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Valvetrain Friction Modeling Of A Motorcycle Engine

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P131652
BENG MOTORCYCLE ENGINEERING

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

Signed:

Date:

The writer would like to make a huge thank you to Malcolm McDonald in the help and assistance provided throughout the final year and the dedication and enthusiasm in which he has shown and without this, it would not have been possible.

ABSTRACT

Engine efficiency has proven to be a key point in modern motorsports with manufactures pushing the limits and exceeding efficiency much greater than that of production vehicles. Friction is a main contributor to engine efficiency and having a clear understanding of It increases performance and lowers emissions. With performance engines, its common that the valvetrain consists of direct acting valve systems to eliminate rotating masses in the engine and in rare cases having electronic valve systems. With motorbikes now exceeding almost 200mph the forces acting on the valve are high and require much stronger components to keep the wear down and care taken when studying the valve spring to keep the valve in contact with the cam.

This paper consists of measured data from an engine and how the forces react at various stages such as valve float, valve acceleration and the friction created. These methods are then validated using various methods and show what realistic figures that would occur. This paper was created in conjunction with an excel model to calculate valvetrain forces and friction within a superbike engine focusing on the cam, inlet valve and cam bucket. A cad package was used to create a model in Solidworks to show a separate method of calculating the extreme forces and validating that of the excel model.

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CHAPTER 1

INTRODUCTION

When looking at the efficiency of a combustion engine it's common for it to be around 25%-30% efficient, that means that on average around 2/3rds of an engines thermal efficiency is wasted. There are many factors contributing to this but much of it is due to the friction, these parts can vary from moving parts in the engine.

Motorcycle engines throughout the years are constantly changing and in some cases reverting to older methods. This can be seen with the Yamaha r1, from 2015 using rocker arms in the cylinder, Yamaha state that

It is far more accurate than a bucket and shim, especially when dealing with big valves, high revs and an aggressive cam profile. This accuracy, as well as its lightness, makes it more reliable. (Yamaha insight of the R1, 2015)

When most motorcycle reach 18,000 miles it involves checking valve clearances due to them becoming loose or tight due to wear. Motorcycles require a much higher maintenance requirement compared to cars and a car engine will out do the mileage of that of a motorcycle. With internal parts wearing at a much faster rate it's imperative that manufacturers focus on the reliability and quality of parts to increase the length of which the engine will last and extend the time between services

The focus on this report will be looking at the friction forces acting upon the inlet cam shafts on a Suzuki GSXR 1000cc motorcycle. The engine had been stripped to give a better access to the parts required and then numerous internal parts measured accordingly for the soul purpose of this report. Using the measurements taken from the components of the engine. A model in Microsoft excel was created to be used to form a friction model for the cams and would show potential improvements that could be made to reduce the friction loss that occurs at various stages though out the cycle.

By the end of the study a full working model should be achieved with various methods of proving the model's relativity.

CHAPTER 2

As an engine goes through its cycle it must have an intake of air to mix with fuel to combust, in this modern 16 valve overhead cam it does this via two cams. The rear cam is for intake which opens a valve letting air get sucked in and exhaust valve letting the burnt gases escape. Having the correct amount of lift is crucial when designing an engine and is the key part of controlling the engines breathing. Having a larger cam to increase lift increase the capacity of air and fuel to be mixed in the engine, air flow becomes capped by the cylinder's capability so too much lift is pointless. When the airflow is capped it becomes bleed to increase the lift further and increases the chance of complications and restrictions such as the valve hitting the piston or interfere from valve springs and retainers.

To the right in figure 1 are examples of the several types of valvetrain examples that are used in modern vehicles. As can be seen in the first image it is an overhead cam directly acting on the valves as is the GSXR engine.

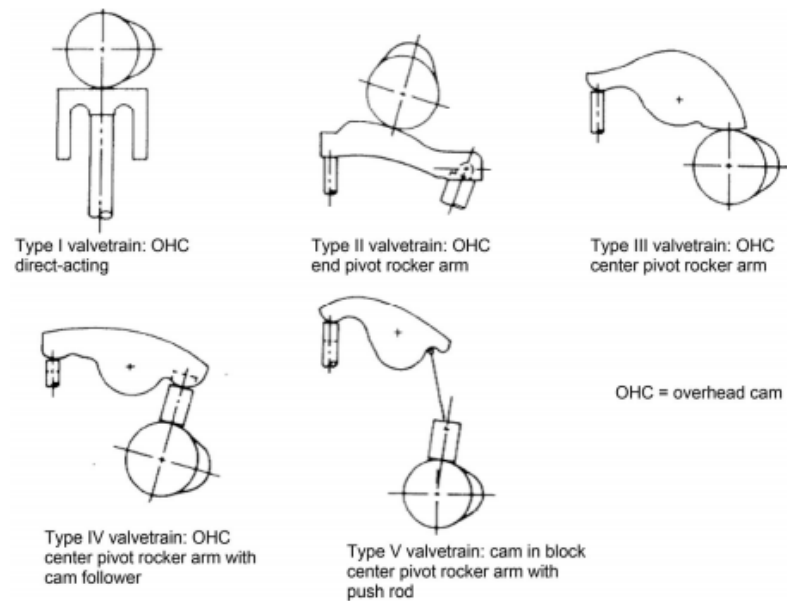


FIGURE 1

VALVE LIFT

The first stage was to measure the cam lobes using a dial gauge. This was done by removing the valves spring and retainer and replacing the spring with a much weaker spring that could be compressed by hand. Turning the cylinder head upside down and cam fitted it was then possible to use a protractor on the cam sprocket and rotate by 1 degree at a time and record the measurement indicated on the dial gauge. This was the most time effective way but not the most accurate due to human error and the dial gauge not staying vertical on the cam lobe and slipping. A more accurate way of measuring the cam would have been to get it scanned and take points at each degree however this would have been more time consuming.

