

COMPLEX NUMBER

1.1 INTRODUCTION TO COMPLEX NUMBERS

1.1.1 Quadratic Equations

Examples of quadratic equations:

1. $2x^2 + 3x - 5 = 0$
 2. $x^2 - x - 6 = 0$
 3. $x^2 = 4$
- $$i^2 = -1$$

The **roots** of an equation are the x-values that make it "work" We can find the roots of a quadratic equation either by using the quadratic formula or by factoring.

We can have 3 situations when solving quadratic equations.

Case 1: Two roots

Example:

$$2x^2 + 3x - 5 = 0$$

We proceed to solve this equation using the quadratic formula as we did earlier:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{9 + 40}}{4} \\ &= \frac{-3 - \sqrt{49}}{4} \text{ or } \frac{-3 + \sqrt{49}}{4} \\ &= -2.5 \text{ or } 1 \end{aligned}$$

Case 2: One root

Example:

$$4x^2 - 12x + 9 = 0$$

Notice what happens when we use the quadratic formula this time. Under the square root we get $144 - 144 = 0$.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{12 \pm \sqrt{144 - 144}}{8} \\
 &= \frac{12}{8} \\
 &= 1.5
 \end{aligned}$$

So it means we only have **one root**. We can also say that this is a **repeated root**, since we are expecting 2 roots.

Case 3: No Real Roots

Example:

$$x^2 - 4x + 20 = 0$$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{4 \pm \sqrt{16 - 80}}{2} \\
 &= \frac{4 \pm \sqrt{-64}}{2}
 \end{aligned}$$

This example gives us a problem. Under the square root, we get $\sqrt{-64}$, and we have been told repeatedly by our teachers that we cannot have the square root of a negative number. Can we find such a root?

1.2 Imaginary Numbers

To allow for these "hidden roots", around the year 1800, the concept of $\sqrt{-1}$ was proposed and is now accepted as an extension of the real number system. The symbol used is $i = \sqrt{-1}$ and i is called an **imaginary number**.

1.1.3 Powers Of i

Since i stands for $\sqrt{-1}$, let us consider some powers of i .

Recall:

$$(\sqrt{a})^2 = a, \text{ for any value of } a.$$

$$\text{And } i = \sqrt{-1}$$

Using these, we can derive the following:

$$i^2 = (\sqrt{-1})^2 = -1$$

$$i^2 = -1$$

$$(-1)^{\text{even power}} = 1$$

$$(-1)^{\text{odd power}} = -1$$

$$\frac{1}{i} = -i$$

Example 1:

Simplify each of the following equation.

$$1. \quad \sqrt{-9} = \sqrt{9(-1)} = \sqrt{9i^2} = 3i$$

$$2. \quad \sqrt{-50} = \sqrt{50(-1)} =$$

$$3. \quad \sqrt{-15}$$

$$4. \quad \sqrt{-25}$$

$$5. \quad -\sqrt{-31}$$

Example 2:

Simplify

$$1. \quad i^2 = -1$$

$$2. \quad i^3 = (i^2)i = (-1)i = -i$$

$$3. \quad i^8 = (i^2)^4 = (-1)^4 = 1$$

$$4. \quad i^{15} = (i^2)^7 i = (-1)^7 i = (-1)i = -i$$

$$5. \quad i^{38}$$

$$6. \quad i^{84}$$

Example 3:

Simplify the expressions below.

$$\begin{aligned}
 1. \quad & i^{17} - 32i^4 \\
 &= (i^2)^8 i - 32(i^2)^2 \\
 &= (-1)^8 i - 32(-1)^2 \\
 &= (1)i - 32(1) \\
 &= i - 32 = -32 + i
 \end{aligned}$$

$$2. \quad -18 + 3i^{23}$$

$$3. \quad 3i^{34} - i^{13}$$

$$4. \quad -2i^3 + 2i^{18} - 3i^{51}$$

1.2 COMPLEX NUMBERS

Complex numbers have a **real part** and an **imaginary part**.

