

11/05/20 - 13:00

Advanced Stress Analysis & FEA (M6X00420)

Ifan Adam Hopkins - 1701085

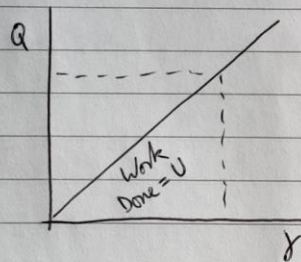
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(2) (a) (i) Shear modulus, $G = \frac{\text{Shear stress}}{\text{Shear strain}}$

$$G = \frac{\tau}{\gamma} \quad \therefore \gamma = \frac{\tau}{G} \quad \text{but} \quad \tau = \frac{Q}{A}$$

$$\therefore \gamma = \frac{Q}{AG}$$

(a) (ii)



$$U = \frac{1}{2} Q \cdot \gamma$$

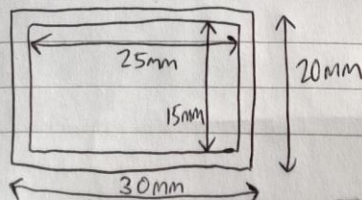
$$U = \frac{1}{2} Q \left(\frac{Q}{AG} \right) \quad \text{for small elements}$$

$$\therefore U = \frac{Q^2}{2AG}$$

$$\text{Total strain energy} = \int_0^L \frac{Q^2}{2AG} ds$$

$$\therefore U = \frac{Q^2 L}{2AG}$$

(b) (i)



$$\text{Area} = (0.03 \times 0.02) - (0.025 \times 0.015)$$

$$\text{Area} = 2.25 \times 10^{-4} \text{ m}^2$$

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$$U = \frac{\sigma^2 AL}{2E} = \frac{(480 \times 10^6)^2 (2.25 \times 10^{-4}) (2.4)}{2(190 \times 10^9)}$$

$$\therefore U = 327.4 \text{ J}$$

(b)(ii) $PE = mgh$ where $W = mg$
 $PE = Wh$

Potential energy is converted to strain energy and is absorbed by the box section.

$$U = \frac{P^2 L}{2AE} = \frac{\sigma^2 AL}{2E} = \frac{\sigma^2 V}{2E}$$

$$\therefore \frac{\sigma^2 V}{2E} = Wh \quad \therefore h = \frac{\sigma^2 V}{2EW}$$

(b)(iii) Volume = $(2.25 \times 10^{-4}) 2.4 = 5.4 \times 10^{-4} \text{ m}^3$
Weight = $(20) 9.81 = 196.2 \text{ N}$

$$\sigma^2 = \frac{2hEW}{V} \quad \therefore \sigma = \sqrt{\frac{2hEW}{V}}$$

$$\sigma = \sqrt{\frac{2(0.45)(190 \times 10^9)(196.2)}{5.4 \times 10^{-4}}} = \sqrt{\frac{3.355 \times 10^{13}}{5.4 \times 10^{-4}}}$$

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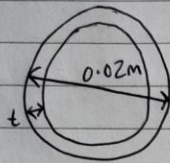
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$$\sigma = 249.26 \times 10^6 \text{ N}$$

(b)(iv)



$$h = \frac{\sigma^2 V}{2EW}$$

$$\therefore V = \frac{2hEW}{\sigma^2}$$

$$V = \frac{2(0.45)(190 \times 10^9)(196.2)}{(249.26 \times 10^6)^2} = \frac{3.355 \times 10^{13}}{(249.26 \times 10^6)^2}$$

$$V = 5.40 \times 10^{-4} \text{ m}^3$$

$$V = \pi(R^2 - r^2) \times L$$

$$5.40 \times 10^{-4} = \pi(0.02^2 - r^2) \times 2.4$$

$$\frac{5.40 \times 10^{-4}}{\pi \times 2.4} = 4 \times 10^{-4} - r^2$$

$$4 \times 10^{-4} - 7.162 \times 10^{-5} = r^2$$

$$3.282 \times 10^{-4} = r^2$$

$$\therefore r = \sqrt{3.282 \times 10^{-4}} = 0.0181 \text{ m}$$

$$\therefore r = 0.0181 \text{ m}$$

$$\therefore \text{Thickness} = 0.02 - 0.0181 = 1.9 \times 10^{-3} \text{ m}$$

$$= 1.9 \text{ mm}$$

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$$(3) (a) \quad \frac{T}{J} = \frac{\tau}{R} \quad \therefore T = \frac{\tau J}{R}$$