The cam profile and measurements was then input in excel and a 360*360 matrix was created and then using a method normally known as trigonometric equation using SOHCAHTOA. A trigonometric function is defined as:

SOHCAHTOA can be used to aid in remembering which function to use in what circumstance - SOH stands for Sine is opposite over hypotenuse; CAH stands for Cosine is adjacent over hypotenuse; and TOA stands for Tangent is opposite over adjacent. (Stephanie Matalone examples 2012)

When looking at figure 2 it shows the angles in which the method in the matrix calculates and shows the highest possible lift at each angle.

The matrix then showed the highest possible valve lift at each degree showed in each column and was then filtered at each degree where the highest lift was taken and valve shim was deducted to make it a more accurate lift and where no lift occurs would equal to zero lift.

The matrix in excel was formed using the formulae
 $\text{Cam profile} * \cos(180 - (\phi + \theta))$

Phi and theta both go from 1-360 and cam profile was 14.25mm. A note was that excel requires the use of converting into radians for angles. At the bottom of each column a designated box known as OB max was created so single out the highest value indicating the maximum possible valve lift

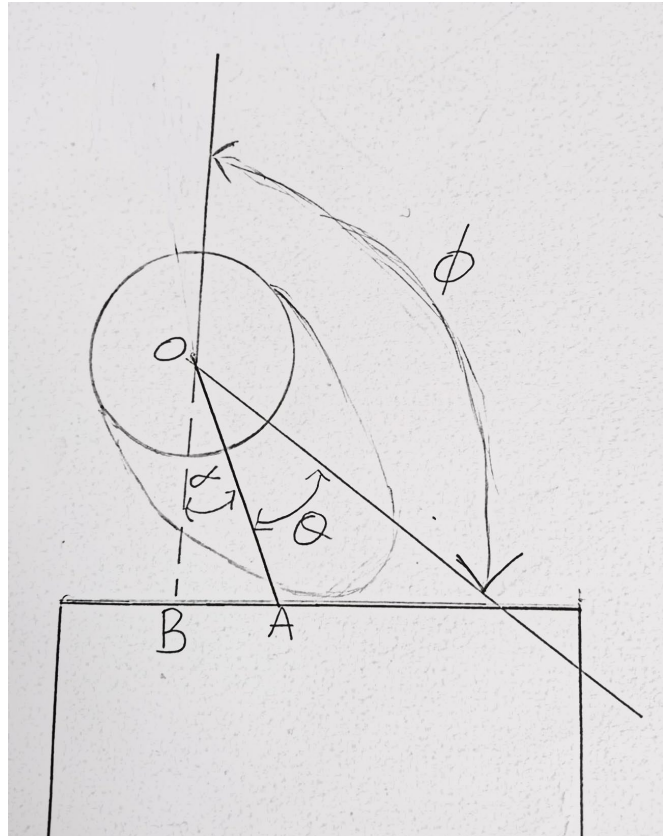
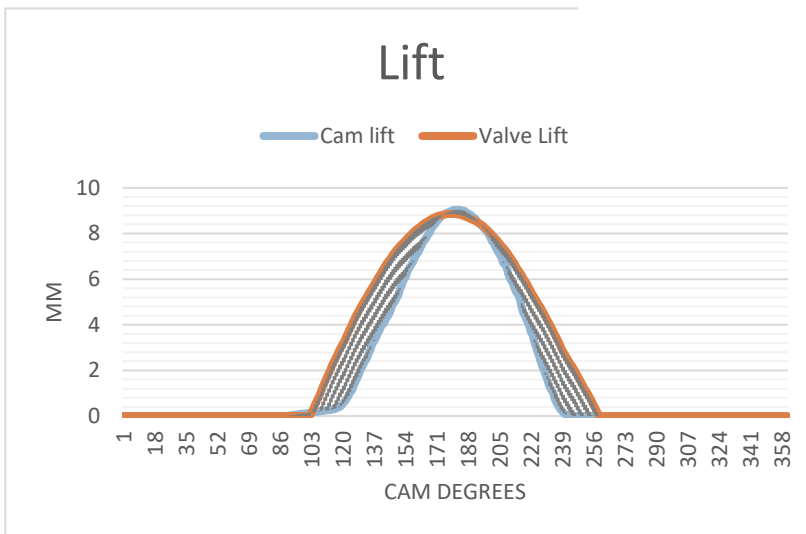


FIGURE 2



GRAPH 1

Using the highest possible lifts from each degree it was then extracted and put into a graph so that it could show cam lift vs valve lift.

The graph shows cam lift being higher than valve lift and valve lift raises at a steeper curve than that of the cam.

VALVE SPEEDS

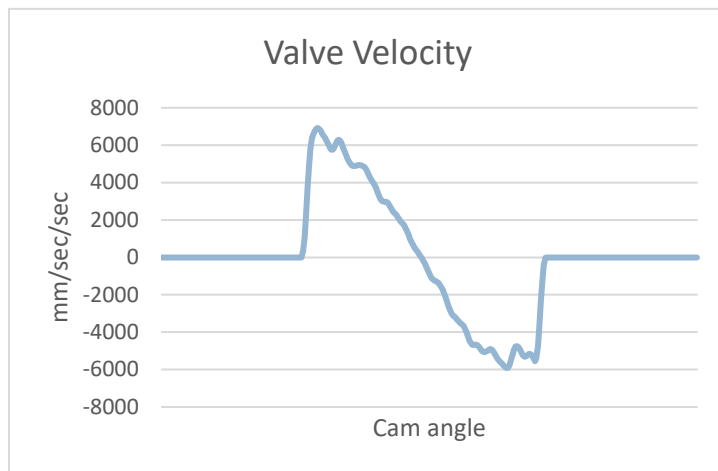
The valve lift is the first part of the model and is used to calculate the valve speeds at various RPM ranges, the faster the RPM is, the faster the valves open and close. The first step is to calculate the valve velocity, this is done by using the difference in valve lift for each degree giving a valve velocity in mm/degree. Acceleration is then calculated by the difference in valve velocity for each degree giving valve acceleration mm/degree/degree. Having the acceleration in this unit meant that it was difficult to put in to future equations

