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Question One

$$\begin{aligned} \epsilon_x &= 23 \times 10^{-6} \\ \epsilon_y &= -14 \times 10^{-6} \\ \nu &= 0.28 \\ E &= 170 \times 10^9 \text{ Pa} \end{aligned}$$

$$\begin{aligned} \text{i) } \sigma_x &= \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{170 \times 10^9}{1-(0.28)^2} ((23 \times 10^{-6}) + (0.28 \times -14 \times 10^{-6})) \\ &= 3519531.25 \text{ N/m}^2 = \underline{3.52 \text{ MPa}} \end{aligned}$$

$$\begin{aligned} \sigma_y &= \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{170 \times 10^9}{1-(0.28)^2} ((-14 \times 10^{-6}) + (0.28 \times 23 \times 10^{-6})) \\ &= -1394531.25 \text{ N/m}^2 = \underline{-1.39 \text{ MPa}} \end{aligned}$$

$$\text{ii) } \begin{aligned} \epsilon_x &= 84 \times 10^{-6} \text{ Pa} \\ \epsilon_y &= \end{aligned}$$

$$\sigma_x = 84 \times 10^6 \text{ Pa}$$

$$\nu = 0.28$$

$$\sigma_y = -43 \times 10^6 \text{ Pa}$$

$$E = 170 \times 10^9 \text{ Pa}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{170 \times 10^9} ((84 \times 10^6) - (0.28 \times -43 \times 10^6))$$

$$= 564.94 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{170 \times 10^9} ((-43 \times 10^6) - (84 \times 10^6 \times 0.28))$$

$$= -391.29 \times 10^{-6}$$

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Question Two

$$\sigma_x = 104 \times 10^6 \text{ Pa}$$

$$\sigma_y = 204 \times 10^6 \text{ Pa}$$

$$\tau_{xy} = 125 \times 10^6 \text{ Pa}$$

$$i) \sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$= \frac{((104 \times 10^6) + (204 \times 10^6))}{2} + \frac{\sqrt{((104 \times 10^6) - (204 \times 10^6))^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 288.63 \times 10^6 \text{ Pa}$$

$$\sigma_1 = 288.63 \text{ MPa}$$

$$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$= \frac{((104 \times 10^6) + (204 \times 10^6))}{2} - \frac{\sqrt{((104 \times 10^6) - (204 \times 10^6))^2 + 4(125 \times 10^6)^2}}{2}$$

$$= 19.37 \times 10^6 \text{ Pa}$$

$$\sigma_2 = 19.37 \text{ MPa}$$

$$ii) \tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{min} = -\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

$$\tau_{max} = \frac{(288.63 \times 10^6 - (19.37 \times 10^6))}{2} = 134.315 \times 10^6 \text{ N/m}^2$$

$$\tau_{max} = 134.32 \text{ MPa}$$

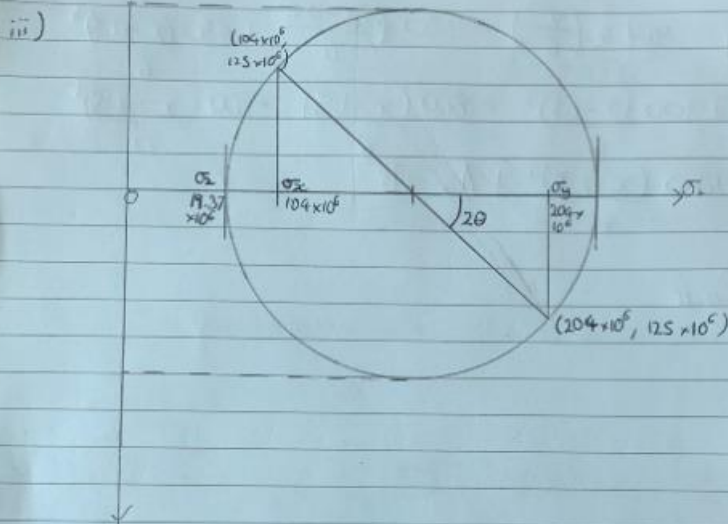
$$\tau_{min} = -\frac{(288.63 \times 10^6 - (19.37 \times 10^6))}{2} = -134.315 \times 10^6 \text{ N/m}^2$$

