

Exam Solutions/Mark Scheme

SECTION A						Qu total
a)	x	f	xf	$f(x-\bar{x})^2$		
	6	7	42	46.957		
	7	21	147	53.090		
	8	25	200	8.703		
	9	19	171	3.194		
	10	14	140	27.833		
	11	9	99	52.273		
	12	5	60	58.141		
	Σ	100	859	250.191	3	
	$\bar{x} = \frac{859}{100} = 8.59$				2	
	$\sigma = \sqrt{\frac{250.191}{100}} = 1.58$				2	7
b)	if $p = 0.6$				2	
	$q = 1 - 0.6 = 0.4$					
(i)	$n=3$	$P(1k) = \binom{3}{1} p^1 q^{3-1}$				
		$= \binom{3}{1} 0.6^1 (0.4)^2$				
		$= 0.288$		5		
(ii)	$n=3$	either 3 or 4				
		$P(3) = \binom{4}{3} (0.6)^3 (0.4)$				
		$= 0.3456$		4		
		$P(4) = \binom{4}{4} (0.6)^4 (0.4)^0$				
		$= 0.1296$		4		
		$\therefore P(3 \text{ or } 4) = 0.3456 + 0.1296$				
		$= 0.4752$		3	18	25

Exam Solutions/Mark Scheme

SECTION B.				Qu total
1.	$R = 20 \quad C = 0.05 \quad E = 20e^{-3t}$ $I(0) = 0$ $RI + \frac{1}{C} \int I dt = E(t)$ $R \frac{dI}{dt} + \frac{1}{C} I dt = \frac{dE}{dt}$ Since $E(t) = 20e^{-3t}$ $\frac{dE}{dt} = -60e^{-3t}$ $20 \frac{dI}{dt} + \frac{1}{0.05} I dt = -60e^{-3t}$ $\frac{dI}{dt} + I dt = -3e^{-3t}$ $\int P dt = \int 1 dt = t$ $IF = e^t$ $\therefore Ie^t = \int e^t (-3e^{-3t}) dt$ $= \int -3e^{-2t} dt$ $Ie^t = \frac{3}{2} e^{-2t} + C$ Since $I(0) = 0$ $0 = \frac{3}{2} e^0 + C$ $C = -\frac{3}{2}$ $\therefore Ie^t = \frac{3}{2} e^{-2t} - \frac{3}{2}$ or $I = \frac{3}{2} e^{-3t} - \frac{3}{2} e^{-t}$			
		2		
		2		
		2		
		2		
		2		
		2		
		3		
		3		
		2		
		2		
		3		25

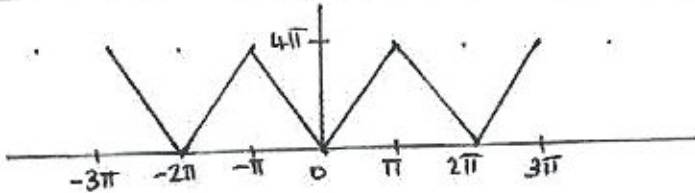
Exam Solutions/Mark Scheme

				Qu total
2.	$y'' + 3y' + 2y = e^t$ $y=0, y'=1$ when $t=0$			
	$(s^2\bar{y} + s y_0 - y_1) + 3(s\bar{y} - y_0) + 2\bar{y} = \frac{1}{s-1}$	3		
	$s^2\bar{y} - 1 + 3s\bar{y} + 2\bar{y} = \frac{1}{s-1}$	2		
	$\bar{y}(s^2 + 3s + 2) = \frac{1}{s-1} + 1$	2		
	$\bar{y}(s+1)(s+2) = \frac{1+s-1}{s-1}$	2		
	$\bar{y} = \frac{s}{(s-1)(s+1)(s+2)}$	2		
	$\frac{s}{(s-1)(s+1)(s+2)} = \frac{A}{(s-1)} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$	1		
	$s = A(s+1)(s+2) + B(s-1)(s+2) + C(s-1)(s+1)$	2		
	put $s = -1$			
	$-1 = A(0) + B(-2)(-1) + C(0)$			
	$-1 = -2B \rightarrow B = 0.5$	2		
	put $s = -2$			
	$-2 = A(0) + B(0) + C(-3)(-1)$			
	$-2 = 3C \rightarrow C = -\frac{2}{3}$	2		
	put $s = 1$			
	$1 = A(2)(3) + B(0) + C(0)$			
	$1 = 6A \rightarrow A = \frac{1}{6}$	2		
	$\therefore \mathcal{L}^{-1} \left\{ \frac{1}{6(s-1)} + \frac{1}{2(s+1)} - \frac{2}{3(s+2)} \right\}$	2		
	$y = \frac{1}{6} e^t + \frac{1}{2} e^{-t} - \frac{2}{3} e^{-2t}$	3		25

