

STUDENT FEEDBACK FORM-JACK SOLOMON

This section details the *extent* to which the assessment criteria are demonstrated by you, which in turn determines your mark. The marks available for each category of skill are shown. Lecturers will use the space provided to comment on the achievement of the task(s), including those areas in which you have performed well and areas that would benefit from development/improvement.

Generic Assessment Criteria	Marks available	Marks awarded
1. Engagement with Literature Skills		
Your work has shown that you researched adequately into the topic and you demonstrated a good understanding of the subject. If you had used other sources such as books, papers, magazines etc, rather than simply relying on the internet would have gained the highest score. The student has clearly shown that he has all-important skills needed to define the essential ideas from sufficient information. The quality of the references needs to be improved.	15	11
2. Knowledge and Understanding Skills		
The report shows good effort and good understanding of the topics covered. Good structure, detail and data presented in clear and appropriate format throughout.	20	18
3. Cognitive and Intellectual Skills		
Your work showed that you can summarise scientific facts into your own words without compromising academic style. There is a coherent flow in the writing and the reader can easily follow the simulations process and the analysis of the results. You have made it interesting to read and have also made attempts at drawing conclusions for the area you studied.	20	18
4. Practical Application Skills		
A good and appropriate application of established methods and techniques. The report clearly demonstrates the student has a clear understanding of studies to date and also shows the student's ability to work as an individual. You could've shown more details about the simulation process (some aspects not being sufficiently carried out).	20	12
5. Transferable Skills for Life and Professional Practice		
Mostly ordered presentation and structure in which relevant concepts are reasonably expressed. Demonstrates the basic skills for employment requiring the exercise of personal responsibility and decision-making, with some areas of minor weakness	25	14

Assessment Mark (Assessment marks are subject to ratification at the Exam Board. These comments and marks are to give feedback on module work and are for guidance only until they are confirmed.)	Late Submission Penalties (tick if appropriate)		73%
	Up to 1 week late (40% Max)		
	Over 1 week late (0%)		

Fourier Series & Fourier Transform

HND Electrical & Electronic Engineering: Year 2

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Fourier Series and the Square Wave form

Fourier series is a tool used to break up any periodic functions into a set of sinusoidal based functions, that can then be analysed/solved individually. It is described as a way to represent a periodic function/signal as a sum of sine or cosine waves. This is done by adding any number of harmonics together to try and approximate the desired complex function. This will give a resulting function. (InMath, 2017)

Below is an example of how a pure square wave can be made up from a number of varied sine waves at different frequencies. The purpose of this is to prove that a pure square wave is the same as a series of sine waves. (AllAboutCircuits)

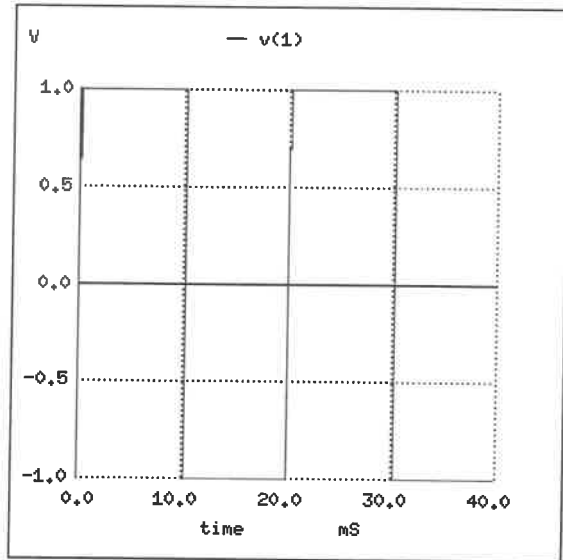


Figure 1; Shows a pure square wave form.

