

4. A 5-metre iron bar is to be produced. The bar may not deviate by more than 1 mm from its stated length. Write a specification for the bar's length x in metres: (a) by using a double inequality; (b) with the aid of an absolute-value sign.

REVIEW PROBLEMS FOR CHAPTER 1

1. (a) What is three times the difference between 50 and x ?
 (b) What is the quotient between x and the sum of y and 100?
 (c) If the price of an item is a including 20% VAT (value added tax), what is the price before VAT?
 (d) A person buys x_1 , x_2 , and x_3 units of three goods whose prices per unit are respectively p_1 , p_2 , and p_3 . What is the total expenditure?
 (e) A rental car costs F dollars per day in fixed charges and b dollars per kilometre. How much must a customer pay to drive x kilometres in 1 day?
 (f) A company has fixed costs of F dollars per year and variable costs of c dollars per unit produced. Find an expression for the total cost per unit (total average cost) incurred by the company if it produces x units in one year.
 (g) A person has an annual salary of $\$L$ and then receives a raise of $p\%$ followed by a further increase of $q\%$. What is the person's new yearly salary?

2. Express as single real numbers in decimal notation:

(a) 5^3	(b) 10^{-3}	(c) $\frac{1}{3^{-3}}$	(d) $\frac{-1}{10^{-3}}$
(e) $3^{-2}3^3$	(f) $(3^{-2})^{-3}$	(g) $-\left(\frac{5}{3}\right)^0$	(h) $\left(-\frac{1}{2}\right)^{-3}$

3. Which of the following expressions are defined, and what are their values?

(a) $(0 + 2)^0$	(b) 0^{-2}	(c) $\frac{(10)^0}{(0 + 1)^0}$	(d) $\frac{(0 + 1)^0}{(0 + 2)^0}$
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4. Simplify:

(a) $(2^3 2^{-5})^3$	(b) $\left(\frac{2}{3}\right)^{-1} - \left(\frac{4}{3}\right)^{-1}$	(c) $(3^{-2} - 5^{-1})^{-1}$	(d) $(1.12)^{-3}(1.12)^3$
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-  5. Simplify:

(a) $(2x)^4$	(b) $(2^{-1} - 4^{-1})^{-1}$	(c) $\frac{24x^3y^2z^3}{4x^2yz^2}$
(d) $[-(-ab^3)^{-3}(a^6b^6)^2]^3$	(e) $\frac{a^5 \cdot a^3 \cdot a^{-2}}{a^{-3} \cdot a^6}$	(f) $\left[\left(\frac{x}{2}\right)^3 \cdot \frac{8}{x^{-2}}\right]^{-3}$

6. Compute: (a) 12% of 300 (b) 5% of 2000 (c) 6.5% of 1500

7. Give economic interpretations to each of the following expressions and then use a calculator to find the approximate values:

(a) $100 \cdot (1.01)^8$ (b) $50\,000 \cdot (1.15)^{10}$ (c) $6000 \cdot (1.03)^{-8}$

8. (a) \$100 000 is deposited into an account earning 8% interest per year. What is the amount after 10 years?

- (b) If the interest rate is 8% each year, how much money should you have deposited in a bank 6 years ago to have \$25 000 today?

- SM** 9. Expand and simplify:

(a) $a(a-1)$ (b) $(x-3)(x+7)$ (c) $-\sqrt{3}(\sqrt{3}-\sqrt{6})$ (d) $(1-\sqrt{2})^2$
 (e) $(x-1)^3$ (f) $(1-b^2)(1+b^2)$ (g) $(1+x+x^2+x^3)(1-x)$ (h) $(1+x)^4$

10. Complete the following:

(a) $x^{-1}y^{-1} = 3$ implies $x^3y^3 = \dots$ (b) $x^7 = 2$ implies $(x^{-3})^6(x^2)^2 = \dots$
 (c) $\left(\frac{xy}{z}\right)^{-2} = 3$ implies $\left(\frac{z}{xy}\right)^6 = \dots$ (d) $a^{-1}b^{-1}c^{-1} = 1/4$ implies $(abc)^4 = \dots$

11. Factor the expressions

(a) $25x - 5$ (b) $3x^2 - x^3y$ (c) $50 - x^2$ (d) $a^3 - 4a^2b + 4ab^2$

- SM** 12. Factor the expressions

(a) $5(x+2y) + a(x+2y)$ (b) $(a+b)c - d(a+b)$ (c) $ax + ay + 2x + 2y$
 (d) $2x^2 - 5yz + 10xz - xy$ (e) $p^2 - q^2 + p - q$ (f) $u^3 + v^3 - u^2v - v^2u$

