

Failure theories of composites

In a Lamina the failure theories are not based on principal stresses, but instead on the stresses seen in the material local axis.

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength (in direction 1),

$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength (in direction 1),

$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength (in direction 2),

$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength (in direction 2), and

$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength (in plane 12).

Maximum Stress Failure Theory

Similar to Tresca and Rankine as applied to isotropic materials. Failure is predicted if any of the normal or shear stresses in the local axis of a lamina exceed those of the unidirectional lamina;

$$-(\sigma_1^C)_{ult} < \sigma_1 < (\sigma_1^T)_{ult}, \text{ or}$$

$$-(\sigma_2^C)_{ult} < \sigma_2 < (\sigma_2^T)_{ult}, \text{ or}$$

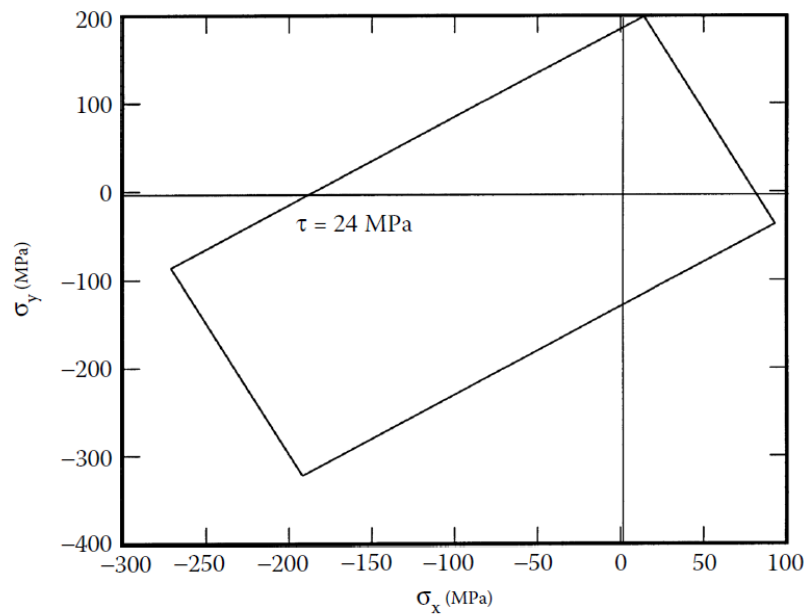
$$-(\tau_{12})_{ult} < \tau_{12} < (\tau_{12})_{ult}$$

Note that each component of stress is compared with the corresponding strength, that is to say that, each component is isolated from each other. Often a STRENGTH RATIO is defined where;

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}.$$

Note that this can be applied to ANY failure theory such that it can be understood how much the load can be increased if the lamina is safe, or how much it should be reduced if it has failed. In addition a FAILURE ENVELOPE may be plotted which is a 3D plot of the combinations of the normal and shear stress that can be applied to an angle lamina before it fails. Due to the fact that drawing 3D graphs is time consuming one may develop failure envelopes for constant shear stress and then use the two normal stress as the two axis.

Then if the applied stress is within the failure envelope the lamina is safe. A failure envelope may look as follows;



A failure envelope for a 60 degree lamina of graphite/epoxy for a constant shear stress of 24Mpa.

Maximum Strain Failure Theory

Similar to Tresca and St. Venant as applied to isotropic materials. Failure is predicted if any of the normal or shear strains in the local axis of a lamina exceed those of the unidirectional lamina;

$$-(\epsilon_1^C)_{ult} < \epsilon_1 < (\epsilon_1^T)_{ult}, \text{ or}$$

$$-(\epsilon_2^C)_{ult} < \epsilon_2 < (\epsilon_2^T)_{ult}, \text{ or}$$

$$-(\gamma_{12})_{ult} < \gamma_{12} < (\gamma_{12})_{ult}$$

Where

$(\epsilon_1^T)_{ult}$ = ultimate longitudinal tensile strain (in direction 1)

$(\epsilon_1^C)_{ult}$ = ultimate longitudinal compressive strain (in direction 1)

$(\epsilon_2^T)_{ult}$ = ultimate transverse tensile strain (in direction 2)

$(\epsilon_2^C)_{ult}$ = ultimate transverse compressive strain (in direction 2)

$(\gamma_{12})_{ult}$ = ultimate in-plane shear strain (in plane 1-2)

Tsai-Hill Failure Theory

This is based on the Von-Mises theory for Isotropic materials. The strain energy in a body consists of two parts

1. Strain energy due to a change in volume – this is called the dilation energy
2. Strain energy due to a change in shape – this is called the distortion energy.

