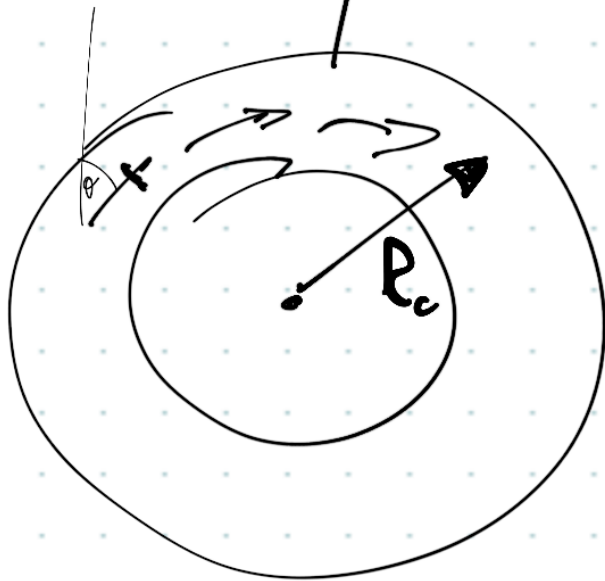


Gyroscopic effects during yaw.

*

What is the yaw rate ?

Ω



$$V = r\omega$$

← wheels

So for yaw rate

$$V = R_c \Omega$$

V = Velocity

R_c = Corner Radius

Ω = yaw rate

$$\omega = V/r$$

$$\Omega = V/R_c$$

So what about gyroscopic torque/moment?

$$G_m = I \omega \Omega \cos \phi$$

During yaw, we can calculate the gyroscopic torque from the wheels

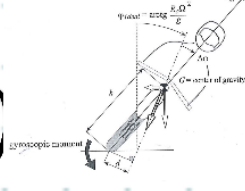
Assume: Wheel Radius $R_f = R_r = 0.32 \text{ m}$

Moment of Inertia $I_f = I_r = 0.6 \text{ kg m}^2$

Mass of Bike 200 kg

Speed $= 40 \text{ ms}^{-1}$

Corner Radius $= 200 \text{ m}$

So the gyro torque is  yaw

$$G_m = I \omega \Omega \cos \phi$$

What do we need?

$$(V = r\omega)$$

wheel angular velocity $\omega = \frac{V}{r} = \frac{40}{0.32}$

So $\omega = 125 \text{ rad/sec}$

Motorcycle yaw velocity $\Omega = \frac{V}{R_c} = \frac{40}{200}$

$\Omega = 0.2 \text{ rad/s}$

Also need to lean angle ---

$$\phi_i = \tan^{-1} \left(\frac{R_c \Omega^2}{g} \right) = \tan^{-1} \left(\frac{v^2}{r g} \right)$$

$$\phi_i = 39.2^\circ$$

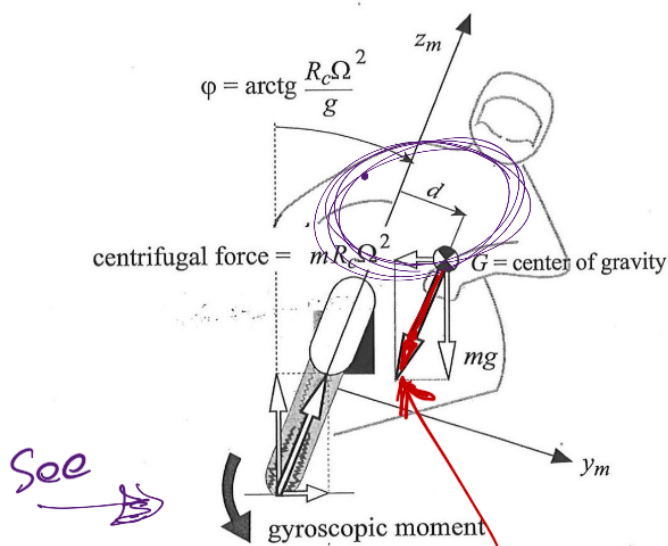
So we can now do $G_m = I \omega \Omega \cos \phi$

$$\text{So } G_m = (2 \times 0.6) \times 125 \times 0.2 \times \cos 39.2$$

$$G_m = \underline{23.25 \text{ Nm}}$$

This gyro equation applies to many dynamic situations
(steering, roll rate, engine effects etc)
which you can look at in isolation.