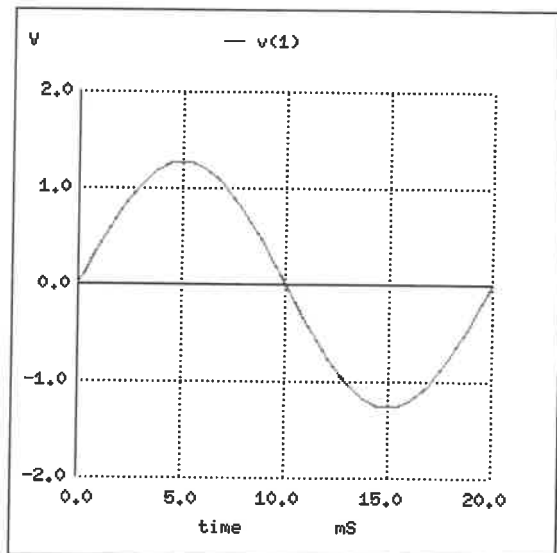


Figure 2; Shows the fundamental frequency or the first harmonics of a sine wave (red).

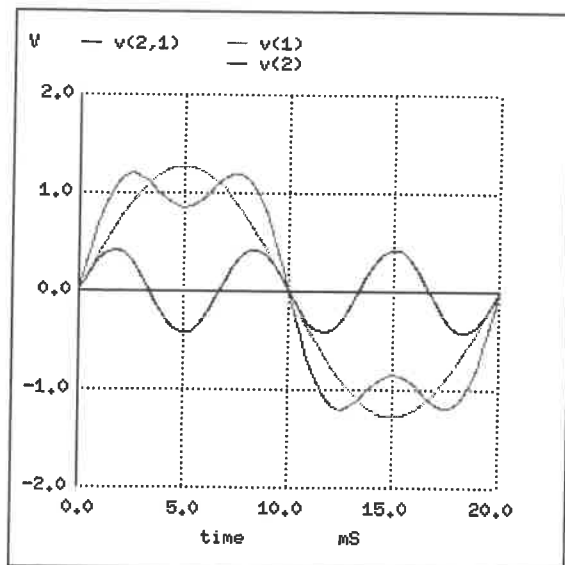


Figure 3; Above illustrates the shape of the resulting function (red) when the third harmonic (blue) is in series with the first harmonic (orange).

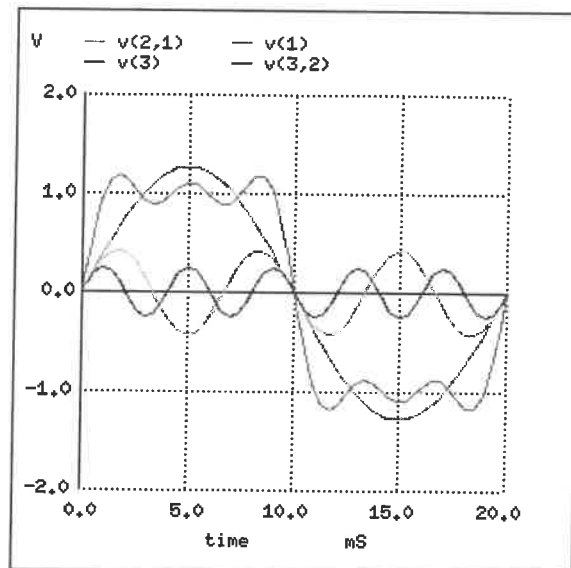


Figure 4; Shows that the addition of the fifth harmonic (blue) has changed the resulting function once again. (AllAboutCircuits)

A square sine wave can only be constructed by using odd frequencies. The greater number of harmonics added means that the resulting function will be a more accurate representation of the desired complex function.

Fourier Transform and its Importance in Engineering.

Fourier transform is a mathematical function that can be used to show the different parts of a continuous signal. Its most common use is to convert a time-based signal into the frequency domain. This is because a complex function can often be better understood in another domain. (Gonzalez-Velasco, 1992)

Fourier series can only be used to break down periodic signals. Periodic signals in the real world are rarely found. This means that Fourier series isn't that useful in the real world. In terms of engineering, Fourier transform however is important because it can be used to breakdown almost any time-based signal imaginable into a set of frequency based functions. This ensures that the frequency signal produced can be analysed more accurately (Wu, 2016).