It is assumed that failure will occur when the distortion energy is greater than the failure distortion energy of the material. Due to the fact that the unidirectional lamina is assumed to be under plane stress, then the simplified version of the equation can be used as shown below, such that the following is used to predict failure;

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}} \right]^2 - \left[\frac{\sigma_1 \sigma_2}{(\sigma_1^T)_{ult}^2} \right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ult}} \right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ult}} \right]^2 < 1.$$

Note the following;

- Unlike max stress and strain theories, Tsai-Hill considers the interaction among the three unidirectional lamina strength parameters.
- It does not distinguish between compressive and tensile strengths which can lead to it underestimating the failure stress as the tensile strength of a unidirectional lamina is generally less than the compressive strength.
- It does not give the mode of failure (for example we saw that the part would fail in shear using the maximum stress failure theory).

Tsai-Wu Failure Theory

This is also based on strain energy like Tsai-Hill but is more general than Tsai-Hill as it accounts for the compressive strengths of the lamina;

$$H_1 \sigma_1 + H_2 \sigma_2 + H_6 \tau_{12} + H_{11} \sigma_1^2 + H_{22} \sigma_2^2 + H_{66} \tau_{12}^2 + 2H_{12} \sigma_1 \sigma_2 < 1$$

Where;

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}},$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}.$$

$$H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}},$$

$$H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}.$$

$$H_6 = 0,$$

$$H_{66} = \frac{1}{(\tau_{12})_{ult}^2}.$$

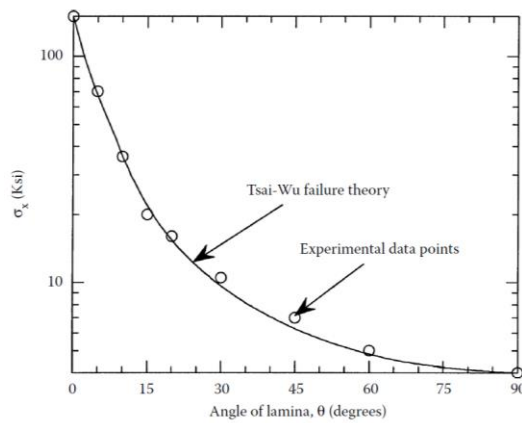
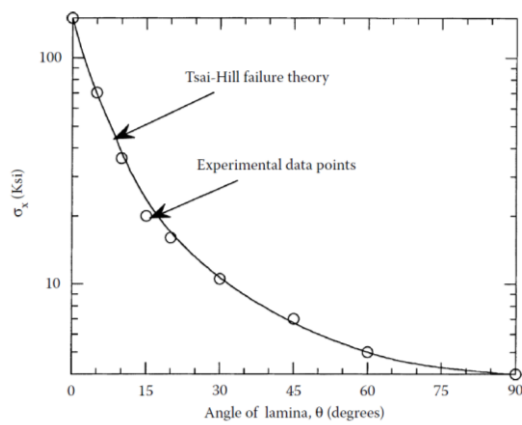
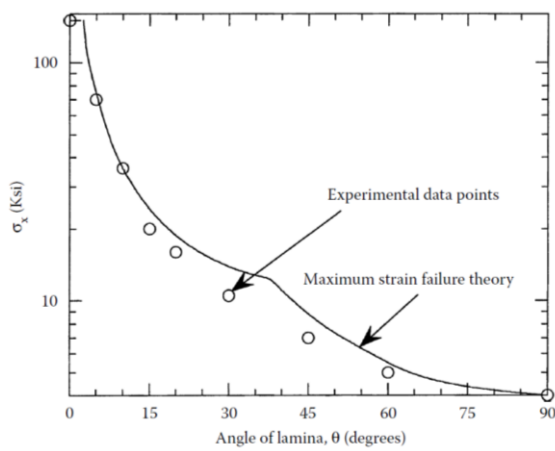
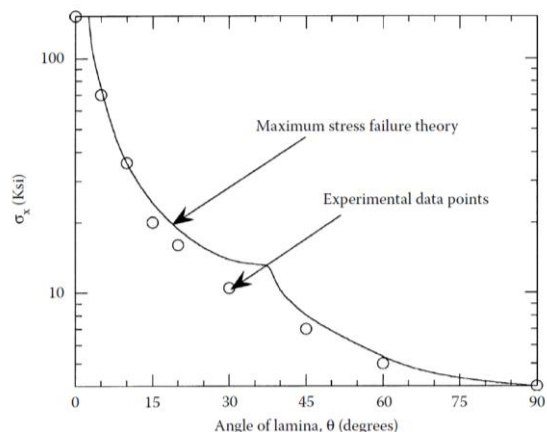
Finding H_{12} can be carried out experimentally or by the following suggested empirical methods

$$H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}^2}, \text{ per Tsai–Hill failure theory}^8$$

$$H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \text{ per Hoffman criterion}^{10}$$

$$H_{12} = -\frac{1}{2} \sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}, \text{ per Mises–Hencky criterion.}^{11}$$