To change the units into seconds the below formulae was used in the model to convert units into mm/sec/sec

$$(\text{valve acceleration mm/deg/deg} / 10^3) * (1000 * 6)^2$$

After the units were converted a graph was plotted for each degree

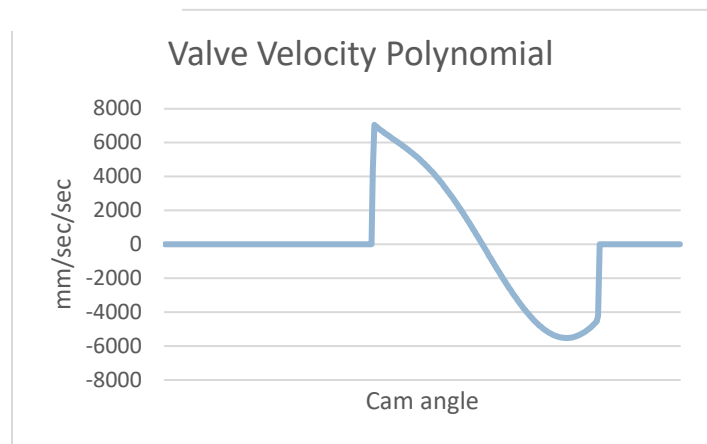
VALVE VELOCITY



GRAPH 2

From looking at the velocity in graph 2 it represents the valve motion, a positive velocity as the valve opens and then negative as it closes. To eliminate the spikes on the graph a perfect measurement is required of the cam load and in this case, it was done inaccurately due to the lines smoothness. By having data in excel, using a polynomial is a fix to give the graph its smooth line however using this method can lead to further inaccurate data due to the ways polynomials work.

The polynomial of the velocity has in this case given an accurate example when comparing the figures and given a much smoother line eliminating the human error. The polynomial was a 6th order and maximum possible decimal places of 25 to increase the accuracy

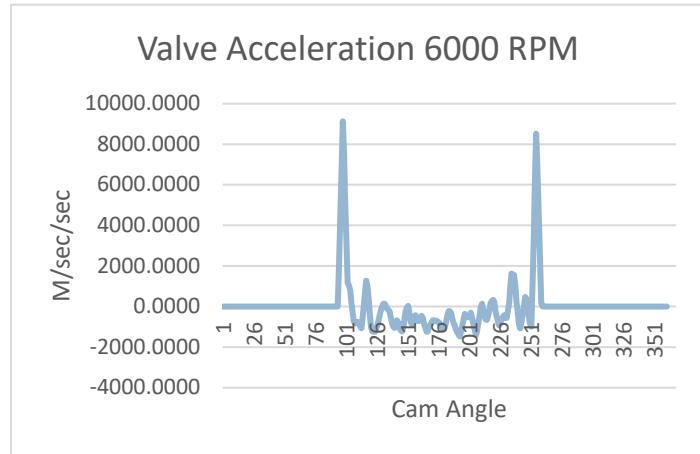


GRAPH 3

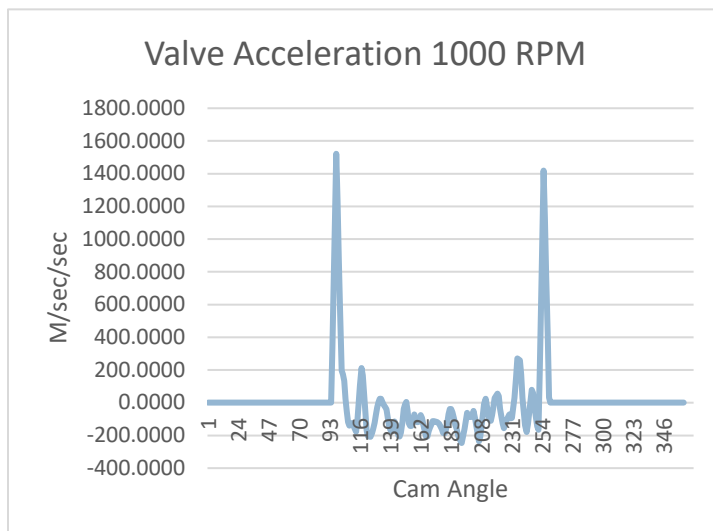
VALVE ACCELERATION

When looking at the acceleration curve it shows a steep curve to start due to the shock of the cam meeting the valve which would need to be controlled as if this shock was too high it would cause breakage and lead to engine failure. There is a higher positive then a negative as the cam forces the valve open and only the spring closes the valve.

Another factor to take in is this is the acceleration at maximum engine speed where forces are highest. When looking at another graph at 1000 RPM the steepness of the curve is significantly lower due to the cam's rotational speed and lead to a much softer shock between the contacts.