Example:

$$1. \quad 5 + 6i$$

Real part: 5, Imaginary part: $6i$

$$2. \quad -3 + 7i$$

Real part: -3, Imaginary part: $7i$

- Some examples of complex numbers are

$$3 - 2i, \quad \frac{1}{2} + 5i, \quad 2 - \frac{1}{3}i, \quad 0 + 3i, \quad 5 + 0i, \quad 0 + 0i$$

NOTE: We can write the complex number $2 + 5i$ as $2 + i5$.

There is no difference in meaning.

1.2.1 Solving Equations with Complex Numbers

We now return to our problem from **above**. We didn't know then what to do with $\sqrt{(-64)}$. Now we can write the solution using complex numbers, as follows:

$$\begin{aligned}x &= \frac{4 \pm \sqrt{-64}}{2} \\&= \frac{4 \pm i\sqrt{64}}{2} \\&= \frac{4 \pm i8}{2} \\&= 2 - 4i @ 2 + 4i\end{aligned}$$

Exercise:

1. Express in terms of i : $\sqrt{-24}$

2. Simplify:

a. $\sqrt{-2}\sqrt{-8}$

b. $\sqrt{(-2)(-8)}$

c. $i^2 - i^6$

d. $(\sqrt{-2})^2 + i^4$

3. $5x^2 - 6x + 5 = 0$

4. $x^2 - 4x + 13 = 0$

5. $x^2 - 6x + 10 = 0$

1.3 ADDITION AND SUBTRACTION OF COMPLEX NUMBERS

If $z = x + yi$ and $w = u + vi$,

Then,

$$\begin{aligned} z + w &= (x + yi) + (u + vi) \\ &= (x + u) + (yi + vi) \end{aligned}$$

$$\begin{aligned} z - w &= (x + yi) - (u + vi) \\ &= (x - u) + (yi - vi) \end{aligned}$$

Example:

Solve the problem below:

$$1. (6 + 7i) + (3 - 5i) = (6 + 3) + (7 - 5)i = 9 + 2i$$

$$2. (12 + 6i) - (4 + 5i) = (12 - 4) + (6 - 5)i = 8 + i$$

1.4 MULTIPLICATION OF COMPLEX NUMBERS

Expand brackets as usual, but care with j^2 !

If, $z = a + bi$ and $w = p + qi$

So,

$$\begin{aligned} z \times w &= (a + bi) \times (p + qi) \\ &= (a \times p) + (a \times qi) + (bi \times p) + (bi \times qi) \\ &= (ap) + (aqi) + (pbi) + (bqi^2) \\ &= (ap) + (aqi) + (pbi) + (bq(-1)) \\ &= (ap) + (aqi) + (pbi) - (bq) \end{aligned}$$

Example:

Solve each equation in the form $a + bi$.

1. $5(2 + 7i)$	2. $(6 - i)(5i)$
3. $(3 + 2i)(3 - 2i)$	4. $(5 + 3i)^2$

1.4.1 Multiplying By The Conjugate

Example 6 is a special case.

$3 + 2i$ is the **conjugate** of $3 - 2i$.

In general:

$x + yi$ is the **conjugate** of $x - yi$ and $x - yi$ is the **conjugate** of $x + yi$.

Notice that when we multiply conjugates, our final answer is **real** only (it does not contain any imaginary terms).

We use the idea of **conjugate** when dividing complex numbers.

