

SGDM5011 Assignment 2

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Mechanical Science 2

SGDM5011

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1.0 Beam Deflection

The Diagram shown in **Figure 1** shows how the loads and supports are arranged on a simply supported beam. First the Reactions at the supports need to be calculated.

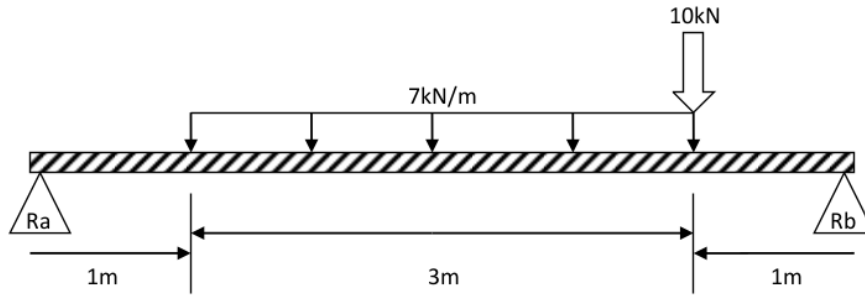


Figure 1 Beam Deflection Loads

$$(7 \times 3 \times 2.5) + 10 \times 4 = R_b \times 5$$

$$(52.5 + 40) / 5 = R_b$$

$$R_b = 18.5$$

$$R_a = ((3 \times 7) + 10) - R_b$$

$$R_a = 12.5$$

$$EI \frac{d^2 y}{dx^2} = 12.5x - 10(x-4) - 7(x-1) \frac{(x-1)}{2} + 7(x-4) \frac{(x-4)}{2}$$

or

$$EI \frac{d^2 y}{dx^2} = 12.5x - 10(x-4) - 7 \frac{(x-1)^2}{2} + 7 \frac{(x-4)^2}{2}$$

$$EI \frac{dy}{dx} = \frac{12.5x^2}{2} - \frac{10(x-4)^2}{2} - 7 \frac{(x-1)^3}{6} + 7 \frac{(x-4)^3}{6} + A$$

$$EI y = \frac{12.5x^3}{6} - \frac{10(x-4)^3}{6} - 7 \frac{(x-1)^4}{24} + 7 \frac{(x-4)^4}{24} + Ax + B$$

Apply known boundary conditions; -

$$\text{When } x=0 \text{ } y=0 \quad 0=0+0+B \quad B=0$$

$$\text{When } x=5 \text{ } y=0$$

$$0=260.4-1.67-74.67+0.292+5A$$

$$-184.352=5A$$

$$-184.352/5=-36.87$$



$$EIy = \frac{12.5x^3}{6} - \frac{10(x-4)^3}{6} - 7\frac{(x-1)^4}{24} + 7\frac{(x-4)^4}{24} - 36.870x$$

$$EIy = \frac{12.5(2.5)^3}{6} - \frac{10(1.5)^3}{6} - 36.870(2.5)$$

$$EIy = 32.55 - 5.625 - 92.175$$

$$EIy = -65.25$$

$$Y = \frac{-65.25 \times 10^3}{(200 \times 10^9) \times (3 \times 10^{-5})}$$

$$Y = -2.637 \times 10^{-3} \text{ m} / 2.637 \text{ mm}$$



2.0 Strut Buckling Analysis

When deciding which method to use (Euler / Rankine Gordon) the graph shown in **Figure 2** should be considered as the Euler method is for long slender struts and the Rankine Gordon method is for intermediate to long as this will cover both crushing and buckling / pure buckling. 

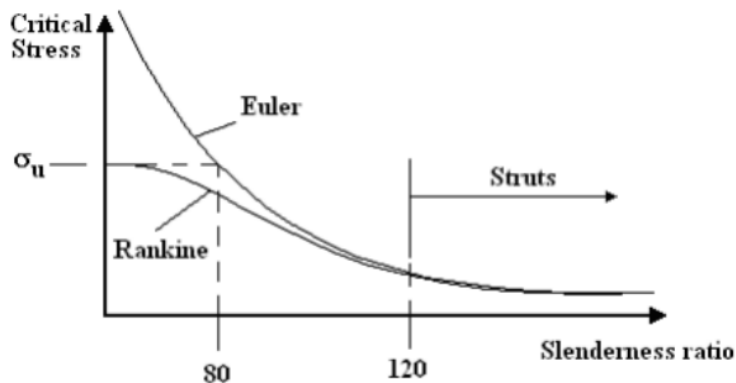


Figure 2 Crushing and Buckling validity graph (Dunn, n.d.)