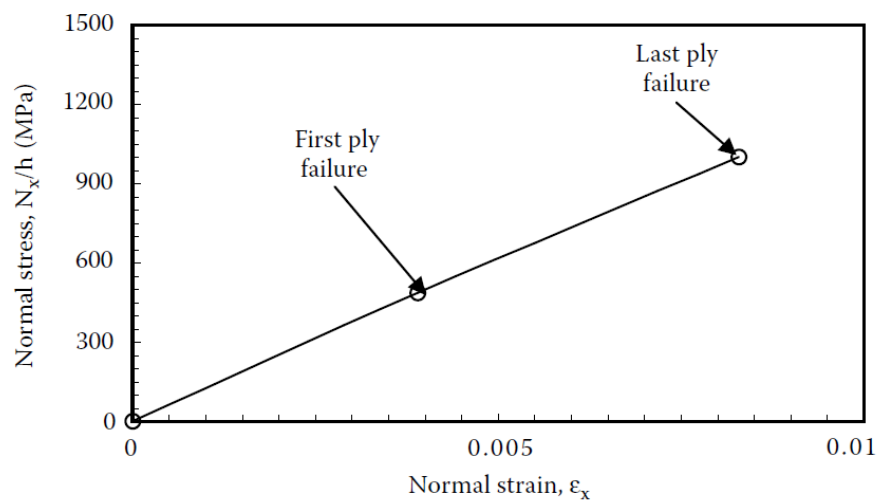
Comparison of the failure theories can be seen below



The following observations can be made;

1. The difference between the maximum stress and maximum strain is quite pronounced
2. Tsai-Hill and Tsai-Wu are in good agreement
3. The variation of the strength of the angle lamina as a function of the angle has cusps in the maximum stress and the maximum strain failure theories due to the change in failure modes.

The above failure theories apply to failure of a single Lamina. A laminate failure is characterised by the fact that even when one lamina fails, the laminate can still carry a load. In fact a bi-linear response is seen as shown below;



Where there is one stiffness up until the First Ply Failure (FPF) and then a reduced stiffness up until the Last Ply Failure (LPF).

Typical values of a unidirectional composite are shown below;

Property	Symbol	Units	Glass/ epoxy	Boron/ epoxy	Graphite/ epoxy
Fiber volume fraction	V_f		0.45	0.50	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	204	181
Transverse elastic modulus	E_2	GPa	8.27	18.50	10.30
Major Poisson's ratio	ν_{12}		0.26	0.23	0.28
Shear modulus	G_{12}	GPa	4.14	5.59	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^t)_{ult}$	MPa	1062	1260	1500
Ultimate longitudinal compressive strength	$(\sigma_1^c)_{ult}$	MPa	610	2500	1500
Ultimate transverse tensile strength	$(\sigma_2^t)_{ult}$	MPa	31	61	40
Ultimate transverse compressive strength	$(\sigma_2^c)_{ult}$	MPa	118	202	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72	67	68
Longitudinal coefficient of thermal expansion	α_1	$\mu\text{m}/\text{m}/^\circ\text{C}$	8.6	6.1	0.02
Transverse coefficient of thermal expansion	α_2	$\mu\text{m}/\text{m}/^\circ\text{C}$	22.1	30.3	22.5
Longitudinal coefficient of moisture expansion	β_1	$\text{m}/\text{m}/\text{kg}/\text{kg}$	0.00	0.00	0.00
Transverse coefficient of moisture expansion	β_2	$\text{m}/\text{m}/\text{kg}/\text{kg}$	0.60	0.60	0.60

Table 1

Questions

For the below questions use the material properties from Table 1.

1.

Find the maximum value of $S > 0$ if a stress of $\sigma_x = 2S$, $\sigma_y = -3S$, and $\tau_{xy} = 4S$ is applied to the 60° lamina of graphite/epoxy.

a. Using the maximum stress failure theory

[16.33MPa]

b. Using the maximum strain failure theory

[16.33MPa]

c. Using the Tsai-Hill failure theory

[10.94MPa]

2.

Assume that one is applying a load of

$$\sigma_x = 2 \text{ MPa}, \sigma_y = -3 \text{ MPa}, \tau_{xy} = 4 \text{ MPa}$$

to a 60° angle lamina of graphite/epoxy. Find the strength ratio using the maximum stress failure theory.

[16.33]

3.

Using the maximum stress failure theory, develop a failure envelope for the 60° lamina of graphite/epoxy for a constant shear stress of $\tau_{xy} = 24 \text{ MPa}$.

[This failure envelope can be seen in these notes]