1.5 DIVISION OF COMPLEX NUMBERS

If, $z = a + bi$ and $w = p + qi$. So,

$$\begin{aligned}\frac{z}{w} &= \frac{(a + bi)}{(p + qi)} \\ &= \frac{(a + bi)}{(p + qi)} \times \frac{(p - qi)}{(p - qi)} \\ &= \frac{(ap) - (aqi) + (pbi) + (bqi^2)}{p^2 - pqi + pqi - q^2i^2} \left\} \begin{array}{l} \text{Multiply with conjugate to get a} \\ \text{real number.} \end{array} \right. \\ &\quad \text{If } w = p + qi, \text{ conjugate of } w \text{ is } \bar{w} = p - qi \\ &= \frac{(ap) - (aqi) + (pbi) + (bqi^2)}{p^2 - pqi + pqi - q^2(-1)} \\ &= \frac{(ap) - (aqi) + (pbi) + (bqi^2)}{p^2 + q^2}\end{aligned}$$

Example:

1. Express $\frac{3-i}{4-2i}$ in the form $x + yi$	2. Simplify: $\frac{1-4i}{2+9i}$
3. $\frac{1-2i}{2+2i}$	4. $\frac{3+5i}{5-3i}$

Exercise:

1. Express in the form $a + bi$:

a) $i^{39} - 23i$

b) $i^{12} + 12i$

c) $1 + i + i^2 + i^3$

d) $(-9.4 - 6.2i)(2.5 + 1.5i)$

e) $\frac{3 + 4i}{2 - i}$

f) $\frac{2}{4 - i}$

g) $\frac{8 + i}{4i}$

h) $(\sqrt{2} + 3i) + (3\sqrt{2} - 5i)$

2. If $z = 2 + 3i$ and $w = 5 - 2i$, solve the following and write each answer in standard form.

a) $z + w$

b) w^2

c) zw

3. Express $\sqrt{-99 + 20i}$ in the form $a + bi$.

1.6 EQUIVALENT COMPLEX NUMBERS

Two complex numbers $x + yi$ and $a + bi$ are equivalent if:

The real parts are equal ($x = a$), **and**
The imaginary parts are equal ($y = b$).

Example:

1. Given that $3 + 2i = a + bi$, then

$$a = 3 \text{ and } b = 2.$$

2. Find the value of x and y , $(x + y) + (x - y)i = 14.8 + 16.2i$

$$(x + y) + (x - y)i = 14.8 + 16.2i$$

$$(x + y) = 14.8 \quad (1)$$

$$(x - y) = 16.2 \quad (2)$$

$$x = 14.8 - y \quad (3)$$

$$(14.8 - y) - y = 16.2$$

$$14.8 - 2y = 16.2$$

$$-2y = 16.2 - 14.8$$

$$-2y = 1.4$$

$$y = \frac{1.4}{-2}$$

$$y = -0.7$$

$$\text{if, } y = -0.7$$

$$x = 14.8 - (-0.7)$$

$$x = 15.5$$

$$\therefore x = 15.5 \quad y = -0.7$$

Exercise:

1. $(x + yi)(-2 + 7i) = -11 - 4i$

ans : $x = -0.1, y = 1.6$

2. $x + yi = \frac{2 + 5i}{1 - i}$

ans : $x = -\frac{3}{2}, y = \frac{7}{2}$

3. $(-5 + 3i) + (x - yi) = 1 + 4i$

ans : $x = 6, y = -1$

4. $(x - yi) - (2 - 3i) = 2 + 5i$

ans : $x = 4, y = -2$

5. $x + yi = (-2 + 7i)^2$

ans : $x = -45, y = -28$

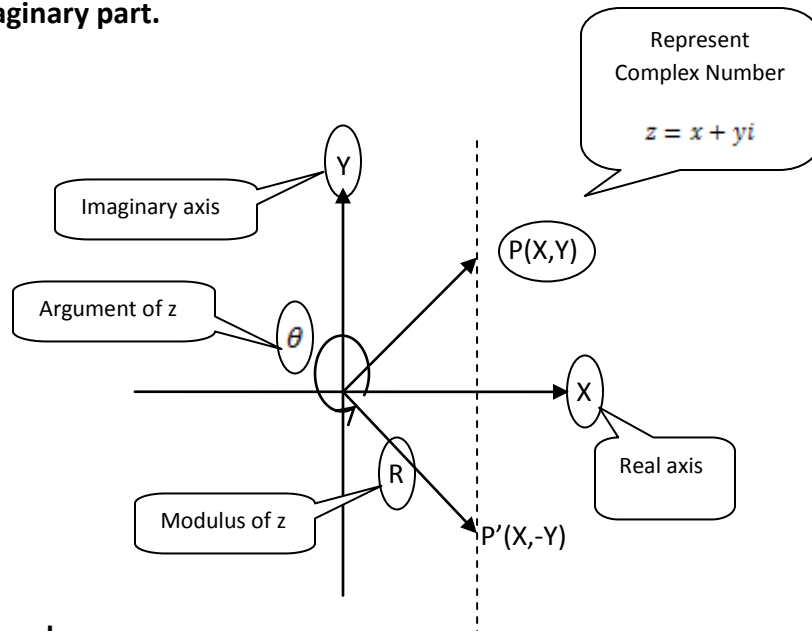
6. $\frac{2}{3}(a + bi) = 7 - 8i$

7. $a + bi = \frac{5 - 3i}{3} + \frac{1}{3 + 2i}$

1.7 GRAPHICAL REPRESENTATION OF COMPLEX NUMBERS

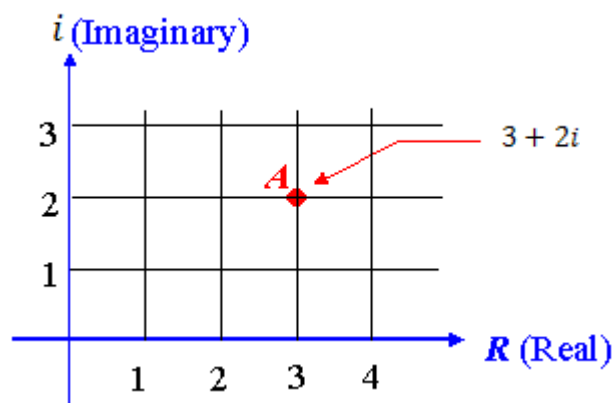
We can represent complex numbers in the **complex plane**.

We use the **horizontal axis** for the **real part** and the **vertical axis** for the **imaginary part**.



Example:

The number $3 + 2i$ is represented by:



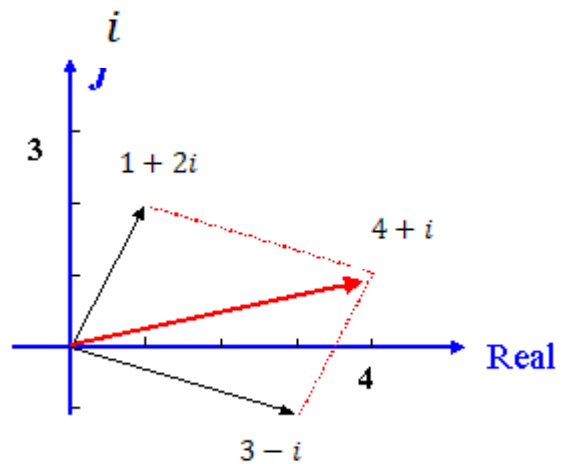
A is the representation of the complex number $3 + 2i$.

1.7.1 ADDING COMPLEX NUMBERS GRAPHICALLY

We can add complex numbers **graphically**.

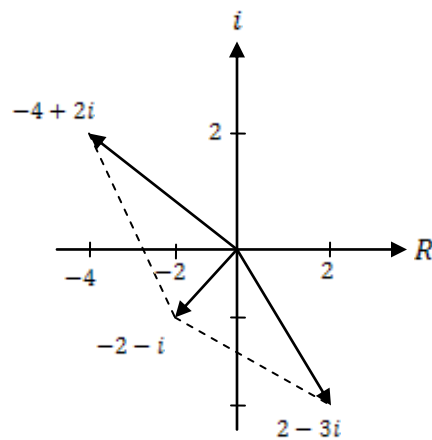
Example:

1. Add $1 + 2i$ and $3 - i$ graphically.



We add the complex numbers by setting up a parallelogram. The solution is $(4 + i)$.