So what does this yaw torque do?



So, due to the torque, we are no longer in equilibrium

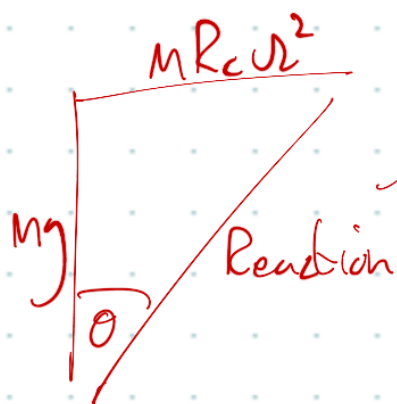
How do we restore equilibrium?

We can hang off a bit!

So how much do we lean off?

This reaction force is equal to

$$R = \sqrt{(mR_c\Omega^2)^2 + (mg)^2}$$



So we know the gyro moment, and want d .

$$G_m = F \times d = d \sqrt{(mR_c\Omega^2)^2 + (mg)^2}$$

So if we want d

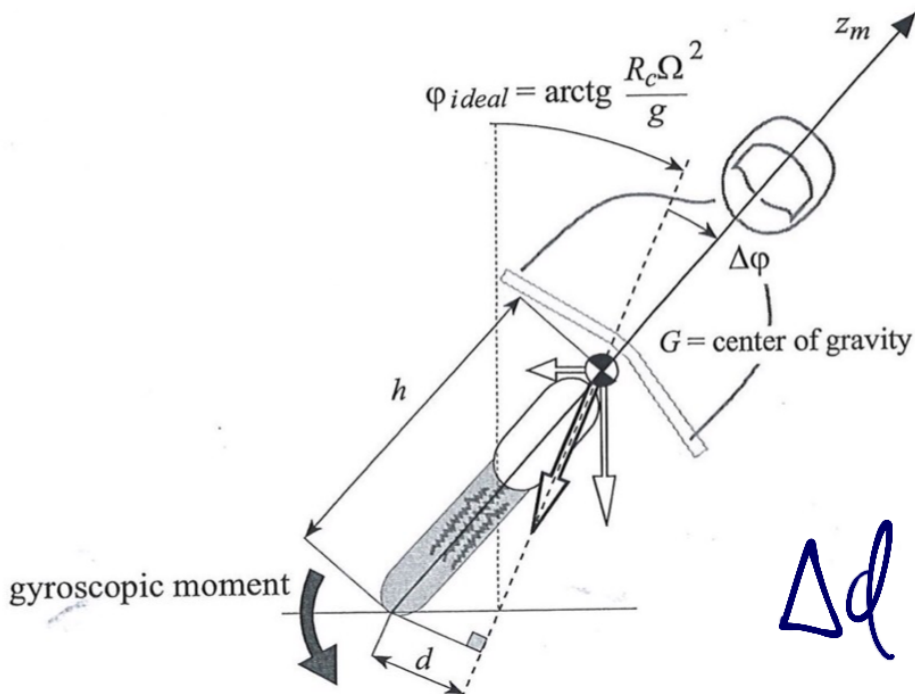
$$d = \frac{G_m}{\sqrt{(mR_c\Omega^2)^2 + (mg)^2}}$$

$$\text{or } d = \frac{I \omega \Omega \cos \phi_i}{\sqrt{(m R_c \Omega^2)^2 + (mg)^2}}$$

So for the example above, what sort of value does d have?

$$d = \frac{23 \cdot 25}{\sqrt{(200 \times 200 \times 0.2^2)^2 + (200g)^2}}$$

$$\text{so } d = 0.0091 = \underline{9 \text{ mm}}$$



Alternatively we
can alter the lean
angle of the bike

$$\Delta \phi = \sin^{-1} \left(\frac{d}{h} \right)$$

So we get a change in angle of:

$$\Delta Q = \sin^{-1} \left(\frac{0.0091}{0.6} \right)$$

Assuming a CofG height of 0.6m

$$\text{so } \underline{\Delta Q = 0.87^\circ}$$

Also interesting, in the same way, are the engine effects during yaw/roll