

2011093

"Analytical Methods"

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$$(Q1) (i) \frac{3x-1}{5} = \frac{6x+5}{9} \Rightarrow 9x(3x-1) = 5(6x+5)$$

$$27x - 9 = 30x + 25$$

$$27x - 30x = 25 + 9$$

$$-3x = 34$$

$$x = -\frac{34}{3}$$

$$(ii) 1 + \frac{5}{x} = \frac{-6}{x^2} \Rightarrow 1 + \frac{5}{x} = \frac{-6}{x^2} = 0$$

$$\frac{x^2 + 5x + 6}{x^2} = 0$$

$$x+3=0, x+2=0$$

$$x_1 = -3, x_2 = -2$$

$$x^2 + 5x + 6 = 0$$

$$x^2 + 3x + 2x + 6 = 0$$

$$x(x+3) + 2(x+3) = 0$$

$$(x+3)x(x+2) = 0$$

$$(iii) 2 \log(3x+1) = \log(x+3) + \log(x+7)$$

$$\log_{10}((3x+1)^2) = \log_{10}((x+3)(x+7))$$

$$\log_{10}((3x+1)^2) = \log_{10}(x^2 + 7x + 3x + 21)$$

$$(3x+1)^2 = x^2 + 7x + 3x + 21$$

$$9x^2 + 6x + 1 = x^2 + 10x + 21$$

$$9x^2 + 6x + 1 - x^2 + 10x + 21 = 0$$

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$$2x^2 - 4x - 20 = 0$$

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$$2x^2 - x - 5 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$x = \frac{1 \pm \sqrt{41}}{4}$$

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(b) (i)

$$8P^3Q^2 - 4P^2Q^3 + 2P^3Q$$

$$-2P^2Q(-4PQ + 2Q^2 - P)$$

$$2P^2Q(4PQ - 2Q^2 + P)$$

(ii)

$$x^2 - y^2$$

$$(x+y)(x-y)$$

(c) (i)

$$x^2 + 10x + 20 = 0$$

$$x \times (x+10) + 2(x+10) = 0$$

$$(x+10)(x+2) = 0$$

$$x_1 = -10$$

$$x_2 = -2$$

(ii)

$$2x^2 + x - 15 = 0$$

$$2x^2 + 6x - 5x - 15 = 0$$

$$2x \times (x+3) - 5(x+3) = 0$$

$$(x+3)(2x-5) = 0$$

$$x_1 = -3$$

$$x_2 = \frac{5}{2}$$

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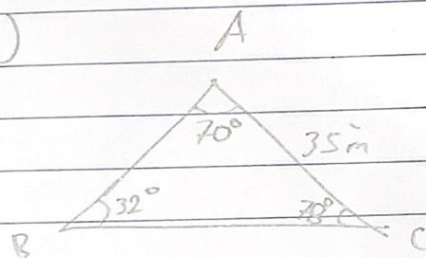
$$(iii) \quad x^2 - 400 = 0$$

$$x^2 = 400$$

$$x = \pm 20$$

$$x_1 = -20 \quad x_2 = 20$$

(Q2) (i)



$$78^\circ + 32^\circ + x^\circ = 180^\circ$$

$$x^\circ = 180 - 110$$

$$x = 78^\circ$$

$$(ii)(iii) \quad \frac{35}{\sin 32^\circ} = \frac{AB}{\sin 78^\circ} = \frac{BC}{\sin 70^\circ}$$

$$\frac{AB}{\sin 78^\circ} = \frac{35}{\sin 32^\circ}$$

$$AB = \frac{35 \sin 78^\circ}{\sin 32^\circ}$$

$$AB = 64.60m$$

$$\frac{BC}{\sin 70^\circ} = \frac{35}{\sin 32^\circ}$$

$$BC = \frac{35 \sin 70^\circ}{\sin 32^\circ}$$

$$BC = 62.06m$$

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(Q3) (i) $\sin x = 0.67$

$$0^\circ \leq x < 360^\circ$$

$$x = \sin^{-1}(0.67)$$

$$x = 42.06$$

$$x = 137.93$$

(ii) $\tan^2 x - 7 \tan x + 10 = 0$

$$\tan x = a$$

$$a^2 - 7a + 10 = 0$$

$$a = \frac{7 \pm \sqrt{49 - 4 \cdot 10}}{14}$$

$$a = \frac{7 \pm 3}{14}$$

$$a_1 = \frac{7+3}{14}$$

$$a_2 = \frac{7-3}{14}$$

$$a_1 = \frac{10}{14}$$

$$a_2 = \frac{-4}{14}$$

$$a_1 = 0.71$$

$$a_2 = -0.29$$

$$\tan x = 0.71$$

$$\tan x = -0.29$$

$$x = 35.37^\circ$$

$$x = 163.88^\circ$$

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(Q4)

$$\begin{cases} y = 5x^2 + 12x - 14 \\ y = 4x^2 + 3x - 4 \end{cases}$$

$$5x^2 + 12x - 14 = 4x^2 + 3x - 4$$

$$x = -10 \quad x = 1$$

$$y = 4 \times (-10)^2 + 3 \times (-10) - 4$$

$$y = 4 \times 1^2 + 3 \times 1 - 4$$

$$y = 366 \quad y = 3$$

$$x_1 = -10, y = 366$$

$$x_2 = 1, y = 3$$

(Q5)

$$z_1 = 3 + 7j, \quad z_2 = 2 - 10j$$

$$(a) (i) \quad z_1 + z_2 = 5 - 3j$$

$$(ii) \quad z_1 - z_2 = (3 + 7j) - (2 - 10j) =$$

$$(iii) \quad \frac{z_1}{z_2} = \frac{3 + 7j}{2 - 10j} \quad \left(\frac{1}{5} \right)$$