2. Subtract $4 - 2i$ from $2 - 3i$ graphically.



Exercise:

1. Perform graphically:

i. $(3-2i)+(-1-i)$	ii. $(-2+2i)-(3+4i)$	iii. $(2+5i)+(-4+i)$
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2. Given $z_1 = 2 + 3i$, $z_2 = -3 + 5i$, $z_3 = -4 - 3i$ and $z_4 = 5 - 4i$, state each of the following function in the Argand Diagram.

- z_1, z_2, z_3, z_4
- $z_1 + z_2$
- $z_3 + z_4$
- $z_1 - z_2$
- $z_4 - z_3$

1.8 MODULUS AND ARGUMENT

$$\text{Modulus, } R = |z| = \sqrt{x^2 + y^2}$$

$$\text{Argument, } \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Example:

Find Modulus and Argument for each Complex Number.

a. $z = 1 - i$	b. $z = -3 + 4i$
c. $z = 3 + 4i$	d. $z = -1 - i$

1.9 FORM OF COMPLEX NUMBERS

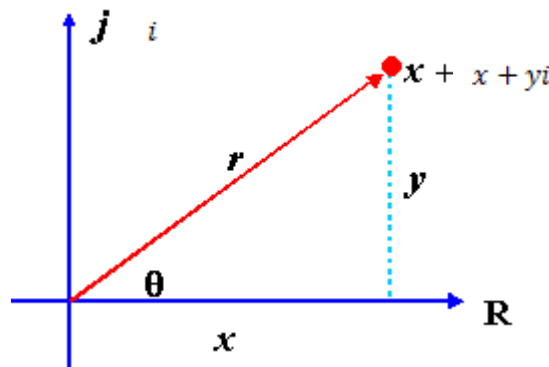
We can write complex numbers in 4 different ways:

Example:

Cartesian form	$x + yi$	$5 + 6i$
Trigonometric form	$r(\cos \theta + i \sin \theta)$	$8(\cos 24^\circ + i \sin 24^\circ)$
Polar form	$z = R \angle \theta^\circ$	$z = 8 \angle 24^\circ$
Exponential form	$z = Re^{i\theta}$ (<i>θ should be in radian</i>)	$z = 6e^{2.5i}$

1.9.1 Polar Form of a Complex Number

Our aim in this section is to write complex numbers in terms of a distance from the origin and a direction (or angle) from the positive horizontal axis.



We find the real (horizontal) and imaginary (vertical) components in terms of r (the length of the vector) and θ (the angle made with the real axis):

From Pythagoras, we have: $r^2 = x^2 + y^2$ and basic trigonometry gives us:

$$\tan \theta = \frac{y}{x}$$

$$x = r \cos \theta \quad y = r \sin \theta$$

Multiplying the last expression throughout by i gives us:

$$yi = i r \sin \theta$$

So we can write the **polar form** of a complex number as:

$$x + yi = r(\cos \theta + i \sin \theta)$$

r is the **absolute value** (or **modulus**) of the complex number

θ is the **argument** of the complex number.

There are two other ways of writing the **polar form** of a complex number:

$$r \text{ cis } \theta \text{ [means } r (\cos \theta + j \sin \theta)]$$

$$r \angle \theta \text{ [means once again, } r (\cos \theta + j \sin \theta)]$$

NOTE: When writing a complex number in polar form, the angle θ can be in DEGREES or RADIANS.

Example 1:

Find the polar form and represent graphically the complex number $7 - 5i$.

Solution:

We need to find r and θ .