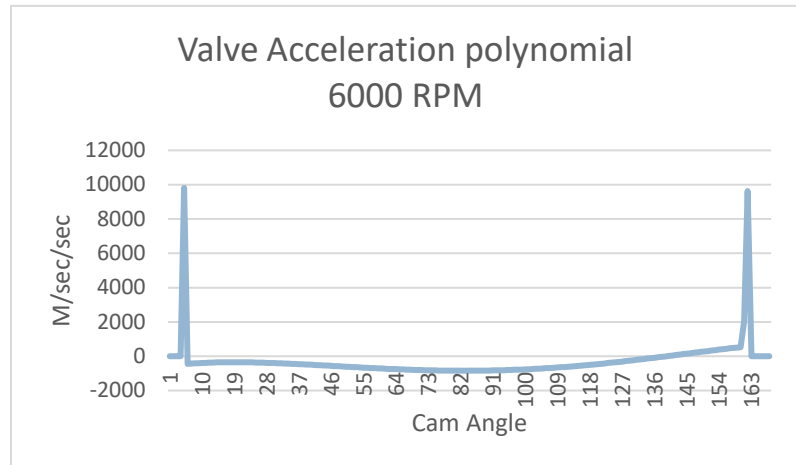
GRAPH 4



GRAPH 5

When looking at valve clearances and shimming it can lead to the valve either not opening enough due to a much larger clearance of the cam leading to a larger shock or if allowed to get to close would mean that the contact remains intact and may cause the valve to hit the piston by not closing fully. Modern sports bikes emit a high pitch tapping sound when the engine requires shimming and leads to lack of power.

A polynomial was also used on acceleration to show the motion however care must be taken to ensure the true resemblance and eliminate errors. Due to the errors from the velocity this carries on further to the acceleration and by using a polynomial it only increases errors further so must be used as a guideline only.



GRAPH 6

VALVE SPRING

The valve spring is a crucial part of the valvetrain model and measuring accurately is a critical requirement to achieve a realistic friction model. The spring was measured using a spring press and measurements taken every 0.1mm-13.2mm, the press used pounds as the recorded force and required converting in newtons. Converting pounds of force in newtons meant multiplying 1 pound by 4.44822 resulting in newtons, this was done for each of the measurements converting all previous calculations.

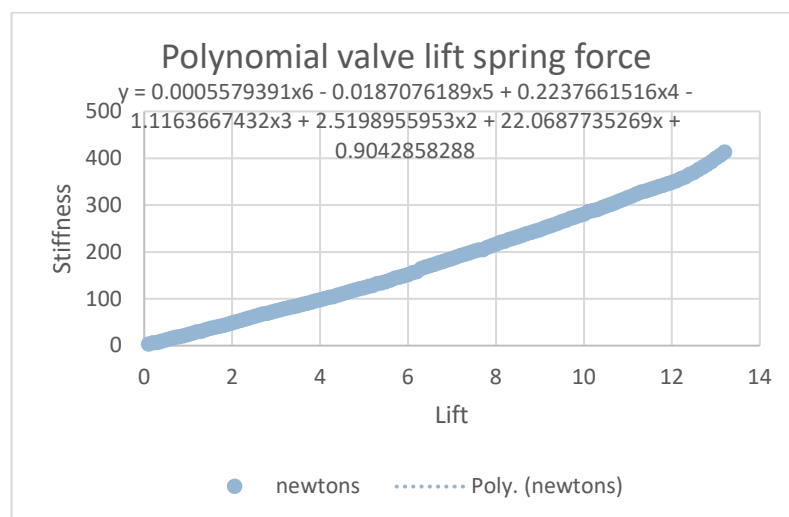
With the valve fitted the spring length was measured, this was also done whilst loose so that the compressed length was recorded and was measured at 5.2mm. Using the force at max lift and compressed length it gives a total compressed at max lift of 14.06mm. with hooks law stating

Hooke's law is valid as long as the elastic material you're dealing with stays elastic — that is, it stays within its elastic limit. If you pull a spring too far, it loses its stretchy ability. As long as a spring stays within its elastic limit, you can say that $F = -kx$. When a spring stays within its elastic limit and obeys Hooke's law, the spring is called an ideal spring. (holzner, 2013, p.152)

$K = f/d$ $k = 395.18/14.06 = 395.18$ $k = 395.18$ as the maximum stiffness at max lift.

Max spring force at full lift is therefore 395.18newtons

A polynomial was made so that spring force could be shown for all valve lift measurements.



GRAPH 7

The mass was recorded using digital kitchen scales and half mass is assumed because the spring is compressed:

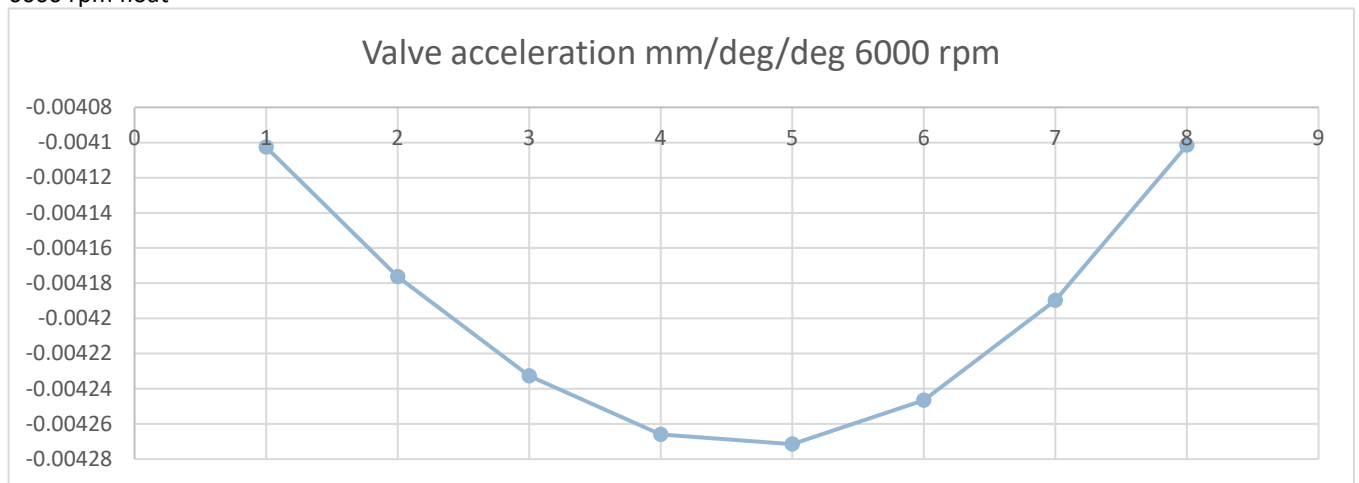
TABLE 1

Weights	Grams	
Inlet valve	33	
Bucket	21	
Spring	20.5	half mass as compressed
Total weight	74.5	grams
	0.0745	kilograms

