



A note on the deep penetration of projectiles into concrete



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ABSTRACT

A simple equation is presented herein to predict the deep penetration of concrete targets struck normally by projectiles within a unified framework. The effects of various parameters such as nose shape, impact velocity and unconfined compressive strength are considered in the formulation by the mean resistive pressure which consists of two parts, namely, cohesive static resistive pressure due to the elastic–plastic deformations and the dynamic resistive pressure arising from velocity effects. It is demonstrated that the present equation is in good agreement with available experimental data for the penetration of concrete targets. It is also demonstrated that the mean penetration resistance of concrete materials with unconfined compressive strengths ranging from 75 MPa to 150 MPa is almost the same which may have serious implication for the design of concrete targets against projectile penetration.

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1. Introduction

Penetration and perforation of targets by projectiles involve highly complex processes which have been investigated experimentally for more than two centuries and analytically mainly during the last few decades. Accounts of this work can be found in the reviews by Backman and Goldsmith [1], Zukas [2], Anderson and Bodner [3], and Corbett et al. [4]. Depending on impact velocity, material and geometric properties of both projectiles and targets, theoretical models (analytical and numerical) have been proposed over the years to predict the level of penetration in thick targets (depth of penetration or DOP) or the impact conditions for the perforation of plates (ballistic limits), as can be seen from these reviews. However, many of the analytical models are single-mechanism models that have so far enjoyed limited applications. Numerical simulations have been successful in predicting the response of targets to projectile impact but, unfortunately, they still require considerable resources in terms of computing time. On the other hand, from the engineering point of view there is still considerable interest in the development of empirical or semi-empirical equations for the penetration and perforation of plates, as noted in Refs. [1,4,5].

Empirical formulae have been suggested to predict penetration depths and ballistic limits of concrete targets subjected to impact by projectiles at normal incidence for more than half a century [6]. Forrestal et al. [7] proposed an empirical equation for the

penetration of ogival-nosed projectiles into concrete targets. The empirical equation was developed on the basis of the assumption that the penetration process can be divided into two regions, i.e. cratering region and tunnelling region. In the cratering region the resistive force was assumed to be proportional to the depth of penetration whilst in the tunnelling region the resistive force had a similar form to that estimated from spherical cavity expansion approximation. The final form of the empirical equation depended on the parameter, S , which was evaluated to be related to the unconfined compressive strength (f'_c) of concrete targets only. On the other hand, Chen and Li [8,9] suggested semi-empirical formulae using two dimensionless parameters, namely, I and N which were defined as impact factor and nose shape factor, respectively. Chen and Li's study represents an extension of the work by Forrestal et al. [7]. The UMIST formulae [6,10], which considered nose shape effects and a rate-dependent characteristic strength of concrete, were developed on the analysis of the test results acquired by nuclear power industry. In other words, the UMIST formulae are most likely applicable to the situation where reinforced concrete slabs are struck normally by large mass low velocity missiles.

The objective of this paper is to present a simple equation to predict the deep penetration of concrete targets struck normally by projectiles with different nose shapes in a wide range of impact velocities within a unified framework. The equation is derived on the basis of the assumption that the mean resistive pressure can be divided into two parts: quasi-static resistive pressure due to the elastic–plastic deformations and dynamic resistive pressure arising from velocity effects. The present equation is compared with available experimental results and discussed.

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2. Formulation of the problem

As discussed in Refs. [11–13], the mean pressure (σ) that is applied normally to the nose surface of a projectile provided by the target material during penetration can be decomposed into two parts: the cohesive static resistive pressure (σ_s) due to the elastic–plastic deformations and the dynamic resistive pressure (σ_d) arising from velocity effects. Thus, one obtains

$$\sigma = \sigma_s + \sigma_d \quad (1)$$

It is further assumed that $\sigma_s = \alpha\sigma_t$ and $\sigma_d = \beta\sqrt{\frac{\rho_t}{\sigma_t}}V_i\sigma_t$, then Equation (1) can be rewritten as

$$\sigma = \sigma_s + \sigma_d = \left(\alpha + \beta\sqrt{\frac{\rho_t}{\sigma_t}}V_i\right)\sigma_t \quad (2)$$