$$\tau_{min} = -134.32 \text{ MPa}$$

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Question TwoQuestion Three

i) $M = \frac{\sigma I}{y}$ $E I \frac{d^2 y}{dx^2} = M = R_1(x) - f_1(x-a) - f_2(x-b)$

about RB $M = (1969.4)(x) - 450(x-0.5) + 450(x-1.5) - 2300(x-1.5) - 800(x-1.5) + 800(x-2.5) - 1000(x-2.5)$

ii) $E I \frac{d^2 y}{dx^2} = 1969.4 \frac{(x)^2}{2} - 450 \frac{(x-0.5)^2}{2} + 450 \frac{(x-1.5)^2}{2} - 2300 \frac{(x-1.5)^2}{2} - 800 \frac{(x-1.5)^2}{2} + 800 \frac{(x-2.5)^2}{2} - 1000 \frac{(x-2.5)^2}{2} + A$

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Question Three

$$\text{iii) } E_y E I_y = 1969.4 \left(\frac{x^3}{6} \right) - 450 \frac{(x-0.5)^3}{6} + 450 \frac{(x-1.5)^3}{6} \\ - 2300 \frac{(x-1.5)^3}{6} - 800 \frac{(x-1.5)^2}{6} + 800 \frac{(x-2.5)^3}{6} \\ - 1000 \frac{(x-2.5)^3}{6} + Ax + B$$

Question Four

$$d = 0.28 \text{ m}$$

$$l = 0.6 \text{ m}$$

$$\eta_L = 54\%$$

$$\eta_C = 63\%$$

$$P = 14.4 \text{ Bar} = 14.4 \times 10^5 \text{ Pa}$$

$$\sigma_{\max} < 150 \times 10^6 \text{ Pa}$$

$$t = \frac{PD}{2\sigma_{\max} \eta_L}$$

$$t = \frac{PD}{4\sigma_{\max} \eta_C}$$

$$\text{i) } t = \frac{14.4 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.54}$$

$$= 2.48 \times 10^{-3} \text{ m}$$

$$\text{or } 2.48 \text{ mm}$$

(based on longitudinal stresses)

$$\text{ii) } t = \frac{14.4 \times 10^5 \times 0.28}{4 \times 150 \times 10^6 \times 0.63}$$

$$= 1.06 \times 10^{-3} \text{ m}$$

$$\text{or } 1.07 \text{ mm}$$

(based on circumferential stresses)

iii) absolute minimum thickness of 2.49 mm (based on longitudinal stresses)

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Question Five

$$l = 1.34 \text{ m}$$

$$d = 19.05 \text{ mm} = 19.05 \times 10^{-3} \text{ m}$$

$$t = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$A = 19.05 - 2(1.2) = 16.65$$

$$i) \quad A_e I = \frac{\pi}{64} ((19.05 \times 10^{-3})^4 - \frac{16.65 \times 10^{-3}}{2} (1.2 \times 10^{-3})^4)$$

$$I = 1.481 \times 10^{-9} \text{ m}^4 \quad 2.692 \times 10^{-9} \text{ m}^4$$

$$ii) \quad n = 0.25$$

$$iii) \quad f_{CR} = \frac{n \pi^2 E I}{L^2} = \frac{0.25 \times \pi^2 \times 200 \times 10^9 \times 2.692 \times 10^{-9}}{1.34^2}$$

$$f_{CR} = 739.8 \text{ N}$$

$$iv) \quad SR = \frac{L}{K}$$

$$K = \sqrt{\frac{EI}{A}} = \sqrt{\frac{2.692 \times 10^{-9}}{6.73 \times 10^{-5}}} = 2.0092$$

$$A = \frac{\pi}{4} (d^2 - d_i^2)$$

$$A = \frac{\pi}{4} ((19.05 \times 10^{-3})^2 - (16.65 \times 10^{-3})^2)$$

$$= 6.73 \times 10^{-5} \text{ m}^2$$

$$K = \sqrt{\frac{EI}{A}} = \sqrt{\frac{2.692 \times 10^{-9}}{6.73 \times 10^{-5}}} = 6.32 \times 10^{-3}$$

$$SR = \frac{1.34}{6.32 \times 10^{-3}} = 212$$