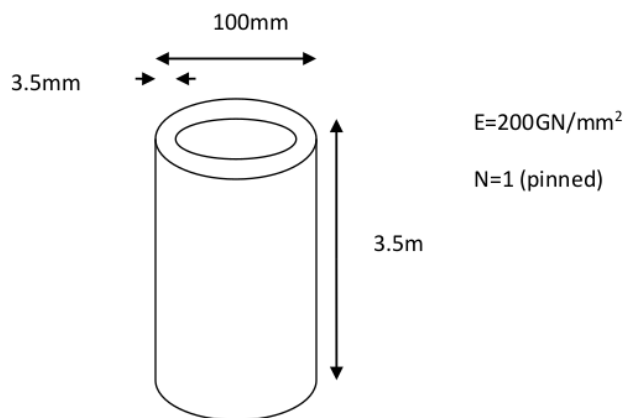


Figure 3 Strut Dimensions

$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$I = \frac{\pi 0.1^4}{64} - \frac{\pi 0.093^4}{64}$$



$$I = 4.91 \times 10^{-6} - 0.0367 = -0.0367 \text{ m}^4$$

$$F_e = \frac{n^2 \pi^2 EI}{L^2}$$

$$F_e = \frac{1^2 \pi^2 \times (200 \times 10^9) \times (-0.0367)}{3.5^2}$$

$$F_e = \frac{-72.443 \times 10^9}{3.5^2}$$

$$F_e = 5.914 \times 10^9 \text{ N}$$



$$\sigma_{critical} = \frac{\pi^2 EI}{(KL)^2}$$

$$K = \sqrt{\frac{I}{A}}$$

$$A = \frac{\pi}{4} (0.1^2 - 0.093^2) = 1.061 \times 10^{-3} \text{ m}^2$$

$$K = \sqrt{\frac{0.0367}{1.061 \times 10^{-3}}} = 5.55$$

3.0 Gyroscopic Motion

According To Bolton “The angular momentum of a body about some axis is the product of its moment of inertia I and its angular velocity ω about that axis.” Also “Gyroscopic couples occur with any spinning rotor when an external torque is applied about an axis perpendicular to the axis of spin.” (Bolton, 2006)

$$I = 1.4 \text{ kgm}^2 / \omega = 1600 \text{ rpm} \times (2\pi/60) = 167.55 / r = 200 / V = 200/3.6 = 55.55$$

$$\Omega = \frac{V}{r}$$

$$\Omega = \frac{55.55}{200} = 0.278$$

$$T = 1.4 \times 167.55 \times 0.278 = 65.159 \text{ Nm}$$



4.0 Balancing Coplanar systems

Bolton explains static balancing as the following; - “A shaft is said to be in static balance if, for off-centre masses, there is no resultant force in any direction due to the weights of the masses and the algebraic sum of their turning moments about the shaft axis is zero” (Bolton, 2006)

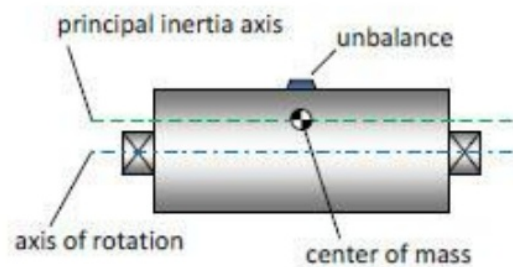
“The diagrams shown in Figure 4 compares dynamic and static balancing, number 1 shows the effect an unbalance located some distance from the crankshaft’s centreline (or axis of rotation) has on the centre of mass. This unbalance results in the crank trying to rotate about a new centreline (the ‘principal inertia axis’). Energy is wasted as the crank tries to run off-centre, creating vibration, stress and consuming power in the process.