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{7^2 + (-5)^2} \\ &= \sqrt{49 + 25} \\ &= \sqrt{74} \approx 8.6 \end{aligned}$$

To find θ , we first find the acute angle α :

$$\alpha = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left(\frac{5}{7} \right) \approx 35.54^\circ$$

Now, $7 - 5i$ is in the **fourth quadrant**, so

$$\theta = 360^\circ - 35.54^\circ = 324.46^\circ$$

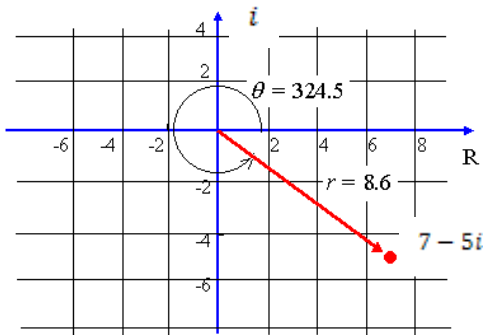
So, expressing $7 - 5i$ in **polar form**, we have:

$$7 - 5j = 8.6 (\cos 324.5^\circ + j \sin 324.5^\circ)$$

We could also write this answer as $7 - 5j = 8.6 \text{ cis } 324.5^\circ$.

Also we could write: $7 - 5j = 8.6 \angle 324.5^\circ$

The graph is as follows:

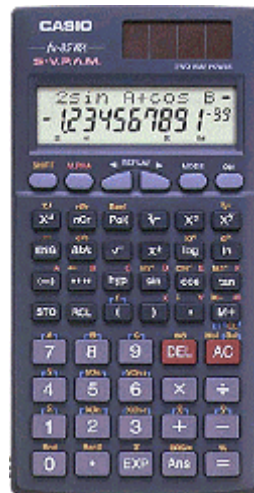


Example 2:

Express $3(\cos 232^\circ + j \sin 232^\circ)$ in cartesian form	Represent graphically and give the cartesian form of $7.32 \angle -270^\circ$	Represent $1 + i\sqrt{3}$ graphically and write it in polar form

Exercises:

1. Represent $\sqrt{2} - i\sqrt{2}$ graphically and write it in polar form
2. Represent graphically and give the cartesian form of $6(\cos 180^\circ + i \sin 180^\circ)$



And the good news is...

Now that you know what it all means, you can use your calculator directly to convert from **cartesian to polar** forms and in the other direction, too.

HOW TO CONVERT USING CALCULATOR

1. Polar Form $\leftrightarrow$ Cartesian Form

Example: Convert $56 \angle 27^\circ$ to rectangular form

Shift	Rec	56	,	27	)	=	RCL	tan
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Answer: $49.9 + 25.4i$

2. Cartesian Form $\leftrightarrow$ Polar Form

Example: Convert $12 - 42i$ to polar form $r\angle\theta^\circ$

Shift	Pol	12	,	-42	)	=	RCL	tan
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Answer: $43.68 \angle -74.05^\circ$

1.9.2 Exponential Form of a Complex Number

IMPORTANT:

In this section, θ MUST be expressed in radians.

The **exponential form** of a complex number is:

$$re^{\theta i}$$

(r is the **absolute value** of the complex number, the same as we had before and θ is in **radians**.)

Example 1:

Express $5(\cos 135^\circ + i \sin 135^\circ)$ in exponential form.

Solution:

We have $r = 5$ from the question.

We must express $\theta = 135^\circ$ in radians.

Recall:

$$1^\circ = \frac{\pi}{180}$$

$$\text{So } 135^\circ = \frac{135\pi}{180} = \frac{3\pi}{4} \text{ (or 2.36 radians)}$$

So we can write

$$5(\cos 135^\circ + i \sin 135^\circ) = 5e^{\frac{3\pi i}{4}} \approx 5e^{2.36i}$$

Example 2:

Express $-1 + 5i$ in exponential form.