VALVE FLOAT

The engine uses a double over head system (DOHC) which the valve is actuated directly from the cam pushing on the bucket and not via a separate rocker arm. As can be seen in figure... the cam starts to contact the valve but at max lift the valve opens further in free fall before the spring pushes it back, this is otherwise known as valve float.

6000 rpm float



GRAPH 8

TABLE 2

valve lift	Valve velocity mm/deg	Valve acceleration mm/deg/deg	acc mm/sec/sec	acc m/s ²
8.744647609	0.029275527	-0.004102532	26300687.8	-5316.88
8.773923136	0.025172995	-0.004176278	-5412456.053	-5412.46
8.799096131	0.020996717	-0.004232745	-5485637.128	-5485.64
8.820092848	0.016763973	-0.004265953	-5528675.523	-5528.68
8.836856821	0.012498019	-0.004271494	-5535856.844	-5535.86
8.84935484	0.008226525	-0.004246529	-5503501.164	-5503.5
8.857581365	0.003979996	-0.004189786	-5429963.018	-5429.96
8.861561361	-0.00020979	-0.004101567	-5315631.405	-5315.63
8.86135157	-0.004311358	-0.003983742	-5162929.79	-5162.93
8.857040213	-0.0082951	-0.00383975	-4976316.101	-4976.32
8.848745113	-0.01213485	-0.003674601	-4762282.73	-4762.28
8.836610263	-0.015809451	-0.003494874	-4529356.536	-4529.36
8.820800812	-0.019304325	-0.003308718	-4288098.838	-4288.1
8.801496488	-0.022613043	-8.756270402	-11348126441	-1.1E+07
8.778883445	-8.778883445	8.778883445	11377432945	11377433

The float speed is shown by using the max valve lift including 7 degrees before and after. When the valve is In float then the maximum engine speed would be:

$$-mass * acceleration = spring force$$

$$RPM = \sqrt{\frac{28.11 \times 14.05 \times 1000}{0.0745 \times 0.00427 \times 36}}$$

$$6132.6 = \text{Cam RPM}$$

$$12265.22 = \text{Engine RPM}$$

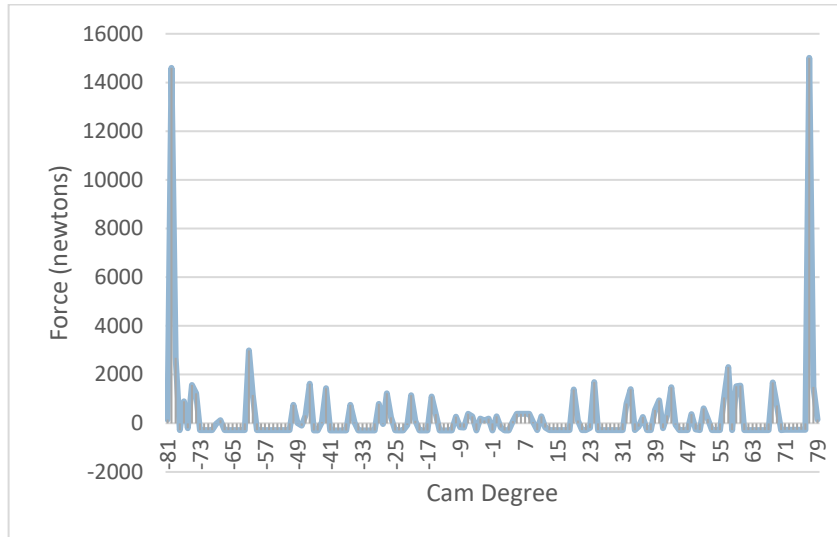
The calculated max rpm of the cam is 6132.6 and because the came moves at half the speed of the engine it shows the max rpm is 12265.22 which is a realistic figure as the maximum rpm of the bike is around 12,000 The calculations are the correckt method for calculating this but there are errors in the recorded data such as the lift would not be 100% accurate due to human error but the results show that the errors are small enough that realistic figures were still achievable.

VALVE FORCES

The maximum force acting upon the valve is in relation to both the spring forces and the valve acceleration. Using newtons second law

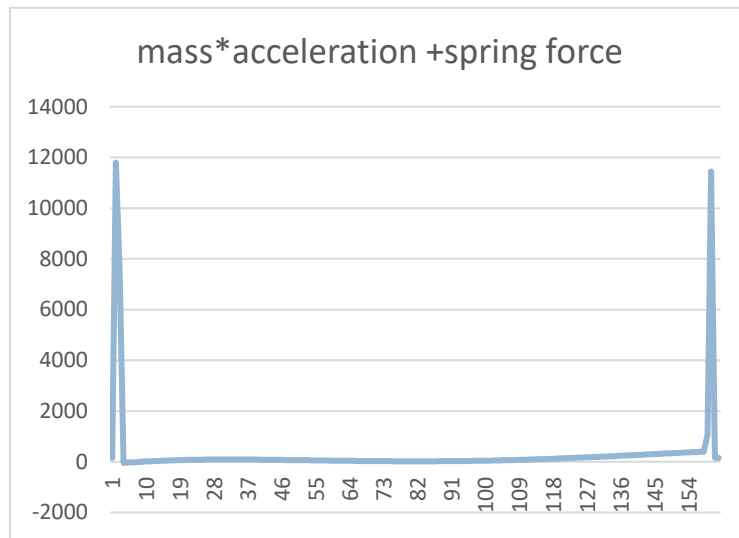
Newton's second law of motion describes the relationship between an object's mass and the amount of force needed to accelerate it. Newton's second law is often stated as $F=ma$, which means the force (F) acting on an object is equal to the mass (m) of an object times its acceleration (a) (hooke,1968)