In the real world, unbalance isn't distributed equally or symmetrically along the crankshaft's length. This results in not only the 'centre of mass' moving away from the crankshaft's centreline, but also a tilt in the 'principal inertia axis' at an angle away from the crankshaft's axis of rotation. Number 2

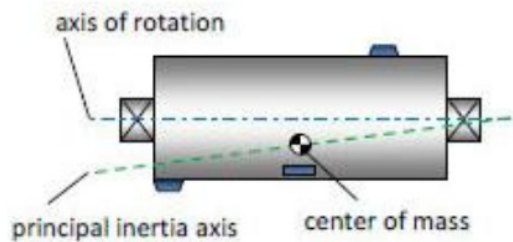
The aim when dynamically balancing a crankshaft is to have the 'principle inertia axis' coincide exactly with the crankshaft's centreline."

(Maher, 2013)

Balancing is important in high speed rotating applications such as mentioned above as in Engine Crankshafts and vehicle driveline components such as propeller/drive shafts.



1. static unbalance



2. dynamic unbalance

Figure 4 Static and Dynamic unbalance

Figure 5 MR Diagram

Plane	Mass	Radius	MR	θ
A	15	0.25	3.75	0°
B	27	0.25	6.75	$45^\circ(180^\circ-135^\circ)$
C	C?	0.25	$C(0.25)$	$C\theta^\circ$

Table 1 Known Balancing Data

4.1 Horizontal element

$$3.75 \cos 0 + 6.75 \cos 45 + 0.25C \cos C\theta = 0$$



$$3.75 + 4.77 + 0.25C \cos C\theta = 0$$

$$8.54 + 0.25C \cos C\theta = 0$$

$$0.25C \cos C\theta = -8.54$$

$$C \cos C\theta = -8.54/0.25$$

$$C \cos C\theta = -34.17$$

4.2 Vertical element

$$3.75 \sin 0 + 6.75 \sin 45 + 0.25C \sin C\theta = 0$$

$$0 + 4.77 + 0.25C \sin C\theta = 0$$

$$0.25C \sin C\theta = -4.77$$

$$C \sin C\theta = -4.77/0.25$$

$$C \sin C\theta = -19.08$$

$$\tan C\theta = \frac{-34.17}{-19.08}$$

$$\tan C\theta = 1.79$$

$$\tan^{-1} 1.79 = 60.81^\circ$$

$$60.81^\circ + 180^\circ = 240.81^\circ$$

$$3.75 \sin 0 + 6.75 \sin 45 + 0.25c \sin 60.81 = 0$$

$$0 + 4.77 + (0.25c \times 0.873) = 0$$

$$0.25c \times 0.873 = -4.77$$

$$0.25c = \frac{-4.77}{0.873} = -5.467$$

$$c = \frac{-5.467}{0.25} = 21.87\text{kg}$$



Plane	Mass	Radius	MR	θ
A	15	0.25	3.75	0°
B	27	0.25	6.75	$45^\circ (180^\circ - 135^\circ)$
C	21.87	0.25	5.47	$60.81^\circ (240.81^\circ - 180^\circ)$

Table 2 Completed balancing data

5.0 Complex Stress

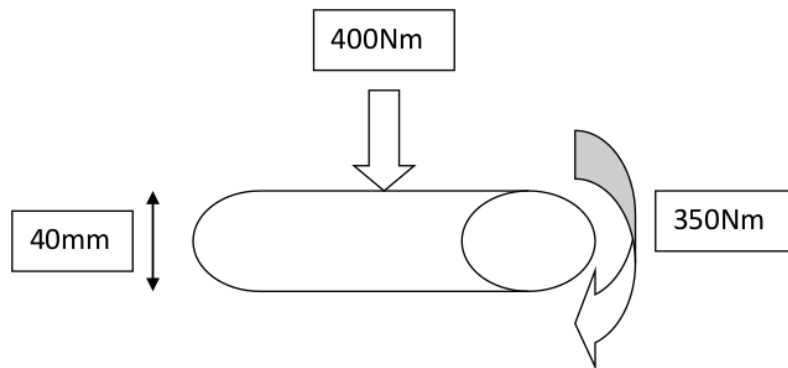


Figure 6 Schematic of the forces acting on shaft

$$R=0.02\text{m}$$

$$M=400\text{Nm}$$

$$T=350\text{Nm}$$

5.1 Stress due to torsion

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$J = \left(\frac{\pi D^4}{32}\right) = \frac{\pi 0.04^4}{32} = 0.25 \times 10^{-6} \text{m}^4$$