Exercises:

1. Express in exponential form:

$$50(\cos 282.3^\circ + i \sin 282.3^\circ)$$

2. Express in exponential form: $-1 - 5i$

3. Express in polar and rectangular forms: $2.50e^{3.84i}$

1.10 Products and Quotients of Complex Numbers

When performing addition and subtraction of complex numbers, use **rectangular form**. (This is because we just add real parts then add imaginary parts; or subtract real parts, subtract imaginary parts.)

When performing multiplication or finding powers and roots of complex numbers, use **polar** and **exponential forms**. (This is because it is a lot easier than using rectangular form.)

We start with an example using exponential form, and then generalise it for polar and rectangular forms.

1.10.1 Multiplying Complex Numbers in Polar Form

We can generalise the example we just did, as follows:

$$(r_1 e^{\theta_1 i})(r_2 e^{\theta_2 i}) = r_1 r_2 e^{(\theta_1 + \theta_2) i}$$

From this, we can develop a formula for multiplying using polar form:

$$\begin{aligned} r_1 (\cos \theta_1 + i \sin \theta_1) &\times r_2 (\cos \theta_2 + i \sin \theta_2) \\ &= r_1 r_2 (\cos [\theta_1 + \theta_2] + i \sin [\theta_1 + \theta_2]) \end{aligned}$$

or with equivalent meaning:

$$r_1 \angle \theta_1 \times r_2 \angle \theta_2 = r_1 r_2 \angle [\theta_1 + \theta_2]$$

In words, all this confusing-looking algebra simply means...

To multiply complex numbers in polar form,

Multiply the *r* parts

Add the **angle** parts

Example 1:

Find $3(\cos 120^\circ + i \sin 120^\circ) \times 5(\cos 45^\circ + i \sin 45^\circ)$

1.10.2 Division

As we did before, we do an example in exponential form first, then generalise it for polar form.

Example in Exponential Form:

$$\begin{aligned}8e^{3.6i} \div 2e^{1.2i} &= 4e^{3.6i-1.2i} \\ &= 4e^{2.4i}\end{aligned}$$

[We divided the number parts, and subtracted the indices, just using normal algebra.]

From this, we can conclude the following:

$$\frac{r_1(\cos \theta_1 + i \sin \theta_1)}{r_2(\cos \theta_2 + i \sin \theta_2)} = \frac{r_1}{r_2}(\cos[\theta_1 - \theta_2] + i \sin[\theta_1 - \theta_2])$$

or

$$\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle [\theta_1 - \theta_2]$$

In words, this simply means...

To divide complex numbers in polar form, **Divide** the **r** parts and **Subtract** the **angle** parts

Example:

- i. Evaluate the following by first converting numerator and denominator into polar form.
- ii. Then check your answer by multiplying numerator and denominator by the *conjugate* of the denominator.

$$\frac{-2+5i}{-1-i}$$

Solution:

Exercise:

Express $5(\cos 59^\circ + i \sin 59^\circ)$ in Cartesian form, polar form and exponential form.	Find $\frac{3 \angle 20^\circ}{9 \angle 60^\circ}$	Find $\frac{8i}{7+2i}$ using polar form
Evaluate: $(0.5 \angle 140^\circ)(6 \angle 110^\circ)$	Evaluate: $\frac{12 \angle 320^\circ}{5 \angle 210^\circ}$	Express $\sqrt{265}e^{i5.45}$ in Cartesian form, polar form and trigonometric form.