$$\text{where } J = \frac{\pi}{32} (D^4 - d^4)$$

$$J = \frac{\pi}{32} (0.075^4 - 0.065^4) = 1.354 \times 10^{-6}$$

$$\therefore T = \frac{\tau (1.354 \times 10^{-6})}{0.0375} \quad \left(\begin{array}{l} \text{Outer radius used} \\ \text{Average radius used } 0.035 \text{ m} \end{array} \right)$$

$$\therefore T = \frac{3.611 \times 10^{-5}}{3.861 \times 10^{-5}} (\tau)$$

TRESCA

$$\sigma_1 - \sigma_3 = \frac{\sigma_y}{n}$$

$$\text{Where } \sigma_1 = \tau_{xy} \quad \text{and} \quad \sigma_3 = -\tau_{xy}$$

$$\tau_{xy} - (-\tau_{xy}) = \frac{325 \times 10^6}{6}$$

$$\therefore 2 \tau_{xy} = 54.167 \times 10^6$$

$$\therefore \tau_{xy} = 27.084 \times 10^6 \text{ N/m}^2$$

$$\text{Since } T = 3.611 \times 10^{-5} \tau_{xy} \quad T = 3.611 \times 10^{-5} \tau_{xy}$$

$$T = (3.611 \times 10^{-5}) (27.084 \times 10^6)$$

$$T = 978.0 \text{ Nm}$$

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(b) VON MISES failure criterion is:

$$2\sigma_y^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad \text{but } \sigma_2 = 0$$

$$2 \left(\frac{\sigma_y}{n} \right)^2 = \left[(\tau_{xy})^2 + (\tau_{xy})^2 + (-\tau_{xy} - \tau_{xy})^2 \right]$$

$$2 \left(\frac{325 \times 10^6}{6} \right)^2 = 6 \tau_{xy}^2$$

$$2 (2.934 \times 10^{15}) = 6 \tau_{xy}^2$$

$$\tau_{xy} = \sqrt{9.78 \times 10^{14}} = 31.273 \times 10^6 \text{ N/m}^2$$

$$\text{Since } T = 3.611 \times 10^{-5} \tau_{xy}$$

$$T = (3.611 \times 10^{-5}) (31.273 \times 10^6)$$

$$T = 1129.3 \text{ Nm}$$

$$(c) \quad T = \frac{\tau J}{R} \quad \text{where } J = \frac{\pi}{32} (D^4 - d^4)$$

$$J = \frac{\pi}{32} (0.05^4 - 0.044^4)$$

$$J = 2.456 \times 10^{-7}$$

$$T = \frac{\tau (2.456 \times 10^{-7})}{(0.05/2)}$$

outer
(Average radius used - ~~0.047~~)

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$$T = \frac{9.824 \times 10^{-6} \tau}{\cancel{1.045 \times 10^{-5}}}$$

VON MISES FAILURE CRITERION IS:

$$2\sigma_y^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad \text{but } \sigma_2 = 0$$

$$2 \left(\frac{\sigma_y}{n} \right)^2 = \left[(\tau_{xy})^2 + (\tau_{xy})^2 + (-\tau_{xy} - \tau_{xy})^2 \right]$$

$$2 \left(\frac{880 \times 10^6}{2.75} \right)^2 = 6 \tau_{xy}^2$$

$$2 (1.024 \times 10^{17}) = 6 \tau_{xy}^2$$

$$\tau_{xy} = \sqrt{3.413 \times 10^{16}} = 184.743 \times 10^6 \text{ N/m}^2$$

Since $\tau = \cancel{1.045 \times 10^{-5}} \tau_{xy}$ $T = 9.824 \times 10^{-6} \tau$

$$T = \frac{9.824 \times 10^{-6}}{\cancel{1.045 \times 10^{-5}}} (184.743 \times 10^6)$$

$$T = \cancel{1930.6 \text{ Nm}} \quad 1814.9 \text{ Nm}$$

- (d) According to von Mises, the low carbon steel hollow shaft could transmit a maximum torque of ~~1129~~ Nm. According to von Mises, the Ti6Al4V titanium alloy 1815 hollow shaft could transmit a maximum torque of ~~1815~~ Nm. Therefore, according to von Mises the titanium alloy shaft can transmit ~~1815~~ approximately

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60.7%

~~larger~~ larger amount of torque compared to the low carbon steel shaft.