"A Fourier transform shows what frequencies are in a signal. For example, consider a sound wave which contains three different musical notes: A, B, and C. Making a graph of the Fourier transform of this sound wave (with the frequency on the x-axis and the intensity on the y-axis) will show a peak at each frequency which corresponds with one of the musical notes." (Remmert, 1991).

Calculations

Below is a sketch of $f(x)$ from the interval -2π to $+2\pi$.

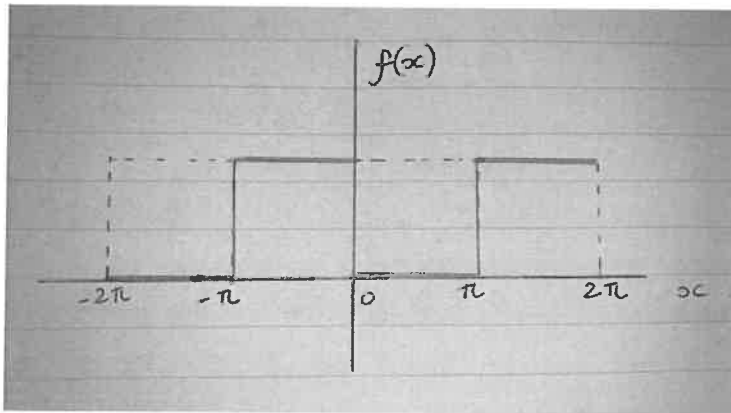


Figure 5.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

The first step is to calculate the DC part of the wave form. This is represented as a_0 .

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^0 (0) f(x) dx + \frac{1}{2\pi} \int_0^{\pi} (1) f(x) dx$$

$$a_0 = \frac{1}{2\pi} \int_0^{\pi} 1 dx$$

$$a_0 = \frac{1}{2\pi} [x]_0^\pi$$

$$a_0 = \frac{1}{2\pi} [\pi] = \frac{\pi}{2\pi} = 0.5 \quad \checkmark \quad \angle$$

The next step is to find out how much cosine is in the wave form. This is represented as a_n .

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 (0) f(x) \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} (1) f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} 1 \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_0^{\pi}$$

$$a_n = \frac{1}{\pi n} (\sin nx)_0^{\pi}$$

$$a_n = \frac{1}{\pi n} (\sin(\pi) - \sin(n0))_0^{\pi}$$

$$a_n = \frac{1}{\pi n} (0)$$

$$a_n = 0 \quad \checkmark$$

The final step is to find out how much sine is in the wave form. This is represented as b_n .

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^0 (0) f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} (1) f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} 1 \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left[-\frac{\cos nx}{n} \right]_0^{\pi}$$

$$b_n = \frac{1}{-n\pi} [\cos nx]_0^{\pi}$$

$$b_n = \frac{1}{-n\pi} [\cos(n\pi) - \cos(n0)]$$

$n = \text{odd}$

$$b_n = \frac{1}{-n\pi} [-1 - 1] = \frac{2}{n\pi} \quad \checkmark$$

$n = \text{even}$

$$b_n = \frac{1}{-n\pi} [1 - 1] = 0 \quad \checkmark$$

The Fourier series representation is;

$$f(x) = \left(\frac{1}{2}\right) + \left(\frac{2}{\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x + \frac{2}{7\pi} \sin 7x + \frac{2}{9\pi} \sin 9x + \frac{2}{11\pi} \sin 11x + \frac{2}{13\pi} \sin 13x\right)$$

Verification of the spectral analysis

Here is the Fourier series formula;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

The formula above is used to solve the function representing a complex wave with a period of 2π (calculations shown above). Below is the Fourier series representation of the function.

$$f(x) = \left(\frac{1}{2}\right) + \left(\frac{2}{\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x + \frac{2}{7\pi} \sin 7x + \frac{2}{9\pi} \sin 9x + \frac{2}{11\pi} \sin 11x + \frac{2}{13\pi} \sin 13x\right)$$

The Fourier trigonometric series of the function produced above can be verified by the use of suitable software. The correct data needs to be entered for the successful verification. Below is the correct data which needs to be entered into the software to create the square wave. The data below is taken from the Fourier series representation of the complex function.