where α and β are constants which can be determined theoretically or experimentally; ρ_t is the density of the target material; V_i is impact velocity. σ_t is a measure of the quasi-static target material strength, which takes different values for different target materials as discussed in Ref. [13]. For metals and alloys σ_t is yield stress; for fibre reinforced plastic laminates, it is linear elastic limit in through-thickness compression; for concrete it is shear strength. The mean resistance force of a projectile penetrating a concrete target can be expressed as

$$F = \sigma\pi(d/2)^2 \quad (3)$$

Hence, from the conversation of energy, one obtains

$$E_k = \int_0^P Fdz = \int_0^P \pi\left(\frac{d}{2}\right)^2 \sigma dz = \frac{\pi d^2 P \sigma}{4} \quad (4)$$

Rearranging Equation (4) gives

$$P_d = \frac{P}{d} = \left(\frac{4}{\pi}\right) \frac{E_k}{\sigma d^3} \quad (5)$$

where P_d is non-dimensional depth of penetration, $E_k = (1/2)MV_i^2$ is initial kinetic energy of the projectile with M being projectile mass and d is projectile diameter.

3. Comparisons and discussion

The values of various parameters in the equations derived in the previous sections can be determined either theoretically or experimentally. For concrete targets [13], the values of the various parameters are given in Table 1.

When a flat-nosed projectile penetrates a semi-infinite concrete target a “dead zone” is usually observed ahead of the flat-nosed projectile [6] [10] [14]. To a first approximation, a flat-nosed

projectile may be treated as a conical-nosed projectile with the same diameter and cone angle of $\theta = 90^\circ$ and, therefore, the values of α and β for a flat-nosed projectile can be estimated by α and β for a conical-nosed missile with $\theta = 90^\circ$.

According to the above discussion, two scenarios may occur during the penetration of a conical-nosed projectile penetrating a semi-infinite concrete target. One scenario is that the cone angle of a conical-nosed projectile is less than 90° (i.e. $\theta < 90^\circ$) and the value of β in Equation (2) is given by $\beta = 2\sin(\theta/2)$ (see Table 1); the other is that the cone angle of a conical-nosed projectile is equal to or greater than 90° (i.e. $\theta \geq 90^\circ$) and in this case the conical-nosed projectile can be treated as that with $\theta = 90^\circ$, the value of β in Equation (2) can be estimated by $\beta_c = 2\sin(90^\circ/2) = \sqrt{2}$ approximately (also see Table 1).

In Table 1, E is the modulus of elasticity of the target material, ν is the Poisson's ratio, Y is the shear strength. If the Young's modulus (E) is not available, it can be determined by the ACI 315-95 and 363 code [15,16], which is given by the following equations

$$E = \begin{cases} 4733\sqrt{f'_c}, & f'_c < 21 \text{ MPa} \\ 3300\sqrt{f'_c} + 6900, & f'_c \geq 21 \text{ MPa} \end{cases} \quad (6)$$

where f'_c is the unconfined compressive strength of the concrete, f'_c and E are in MPa. Hence, the only parameter to be determined is Y which can be estimated by the following equations on the basis of the analysis of available test data, namely

$$Y = \begin{cases} 1.4f'_c + 45, & f'_c \leq 75 \text{ MPa} \\ 150, & 75 \text{ MPa} < f'_c < 150 \text{ MPa} \\ f'_c, & f'_c \geq 150 \text{ MPa} \end{cases} \quad (7)$$

Equation (7) can be presented graphically in Fig. 1.

