} - \frac{1}{9} \times (-\cos(t)) \right) \quad \left(\frac{4x \times \sin(3x)}{3} - \frac{1}{9} \times (-\cos(3x)) \right)$$

$$\frac{4x \times \sin(3x)}{3} + \frac{4 \cos(3x)}{9} + C$$

$$(i) \int_1^2 3x e^{2x} dx$$

$$\int 3x e^{2x} dx \quad 3 \times \int x e^{2x} dx$$

$$3 \left(x \times \frac{e^{2x}}{2} - \frac{1}{2} \times \int e^{2x} dx \right)$$

$$3 \left(x \times \frac{e^{2x}}{2} - \frac{1}{2} \times \frac{1}{2} e^{2x} \right)$$

$$\frac{3x e^{2x}}{2} - \frac{3e^{2x}}{4}$$

$$\left(\frac{3x e^{2x}}{2} - \frac{3e^{2x}}{4} \right) \Big|_1^2$$

$$\frac{3 \times 2 e^{2 \times 2}}{2} - \frac{3e^{2 \times 2}}{4} - \left(\frac{3 \times 1 e^{2 \times 1}}{2} - \frac{3e^{2 \times 1}}{4} \right)$$

$$= \frac{9e^4 - 3e^2}{4}$$

$$c(i) \int \frac{2x^2 + 10x - 3}{7x^3 + 5x^2 - 3x - 1} dx \quad \int \frac{1}{t} dt$$

$$t = 7x^3 + 5x^2 - 3x - 1 \quad \int \frac{1}{t} dx = \ln(f(x))$$

$$= \ln(7x^3 + 5x^2 - 3x - 1) + C$$

$$(ii) \int \frac{8}{3x+10} dx$$

$$\int \frac{8}{3t} dt$$

$$\frac{8}{3} \times \int \frac{1}{t} dt$$

$$\frac{8}{3} \times \ln(3x+10)$$

$$= \frac{8}{3} \times \ln(3x+10) + C$$

$$(iii) \int \frac{1}{(3 \tan u)^2 + 9} \times 3 \sec^2 u \, du$$

$$\int \frac{1}{3} du \quad \frac{u}{3} \quad u = \tan^{-1}\left(\frac{1}{3}x\right) \quad \frac{\tan^{-1}\left(\frac{x}{3}\right) + C}{3}$$

$$= \frac{\tan^{-1}\left(\frac{x}{3}\right) + C}{3}$$

d)

e)

(vii) Not mathematically possible

~~16)~~