<u>Harmonics</u>	0	1	2	3	4	5	6	7
<u>Frequency</u>	-	10	30	50	70	90	110	130
<u>Amplitude</u>	-	0.6366	0.2122	0.0909	0.0909	0.0707	0.0579	0.0490
<u>Offset</u>	0.5	-	-	-	-	-	-	-
<u>Wave</u>	Consistent	Sine	Sine	Sine	Sine	Sine	Sine	Sine

Table 1.

Harmonics - It was specified in the brief that only the first 7 harmonics were used. ✓

Frequency - Only odd frequency values can be used, would not have been possible to use even values. ✓

Amplitude - This is worked out by the Fourier series representation. For example, $\frac{2}{\pi} = 0.6366$. The values given have been rounded with 4 decimal places. ✓

Offset - This is the DC representation in the formula. ✓

Below is the square wave form of 7 harmonics which has been produced using the software.

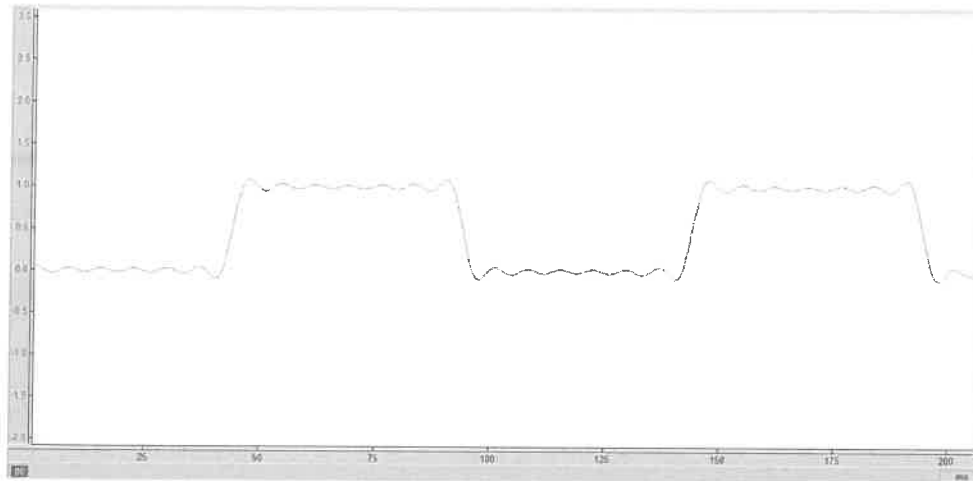


Figure 6.

More Harmonics or Less Harmonics

The following images have been taken to show what will happen to the shape of the square wave when more than 7 harmonics are present and also when there are less than 7 harmonics present.