Practice Exercise

A. Algebraic Operations Of Complex Number

1. Simplify $\frac{(3-2i)(7-5i)}{2-i}$
2. Given $u = 3 + 2i$, $v = -1 - 3i$, and $w = -2 + 5i$
 - i. $2u - v$
 - ii. $\frac{w}{u}$
 - iii. $vw + u$
3. State each of the complex number in form $a + ib$
 - i. $3 + \sqrt{-1}$
 - ii. $2 - \sqrt{-4}$
4. State the value of $Z_1 + Z_2$ and $Z_1 - Z_2$ in each case
 - i. $Z_1 = 5 + 3i$; $Z_2 = 2 + 2i$
 - ii. $Z_1 = 4 - \sqrt{2}i$; $Z_2 = 6 + 3\sqrt{2}i$
5. Given $z = -13 + 7i$ and $w = 2 - 5i$, find:
 - i. $z + w$
 - ii. $z - w$
 - iii. $\frac{z}{w}$
6. Simplify each of the complex numbers:
 - i. $(5 + 7i) + (3 - 4i) - (6 - 3i)$
 - ii. $\sqrt{-12}$
 - iii. $\frac{2 + 2i}{1 - i}$
7. Given that $(x + 2y) + (x - y)i = -180 + 115i$. Find the value of x and y .

8. If $z = \frac{2+3i}{2} + \frac{1}{1-2i}$, find z in form $a + ib$.
9. Simplify each of the complex numbers in form $a + jb$.

i. $\frac{6j}{5-7j}$

ii. $\frac{4-5j}{2+3j}$

iii. $(3+7j)(4-5j)$

B. Graphical Representation and other Form of Complex Numbers

10. Given $Z_1 = 3 - 2i$ and $Z_2 = -1 + i$. Find the modulus and argument of:

i. $Z_1 + Z_2$

ii. $Z_1 - Z_2$

11. Given $Z_1 = 4 + i$, $Z_2 = 2\angle 10^\circ$, and $Z_3 = 4e^{0.35i}$. Find $\frac{Z_1 Z_3}{Z_2}$ in the polar expression.

(Given $\theta^\circ = \frac{\theta_{rad}}{\pi} \times 180$).

12. Given $z_1 = 2 + i$, $z_2 = -3 + 2i$

i. Calculate modulus and argument of z_1 , z_2 and $z_1 z_2$.

ii. Sketch Argand diagram for $z_1 z_2$.

13. Find the modulus and argument for each complex number:

i. $-4 + 3i$

ii. $-1 - \sqrt{3}i$

14. Given $z_1 = 18(\cos 175^\circ + i \sin 175^\circ)$ dan $z_2 = 6(\cos 54^\circ + i \sin 54^\circ)$. Find:

i. $z_1 \bullet z_2$

ii. $\frac{z_1}{z_2}$

iii. Find the sum of $z_1 = 3 + 4i$ and $z_2 = 5 + 3i$ by using an Argand diagram.

15. Given $(x + yj) = \frac{-j}{(3 - j)}$, where x and y are real numbers.

- i. Find the value of x and y
- ii. Refer to the answer in (i). Find the modulus and the argument.

16. Given $Z = 25(\cos 137^\circ + i \sin 137^\circ)$, express Z in:

- i. Cartesian/ Rectangular Form.
- ii. Exponential Form.

17. Simplify

i.
$$\frac{5(\cos 60^\circ + i \sin 60^\circ) \bullet 4(\cos 30^\circ + i \sin 30^\circ)}{2(\cos 50^\circ + i \sin 50^\circ)}$$

18. State each of the complex number in form $a + ib$

- i. $(6 + j7) + (8 + j5) - (4 - j10)$
- ii. $(4 + j4)(4 + j3)$
- iii. $(20 \angle 36^\circ)^2 + (10 + j5)$
- iv. $(3 + j4) + (2 - j5)(1 - j2)$
- v. $\frac{-2 + j3}{4 + j2} + 5 \angle 45^\circ$

19. Given $z_1 = 4 - 3i$ and $z_2 = 4 + 3i$.

Find the modulus and argument for each complex number and show it in Argand diagram:

i.
$$\frac{1}{2}(z_1 + z_2)$$

ii.
$$\frac{2}{3}z_1 + \frac{1}{2}z_2$$

20. Express each complex numbers to the polar form.

- i. $z = -5 + 8j$
- ii. $z = 3 + 4j$