$$\tau = \frac{Tr}{J}$$

$$\tau = \frac{350 \times 0.02}{0.25 \times 10^{-6}} = 27.85 \text{MN/m}^2$$

5.2 Stress due to bending

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$I = \left(\frac{\pi D^4}{64} \right) = \frac{\pi 0.04^4}{64} = 0.126 \times 10^{-6} \text{m}^4$$

$$\sigma_x = \frac{MY}{I}$$

$$\sigma_x = \frac{400 \times 0.02}{0.126 \times 10^{-6}} = 63.66 \text{MN/m}^2$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_{1,2} = \frac{63.66 \times 10^6 + 0}{2} \pm \frac{1}{2} \sqrt{(63.66 \times 10^6 - 0)^2 + 4 \times 27.85 \times 10^6^2}$$

$$\sigma_{1,2} = 31.83 \times 10^6 \pm \frac{1}{2} 84.59 \times 10^6$$

$$74 \times 10^6 \text{N/m}^2 = \sigma_1$$

$$-10.47 \times 10^6 \text{N/m}^2 = \sigma_2$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau = \frac{74 - (-10.47)}{2} = 42.3 \times 10^6 \text{N/m}^2$$



$$\tan 2\vartheta = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$\tan 2\vartheta = \frac{2 \times 27.85}{63.66 - 0}$$

$$\tan 2\vartheta = 0.875$$

$$2\vartheta = \tan^{-1} 0.875$$

$$2\vartheta = 41.186^\circ$$

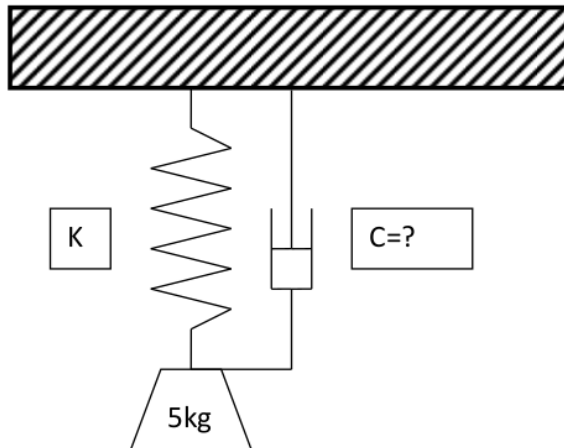
$$\vartheta = (41.186/2) = 20.59^\circ$$

$$\vartheta_1 = 20.59^\circ$$

$$\vartheta_2 = (20.59 + 90) = 110.59^\circ$$

6.0 Logarithmic Decrement

Logarithmic decrement according to Babn, "It is defined as the natural logarithm of the ratio of two successive amplitudes on the same side of the mean line." (babn, 2016)



$$\frac{4}{10} = 0.4 \text{ Sec /oscillation}$$

$$F = \frac{1}{0.4} = 2.5\text{hz}$$

$$\frac{x_1}{x_6} = 2$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\omega_n = 2.5 \times 2\pi = 15.71 \text{rads/s}$$

$$15.71 = \sqrt{\frac{k}{5}}$$

$$15.71^2 = \frac{k}{5}$$

$$246.7 \times 5 = k$$

$$K = 1233 \text{ N/m (spring Stiffness)}$$

$$C_c = 2 \cdot m \cdot \omega_n$$

$$C_c = 2 \times 5 \times 15.71 = 157 \text{ N/m/s}$$

$$\ln \frac{x_1}{x_6} = \frac{2\pi\epsilon M}{\sqrt{1 - \epsilon^2}}$$

$$\ln 2 = \frac{2\pi\epsilon 5}{\sqrt{1 - \epsilon^2}}$$

$$\sqrt{1 - \epsilon^2} = \frac{2\pi\epsilon 5}{0.693}$$

$$1 - \epsilon^2 = \left(\frac{2\pi\epsilon 5}{0.693}\right)^2$$

$$1 - \epsilon^2 = \left(\frac{4\pi^2\epsilon 25}{0.48}\right)$$

$$1 - \epsilon^2 = 2054\epsilon^2$$

$$1 = 2054\epsilon^2$$

$$\frac{1}{2054} = \epsilon^2$$

$$\sqrt{0.4868 \times 10^{-3}} = \epsilon$$

$$0.02206 = \epsilon \text{ (Damping Coefficient)}$$