The mass is as previously stated at 74.5 grams but will need to be converted in kilograms equaling 0.0745kg and acceleration is at maximum which is 193982m/sec/sec and at its highest which is when it contacts the valve. The maximum contact force is then theoretically 14451.66 newtons+ the spring force.



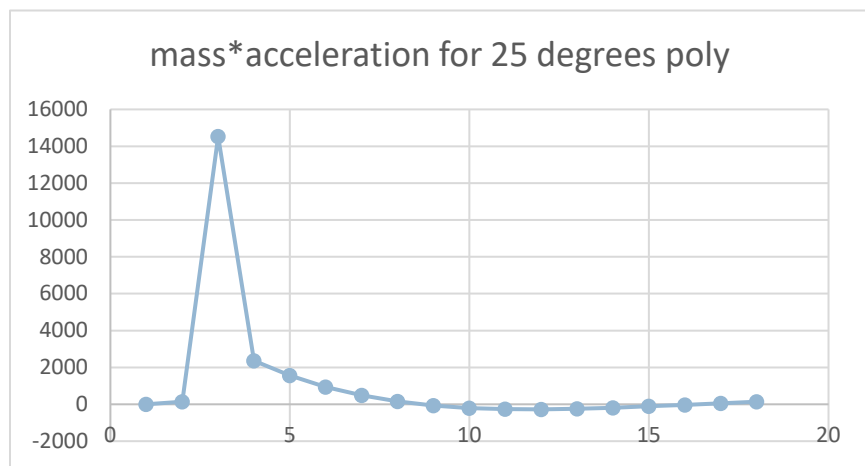
GRAPH 9

When looking at the graph above it shows spikes throughout and that this was caused by human error so a 6th order polynomial was used on the of lift to give it a much smoother curve and hopefully stay close to the raw data.



GRAPH 10

With the polynomial only using the first 25 degrees of lift it has given a more accurate answer and a better understanding on the forces acting on the valve without the spikes and a more textbook example of what the true forces look like



GRAPH 11

CHAPTER 3

An engines valvetrain is a complex set of parts and analyzing stresses by hand calculations would neither be time efficient or accurate. A solution to this complex part solving is to use a Finite Element Analyses (F.E.A), this is a numerical technique that solves equations in matrices to solve equations simultaneously. Doing these calculations via a computer is the most sufficient and beneficial way of solving the equations.

Models of parts can be created using a cad package and be broken down into further smaller parts that can be any size, these can be defined as elements which work as parts of a puzzle and model is a complete puzzle. These puzzle bits or elements can be used at any size and are used as a mesh of the part.

The elements of the part are all connected and are called nodes, these nodes are what form the elements and the more nodes the smaller the elements are. The more nodes and elements on the part the longer it will take to run an analysis.

Elements are connected at defined points called Nodes, for linear Elements the Nodes are placed at the extremities of the Element. As already stated there will be no deformation between the Nodes. The response of this element can be calculated by solving the differential equations at each node of the element. The results of the equations will be averaged to predict the displacement of the element.

Using Solidworks as the chosen FEA package the main method of meshing is tetrahedrals as pictured to the right. By solid works using this in meshing the elements must be smaller around high concentrations and is described as

`Meshes of triangles or tetrahedra have many applications, including interpolation, rendering, and numerical methods such as the finite element method. Most such applications demand more than just a triangulation of the object or domain being rendered or simulated. To ensure accurate results, the triangles or tetrahedra must be "well shaped," having small aspect ratios or bounds on their smallest and largest angles` (Brancheau, J, 2005)

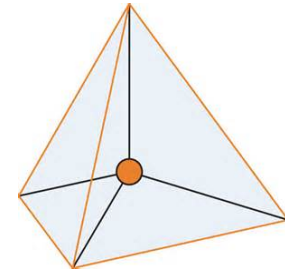


FIGURE 3

SOLIDWORKS

Solidworks is a cad package that can be used to run finite element methods, in this case it was used more of a validation tool to validate the excel model. The first step was to create various parts in excel, the first being the cam shaft. The camshaft was done by first starting with a single cam lobe and then inserting 360 lines in a rotational pattern all at the cam lobes minimum length and then changing them to match the recorded lengths from using the dial gauge as previously stated. This a length procedure but was necessary so that it was matching the model and that any incorrect measurements would also be represented on the model and can be clearly shown below

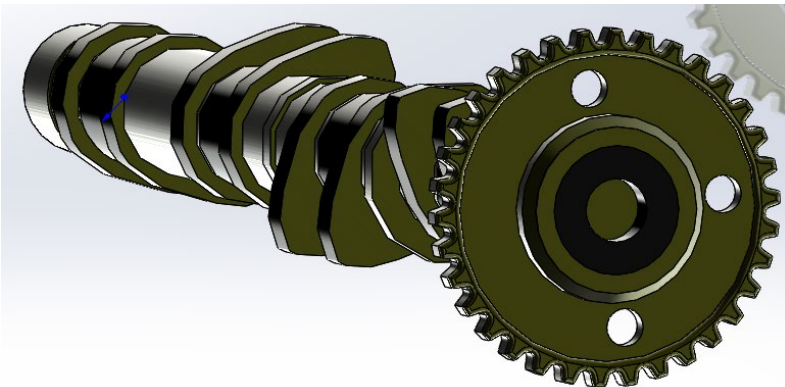


FIGURE 4

As can be seen from figure 4 there are a several measurements that are not accurate and can be see clearer in the picture than in the excel. The rest of the cam was measured using a Vernier and made to look more appropriate due to not needing to be as accurate as only the cam lobes were being tested. The rest of valvetrain parts were created in the hope of doing further motion simulations however was found best not as a static study would be more beneficial to link it to the excel model.