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(4) (a) Complete section

$$\begin{aligned} I_{XX} &= 2 \times I_{XX-\text{PLATES}} + 2 \times I_{XX-\text{C-CHANNEL}} \\ &= 2 \left[\frac{bd^3}{12} + Ah^2 \right] + 2 (26.2 \times 10^6) \\ &= 2 \left[\frac{300(12)^3}{12} + (3600(106)^2) \right] + 52.4 \times 10^6 \\ &= 2 (43200 + 40.4496 \times 10^6) + 52.4 \times 10^6 \\ &= \underline{\underline{133.39 \times 10^6 \text{ mm}^4}} \end{aligned}$$

$$\begin{aligned} I_{YY} &= 2 \times I_{YY-\text{PLATES}} + 2 \times I_{YY-\text{C-SECTION}} \\ &= 2 \left(\frac{bd^3}{12} \right) + 2 \left[1.57 \times 10^6 + (4400(79)^2) \right] \\ &= 2 \left(\frac{12(300)^3}{12} \right) + 2 (1.57 \times 10^6 + 27.4604 \times 10^6) \\ &= 2 (27 \times 10^6) + 2 (29.0304 \times 10^6) \\ &= \underline{\underline{112.06 \times 10^6 \text{ mm}^4}} \end{aligned}$$

$I_{XX} > I_{YY}$ Therefore, the strut will tend to buckle about the YY axis.

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$$(b) \quad k = \sqrt{\frac{I}{A}}$$

$$\text{Total area} = 2(300 \times 12) + 2(4400) \\ = 16000 \text{ mm}^2$$

$$k = \sqrt{\frac{112.06 \times 10^6}{16000}}$$

$$\therefore k = \underline{\underline{83.69 \text{ mm}}}$$

$$(c) \quad \text{Slenderness ratio} = \frac{L}{k} = \frac{8}{8.369 \times 10^{-2}}$$

$$\therefore \text{slenderness ratio} = 95.59$$

(d) PINNED - PINNED

$$I_{yy} = 112.06 \times 10^{-6} \text{ m}^4$$

$$P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (112.06 \times 10^{-6})}{8^2}$$

$$\therefore P_e = \underline{\underline{3.456 \times 10^6 \text{ N}}}$$

$$(e) \quad P_R = \frac{\sigma_y A}{1 + \left[a \left(\frac{L}{k} \right)^2 \right]}$$

$$\text{Total area} = 16000 \text{ mm}^2 \\ = 0.016 \text{ m}^2$$

$$P_R = \frac{(320 \times 10^6) 0.016}{1 + \left[\frac{1}{7500} (95.59)^2 \right]}$$

$$P_R = \underline{\underline{2.308 \times 10^6 \text{ N}}}$$

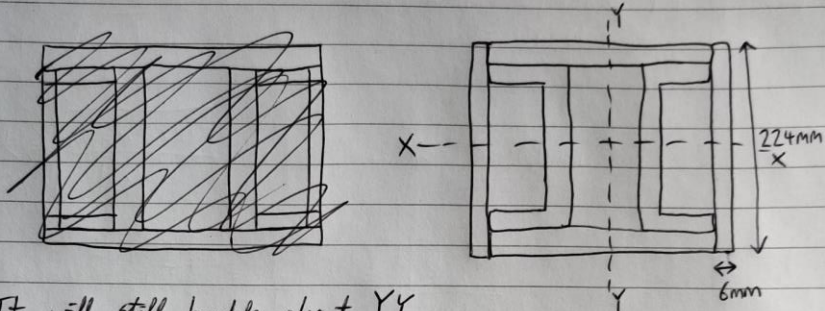
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(b)



It will still buckle about YY.

~~$I_{XX} = 2 \times I_{XX-PLATES}$~~

$$I_{YY} = 2 \times I_{YY-PLATES} + 2 \times I_{YY-SECTION} + 2 \times I_{YY-VERTICAL PLATES}$$

$$= 112.06 \times 10^6 + 2 \left(\frac{bd^3}{12} + Ah^2 \right)$$

$$= 112.06 \times 10^6 + 2 \left(\frac{224(6)^3}{12} + (224 \times 6)(153)^2 \right)$$

$$= 112.06 \times 10^6 + 2 (31.466 \times 10^6)$$

$$= 174.992 \times 10^6 \text{ mm}^4$$

$$174.992 \times 10^{-6} \text{ m}^4$$

~~Brigging~~ fixed

$$P_c = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (174.992 \times 10^{-6})}{8^2}$$

$$P_c = 5.397 \times 10^6 \text{ N}$$

$\therefore$ An increase of 56.2%