Exam Solutions/Mark Scheme

			Qu total
$\frac{dI}{dt} = -0.858 A e^{-0.858t} - 29.142 B e^{-29.142t}$	4		
$I'(0) = 10$			
$10 = -0.858 A e^0 - 29.142 B e^0$	2		
$10 = -0.858 A - 29.142 B$	1		
$\textcircled{1} \times 0.858$			
$0 = 0.858 A + 0.858 B$	1		
$10 = -28.284 B$			
$B = -0.354$	1		
$0 = A - 0.354$			
$A = 0.354$	1		
$\therefore I = 0.354 e^{-0.858t} - 0.354 e^{-29.142t}$	2		25

Exam Solutions/Mark Scheme

			Qu total
4	 Even function $\therefore b_n = 0.$ $a_0 = \frac{2}{\pi} \int_0^{\pi} 4x \, dx$ $= \frac{2}{\pi} \left[\frac{4x^2}{2} \right]_0^{\pi}$ $= \frac{2}{\pi} \left[2(\pi)^2 \right] - \frac{2}{\pi} \left[2(0) \right]$ $a_0 = 4\pi$ $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$ $= \frac{2}{\pi} \int_0^{\pi} 4x \cos nx \, dx$ Let $u = 4x$ $\frac{dv}{dx} = \cos nx$ $\frac{du}{dx} = 4$ $v = \frac{1}{n} \sin nx$ By parts: $\int 4x \cos nx \, dx = (4x) \left(\frac{1}{n} \sin nx \right)$ $- \int \frac{1}{n} \sin nx \cdot 4 \, dx$	3 1 2 1 4 3	

Exam Solutions/Mark Scheme

		Qu total
$= \frac{4x}{n} \sin nx - \int \frac{4}{n} \sin nx dx$		
$= \frac{4x}{n} \sin nx + \frac{4}{n^2} \cos nx$	3	
$\therefore a_n = \frac{2}{\pi} \left[\frac{4x}{n} \sin nx + \frac{4}{n^2} \cos nx \right]_0^{\pi}$	2	
$= \frac{2}{\pi} \left[\frac{4\pi}{n} \sin(n\pi) - \frac{4}{n^2} \cos(n\pi) \right]$		
$- \frac{2}{\pi} \left[\frac{4\pi}{n} \sin 0 + \frac{4}{n^2} \cos 0 \right]$	2	
$= \frac{2}{\pi} \left[\frac{4}{n^2} \cos(n\pi) \right] - \frac{2}{\pi} \left[\frac{4}{n^2} \right]$		
$a_n = \frac{8}{\pi n^2} [\cos(n\pi) - 1]$	1	
$n=1, a_1 = \frac{8}{\pi(1)^2} [\cos \pi - 1] = -\frac{16}{\pi(1)^2}$		
$n=2, a_2 = \frac{8}{\pi(2)^2} [\cos 2\pi - 1] = 0$		
$n=3, a_3 = \frac{8}{\pi(3)^2} [\cos 3\pi - 1] = -\frac{16}{\pi(3)^2}$		
$a_4 = 0.$	2	
$\therefore f(x) = \frac{1}{2}(4\pi) - \frac{16}{\pi(1)^2} \cos x$		
$- \frac{16}{\pi(3)^2} \cos 3\pi - \frac{16}{\pi(5)^2} \cos 5x$		
$= 2\pi - \frac{16}{\pi} \cos x - \frac{16}{9\pi} \cos 3\pi - \frac{16}{25\pi} \cos 5x \dots$	1	
		25