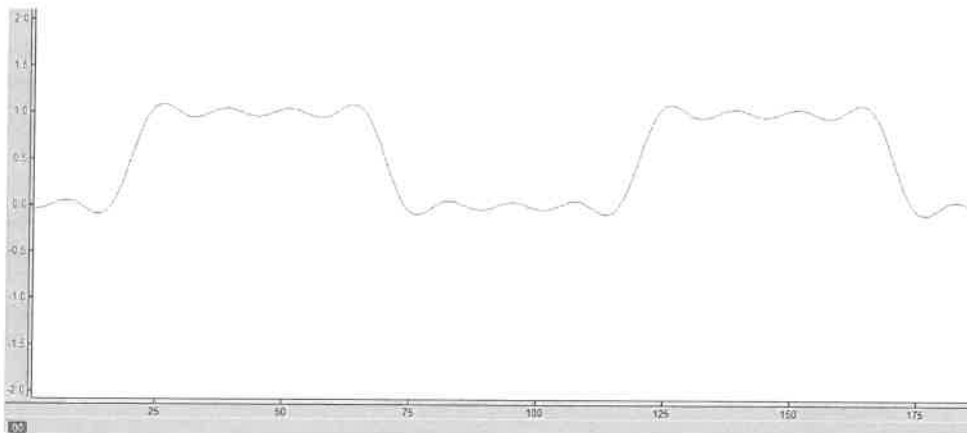


Figure 7; Shows the results of the wave form when there are only 4 harmonics present.

It is clear that the shape of this wave is not as good as the wave shown in figure 6.

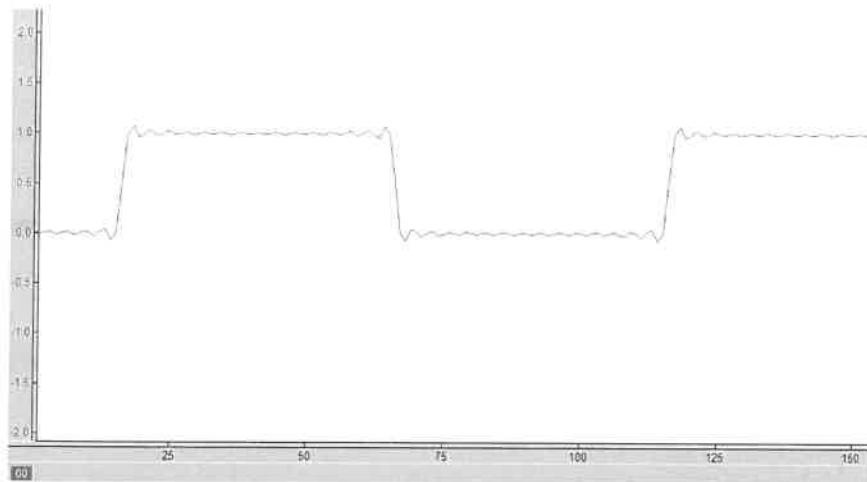


Figure 8; Shows the effect of using a greater number of harmonics.

There are 15 harmonics used to make the square wave below and it is a much better representation of a square wave. It is clear that the more harmonics which are present, will improve the shape of the wave form therefore improve the representation of a square wave. ✓

Greater or Fewer Decimal Places

The results shown above are all with 4 decimal places. Unfortunately the software doesn't allow any more decimal places to be added. Below will be the results of having fewer decimal places. The results will have the last decimal rounded up/down.

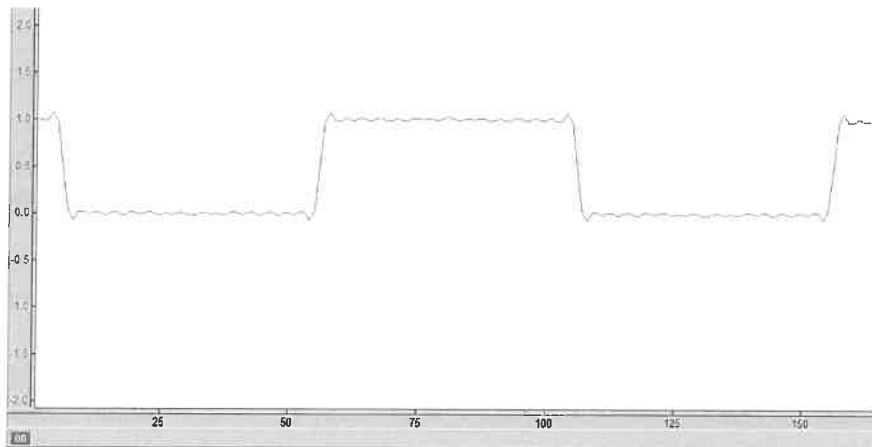


Figure 9; Shows the results of having 2 decimal places and 15 harmonics.