(b)

$$7x^2 = -2x - 11$$

$$7x^2 + 2x + 11 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 7 \times 11}}{2 \times 7}$$

$$x = \frac{-2 \pm \sqrt{4 - 308}}{14}$$

$$x = \frac{-2 \pm \sqrt{304}}{14}$$

$$x = \frac{-2 + 4\sqrt{19}i}{14}$$

$$x = \frac{-2 - 4\sqrt{19}i}{14}$$

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$$x_1 = -\frac{1}{7} + \frac{2\sqrt{19}i}{7}$$

$$x_2 = -\frac{1}{7} - \frac{2\sqrt{19}i}{7}$$

$$(c) (5+3j)^4$$

$$z = 5+3j$$

$$|z| = \sqrt{(5)^2 + (3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34}$$

$$= 5.83$$

$$r = 5.83$$

$$\cos \theta = \frac{5}{5.83}$$

$$\cos \theta = 0.857$$

$$\sin \theta = \frac{3}{5.83}$$

$$\sin \theta = 0.514$$

$$\tan \theta = \frac{0.857}{0.514}$$

$$\tan \theta = 1.667$$

$$\theta = \arctan(1.667)$$

$$\theta = 1.030 \quad \theta^\circ = 59.04$$

$$z = 5.83(\cos 59.04^\circ + j \sin 59.04^\circ)$$

$$(5.83)^4 (\cos 4 \times 59.04^\circ + j \sin 4 \times 59.04^\circ)$$

$$1.155 \cdot 25 (-0.56 + j(-0.83))$$

$$(-646.94 - 959.54j)$$

$$(2.6) (a) (i) y = 9x^3 + 7x^2 - x + 4$$

$$y = 9 \times 0^3 + 7 \times 0^2 - 0 + 4$$

$$y = 4$$

$$(ii) y = (2x+3)(3x-5)$$

$$(2x+3) \times (3x-5) = 0$$

$$2x+5=0 \quad 3x-5=0$$

$$x_1 = -\frac{5}{2} \quad x_2 = \frac{5}{3}$$

$$(iii) y = 2 \ln(x) + \frac{5}{x}$$

$$y = 2 \ln$$

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$$(iv) \quad y = 7 \sin x + 3 \cos 2x + \tan \frac{x}{2}$$

$$y = 7 \sin(0) + 3 \cos(2 \times 0) + \tan\left(\frac{0}{2}\right) \quad (y=3)$$

$$(v) \quad y = x^5 \sin 2x$$

$$x^5 \sin 2x = 0$$

$$x^5 = 0 \quad \sin 2x = 0 \quad x = 0 \quad x = \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

$$(vi) \quad y = \frac{e^{3x}}{x^3}$$

$$0 = \frac{e^{3x}}{x^3}$$

$$e^{3x} = 0 \quad x \in \emptyset$$

$$(vii) \quad y = (5x^2 - 2x + 1)^5$$

$$(5x^2 - 2x + 1)^5 = 0$$

$$5x^2 - 2x + 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 5 \times 1}}{2 \times 5}$$

$$x = \frac{2 \pm \sqrt{-16}}{10} \quad x \in \mathbb{R}$$

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Q7 @ (i) $\int x^5 + 2x^2 + x - 10 \, dx$

$$\frac{x^6}{6} + \frac{2x^3}{3} + \frac{x^2}{2} - 10x + C, \text{ C.R.}$$

(ii) $\int (2x+5)(x^2-1) \, dx$

$$\int 2x^3 - 2x + 5x^2 - 5 \, dx$$

$$\frac{x^4}{2} - x^2 + \frac{5x^3}{3} - 5x + C, \text{ C.R.}$$

(iii) $\int_2^5 2x^2(x^2+3) \, dx$

$$2x \int x^2(x^2+3) \, dx$$

$$2x \int x^4 + 3x^2 \, dx$$

$$2 \left(\int x^4 \, dx + \int 3x^2 \, dx \right)$$

$$2 \left(\frac{x^5}{5} + x^3 \right)$$

$$\frac{2x^5}{5} + 2x^3$$

$$\left(\frac{2x^5}{5} + 2x^3 \right) \Big|_2^5$$

$$\frac{2 \times 5^5}{5} + 2 \times 5^3 - \left(\frac{2 \times 2^5}{5} + 2 \times 2^3 \right) = \frac{7356}{5}$$

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$$(iv) \int 5 \sin 3x - \cos \frac{x}{3} dx$$

$$\int 5 \sin 3x dx - \int \cos \frac{x}{3} dx$$

$$-\frac{5 \cos(3x)}{3} - 3 \sin \frac{x}{3} + C, C \in \mathbb{R}$$