Comparisons are made between the normalized experimental data available in the literature and the normalized depths of penetration predicted by the present model (Equation (5)) in Fig. 2 for flat-nosed, conical-nosed projectiles and in Fig. 3 for ogival-nosed missiles. It is seen from these figures that good agreement is obtained between the present model predictions (Equation (5)) and the available test data. It should be mentioned here that the influence of reinforcing steel has not been taken into account in the present formulation based on the assumption that the only function of reinforcing steel bar is to control radial cracks from circumferential tensile stresses and virtually has no influence on

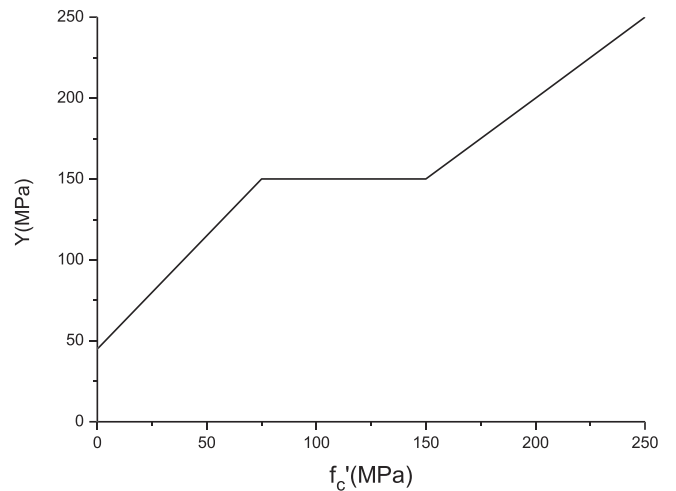
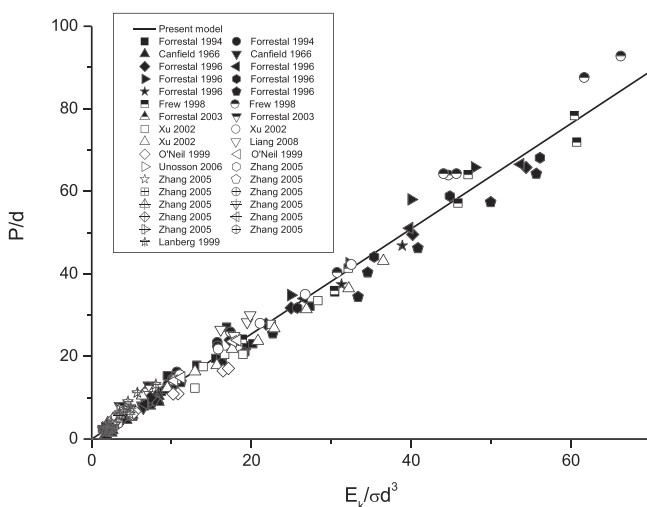
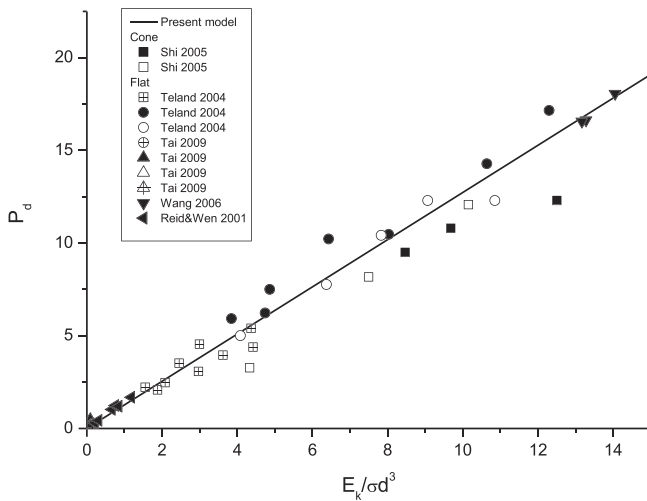


Fig. 1. Relationship between Y and f'_c .

Table 1
Values of various parameters for concrete targets.

	α	β	σ_t
Conical-nosed ($\theta < 90^\circ$)	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$2\sin\theta/2$	Y
Conical-nosed ($90^\circ \leq \theta < 180^\circ$)	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$\sqrt{2}$	Y
Flat-nosed	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$\sqrt{2}$	Y
Ogival-nosed	$\frac{2}{3} \left[1 + \ln \frac{E}{3(1-\nu)Y} \right]$	$3/4\psi$	Y
Hemispherical-nosed	$\frac{2}{3} \left[1 + \ln \frac{E}{3(1-\nu)Y} \right]$	$3/2$	Y



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