The results are surprising as there isn't much of a difference to having more decimal places.

FFT

The software can then be used to convert the square wave above from the time domain and into the frequency domain. This is called Fourier transform.

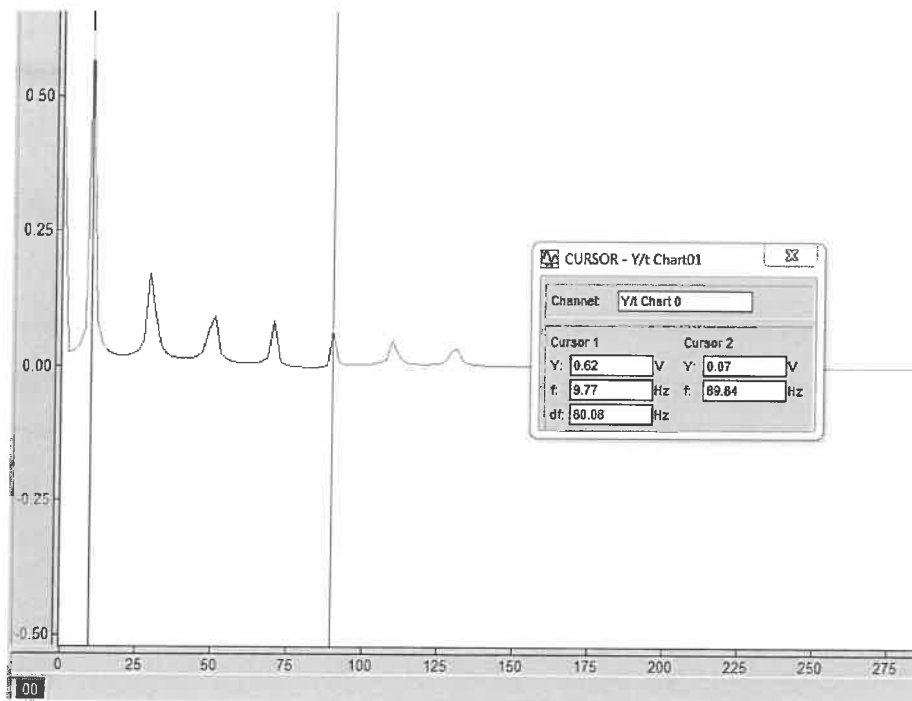


Figure 10; Demonstrates what the square wave looks like in the frequency domain.

The spikes in figure 10 shows the amplitudes at different frequencies. The vertical lines on the graph above (cursor 1 & 2) show the values of the amplitudes at different frequencies. These amplitudes match the readings which are in the table on page 7.

For example, cursor 1 is on the first harmonic, it shows when the frequency is 9.77Hz and the amplitude is 0.62V. In the table on page 7, when the frequency is 10Hz, the amplitude is 0.6366. Cursor 2 is on the fifth harmonic, it shows when the frequency is 89.84Hz the amplitude is 0.07. In the table, when the frequency is 90Hz the amplitude is 0.0707.

Evaluation

In conclusion, the results from the production of the square wave show that the more harmonics used result a far greater representation of square wave. The number of decimal places in fact, doesn't change the shape of the wave form.

In terms of the software, it was difficult to move the cursor more accurately. For example, it may have been more accurate to move the cursor on the exact frequency spike to see if the readings match the data which was inputted. i.e instead of having the cursor on 9.77Hz, move it to 10Hz to see if the amplitudes readings are the same as the ones which were calculated in Table 1.

Also, it would have been interesting to add more harmonics and have the ability to have greater decimal places to see how the square wave may change.

Bibliography

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<https://www.allaboutcircuits.com/textbook/alternating-current/chpt-7/square-wave-signals/>

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