The first static study was to use the force calculated in the excel model and apply it on Solidworks to resemble the actual force acting upon the cam. Figure 5 shows the cm lobe being locked in placed as it would in the engine and where the cam guides would be supportive. A static study was then used forcing the cam into the bucket applying the same force as that calculated in the excel.

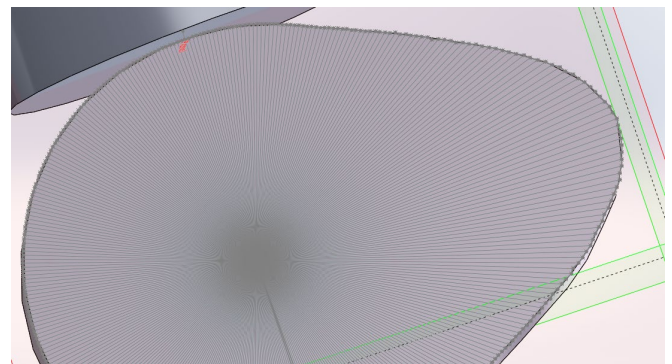


FIGURE 5



FIGURE 6

Results from 3 separate simulations

TABLE 3

	Simulation 1	Simulation 2	Simulation 3
<i>No of nodes</i>	18796	2659424	50600
<i>No of elements</i>	9258	1742177	27625
<i>No of deg dof</i>	55902	7972224	133764
<i>Simulation time</i>	0:03	4.15	0:06
<i>Highest stress concentration</i>	310.4	290.4	292.41

Running a static simulation can vary from a few seconds through to hours depending on the limits defined by the user on the model and the scale of the model and the general rule is the larger the model and finer the mesh the longer a simulation requires running. Evidence of this can be seen when looking at table ,,,,,, showing the results from the previous three simulations. The first simulation took 3 seconds compared simulation 2 taking 4:15mins. Knowing where the highest stress point can help as input of mesh controls can be inserted into the region in advance of the simulation.

Looking at the table it shows that the more elements used the longer a simulation takes but not necessarily the most accurate. As can be seen in table 1 column 2 the mesh was fine over the entire part and yet gave a max stress point of 290.4n/mm however by refining the mesh to around the stress points this minimalized the overall elements on the model and increased them around stress points minimizing the hotspots and allowing the mesh to give a higher stress. This is due to the way meshing elements work, for an example on a larger mesh the stress hot spots are larger and need to be more defined to lower the stress hot spots.

HERTZIAN

Hertzian contact force equations is an appropriate method of which hand calculations can be used as a way of validating the F.E.A method used in Solidworks and can show the relation between the two. This method use the diameter of the cam (a radius of where the cams max force is) and the elastic modulus of both the cam and the bucket. Solidworks simulations can be a great tool however errors can occur which in turn give inaccurate results that the user must understand and using Hertzian method it can show proof that that the simulations results are accurate.

Stresses formed by the contact of two radiuses can cause extremely high surface stresses. The application of Hertzian Contact stress equations can estimate maximum stresses produced. These stresses can then be analyzed in context of the application. (Lakes,R 2011)

This is calculated using the below formulas:

An elastic sphere of radius R indents and elastic half-space to depth d and thus creates a contact area of radius

$$\frac{1}{E^+} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

E_1 E_2 are the elastic moduli and ν_1 ν_2 the poissons ratios associated with each body

The distribution of normal pressure in the contact area as a function of distance from the center of the circle is

$$a = \sqrt{Rd}$$

$$d = \frac{a^2}{R}$$

p_0 is the maxium contact stress

$$p_o = \frac{3F}{2\pi a^2} = \frac{1}{\pi} \left(\frac{6FE^2}{R^2} \right)^{1/3}$$

The young's modulus units and figure were taken off the Solidworks material properties tab to ensure accuracy between the two models and to eliminate any errors. Poissons ratio was also taken from this tab and is defined as

Poisson's ratio is the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative. The definition of Poisson's ratio contains a minus sign so that normal materials have a positive ratio. Poisson's ratio, also called Poisson ratio or the Poisson coefficient, or coefficient de Poisson, is usually represented as a lower case Greek nu, ν . (lakes.R. 2011)

The formulas and method for Hertzian was used in the excel model where it is possible to change values with ease and a more user-friendly interface.