13. Compute the following without using a calculator:

(a) $16^{1/4}$ (b) $243^{-1/5}$ (c) $5^{1/7} \cdot 5^{6/7}$ (d) $(4^8)^{-3/16}$
 (e) $64^{1/3} + \sqrt[3]{125}$ (f) $(-8/27)^{2/3}$ (g) $(-1/8)^{-2/3} + (1/27)^{-2/3}$ (h) $\frac{1000^{-2/3}}{\sqrt[3]{5^{-3}}}$

14. Solve the following equations for x :

(a) $2^{2x} = 8$ (b) $3^{3x+1} = 1/81$ (c) $10^{x^2-2x+2} = 100$

15. Find the unknown x in each of the following equations:

(a) $25^5 \cdot 25^x = 25^3$ (b) $3^x - 3^{x-2} = 24$ (c) $3^x \cdot 3^{x-1} = 81$
 (d) $3^5 + 3^5 + 3^5 = 3^x$ (e) $4^{-6} + 4^{-6} + 4^{-6} + 4^{-6} = 4^x$ (f) $\frac{2^{26} - 2^{23}}{2^{26} + 2^{23}} = \frac{x}{9}$

SM 16. Simplify: (a) $\frac{s}{2s-1} - \frac{s}{2s+1}$ (b) $\frac{x}{3-x} - \frac{1-x}{x+3} - \frac{24}{x^2-9}$ (c) $\frac{\frac{1}{x^2y} - \frac{1}{xy^2}}{\frac{1}{x^2} - \frac{1}{y^2}}$

SM 17. Reduce the following fractions:

$$(a) \frac{25a^3b^2}{125ab} \quad (b) \frac{x^2 - y^2}{x + y} \quad (c) \frac{4a^2 - 12ab + 9b^2}{4a^2 - 9b^2} \quad (d) \frac{4x - x^3}{4 - 4x + x^2}$$

18. Solve the following inequalities:

$$(a) 2(x - 4) < 5 \quad (b) \frac{1}{3}(y - 3) + 4 \geq 2 \quad (c) 8 - 0.2x \leq \frac{4 - 0.1x}{0.5}$$

$$(d) \frac{x - 1}{-3} > \frac{-3x + 8}{-5} \quad (e) |5 - 3x| \leq 8 \quad (f) |x^2 - 4| \leq 2$$

19. Using a mobile phone costs \$30 per month, and an additional \$0.16 per minute of use.

- (a) What is the cost for one month if the phone is used for a total of x minutes?
 (b) What are the smallest and largest numbers of *hours* you can use the phone in a month if the monthly telephone bill is to be between \$102 and \$126?

20. If a rope could be wrapped around the Earth's surface at the equator, it would be approximately circular and about 40 million metres long. Suppose we wanted to extend the rope to make it 1 metre above the equator at every point. How many more metres of rope would be needed? (The circumference of a circle with radius r is $2\pi r$.)

21. (a) Prove that $a + \frac{a \cdot p}{100} - \frac{\left(a + \frac{a \cdot p}{100}\right) \cdot p}{100} = a \left[1 - \left(\frac{p}{100}\right)^2\right]$.

- (b) An item initially costs \$2000 and then its price is increased by 5%. Afterwards the price is lowered by 5%. What is the final price?
 (c) An item initially costs a dollars and then its price is increased by $p\%$. Afterwards the (new) price is lowered by $p\%$. What is the final price of the item? (After considering this problem, look at the expression in part (a).)
 (d) What is the result if one first *lowers* a price by $p\%$ and then *increases* it by $p\%$?

22. (a) If $a > b$, is it necessarily true that $a^2 > b^2$?

- (b) Show that if $a + b > 0$, then $a > b$ implies $a^2 > b^2$.

23. (a) If $a > b$, use numerical examples to check whether $1/a > 1/b$, or $1/a < 1/b$.

- (b) Prove that if $a > b$ and $ab > 0$, then $1/b > 1/a$.

24. Prove that (i) $|ab| = |a| \cdot |b|$ and (ii) $|a + b| \leq |a| + |b|$, for all real numbers a and b . (The inequality in (ii) is called the **triangle inequality**.)

SM 25. Consider an equilateral triangle, and let P be an arbitrary point within the triangle. Let h_1 , h_2 , and h_3 be the shortest distances from P to each of the three sides. Show that the sum $h_1 + h_2 + h_3$ is independent of where point P is placed in the triangle. (*Hint*: Compute the area of the triangle as the sum of three triangles.)