TABLE 4

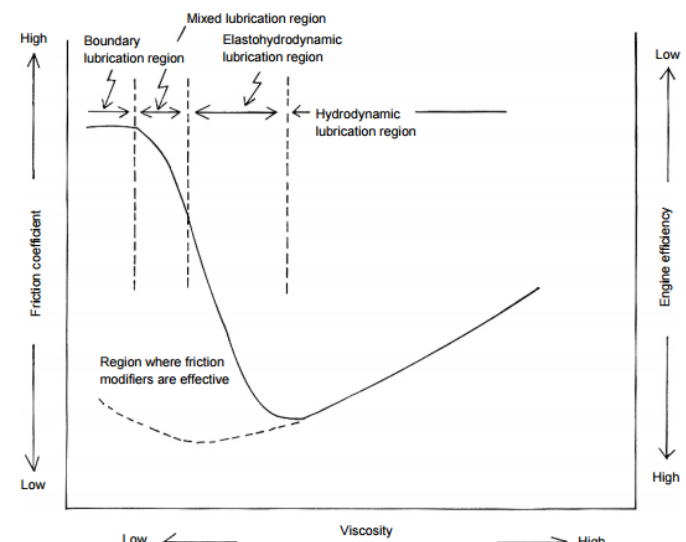
Cam Radius	R1	16	mm
Bucket radius	R2	0	
Young's modulus cam	E1	190000	mpa
Young's modulus bucket	E2	210000	mpa
Poissons ratio cam	V1	0.211	μ
Poissons ratio bucket	V2	0.305	μ
Force	F	15000	n
Equivalent radius of curvature	R1	16	mm
Length	L	8	mm
Equivalent elastic modulus			
1/E	9.3478E-06		
E	106977.439		
depth of indentation	2.30E-04	d	
area of radius	0.06071552	a	
maximum contact pressure	260.574427		

CHAPTER 4

FRICTION LOSS

When two surfaces are in relative motion with each other it creates a friction, depending on the surface material and lubricant depends on the friction that is created. In a high-performance engine such as a GSXR engine it's common for there to be a higher friction at low RPM from the valve train due to the valve spring resisting the cam however this is lowered at higher RPM as the cam is moving at a much greater speed and creates a valve float which makes the bucket and cam separate.

The friction is calculated using the mass*acceleration+spring force as previously calculate and then multiplied by a coefficient of friction defined as mu (μ) This method therefore calculates the friction for each degree of the cam showing where the highest points on the cam load are and in theory can prove this by showing a relation to the cam itself with pictures of wear. The co-efficient used was 0.11 for this model however this is only an estimate due to no way of realistically getting this figure without further time required. As stated a coefficient of 0.1 was used, when comparing with a Steinbeck curve this figure changes depending on the speed and temperature of the engine.



GRAPH 12

Work done

$$((f_1+f_2)/2)*(cr_1-cr_2)$$

F=friction force

cr=cam rotation distance

1 and 2 refer to crank angle values

Using equations for the sum of work done it was totaled for the entire column giving a total amount of work done for one revolution of the cam. With the total amount of work done the it is multiplied by the quantity of cam lobes and in the engine, being 8 in this case as it's a 16 valve DOHC engine and half of those being intake valves.

$$total\ work\ done/rev\ of\ cam = \sum(\Delta \omega)$$

$$total\ work\ done/rev\ of\ engine = \frac{1}{2} \sum(\Delta \omega)$$

$$total\ work\ done/2\ engine\ revs = \sum(\Delta \omega)$$

$$\sum(\Delta \omega) = fmep \times eng.\ displacement$$

$$fmep = \frac{\sum(\Delta \omega)}{1000 \times 10^{-6}} . PA$$

$$= \sum \frac{(\Delta \omega)}{100} BAR = \sum \frac{(\Delta \omega)}{1000 \times 10^{-6}} \times 10^{-5}$$

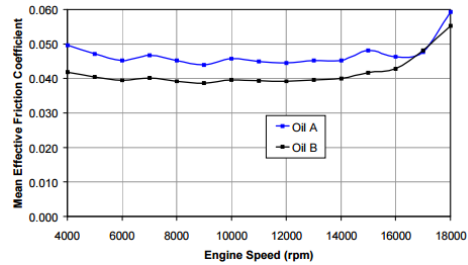
Mew 0.11

sum work done	19.33322502
fmep bar	0.19333225
fmep bar 8 valves	1.546658002

The total calculate friction in the model for the inlet valves come to a total of 1.55 bar and he total valvetrain being intake and exhaust would come to a total of 3.1 bar assuming exhausts valves would be identical. There are numerous ways in which to lower this including changing the materials to create a lighter valve or spring. Another way could be to change the way the cam is made and perhaps consider surface treatments to harden the cam lobe to lower the risk of wearing and flattening off the lobe

RICARDO RESEARCH

With extensive research a paper from Ricardo engineering was discovered with a calculated mean coefficient of a the valvetrain. They state two separate coefficients and this is due to different oils used. By using different oils and dropping the mean friction this is proof that some oils can provide a better lubricant and can pro long the life of an engine.



GRAPH 13

sum work done	8.787829556
fmep bar	0.087878296
fmep bar 8 valves	0.703026364

CONCLUSION

The primary objective of this report it to create a model representing the effects of friction in a valve train model. When the model and Solidworks showed correlation, it showed that the objective was being achieved. Measurements were made taken from the engine and put into the model resulting in realistic results from those figures. Although at various stages results were fluctuated due to raw data inaccuracy, polynomials were used in place to counter this. The model contains various mathematical methods that were used to give insight to the speeds and forces within the valvetrain. At the start of the project the highest effort was to measure components of the valve train components which led to a more accurate model. An FEA model was created using measurements recorded and static studies undertaken resulting in realistic answers that also relate to the excel model. The technique used to model the cam was to use the measured data and then input forces calculated in excel so that if the showed correlation that it would be evidence of a working model. The model showed data that would present the possible dangers and wear of parts and a when they would most likely occur and why that would be. With the excel model in its finished state it is simple to change the cams and identify possible improvements. To further the model, it could go on to show sources of friction including the piston or the crank etc. In conclusion, the project has reached some intriguing results and discovery's which were then verified using various methods.

FURTHER WORK

To further this report there are several key areas that could be addressed, firstly would be to use a dyno and change oils to see the affects this may have of the work required to turn the cams. By having a running dyno, it would be possible to calculate the force required to turn a cam 1000 times for an example and then doing the same again but with a different oil. Running this experiment would also help in creating a Steinbeck curve depending on the oil viscosity and engine speed and would be extremely beneficial as this is rare subject to research upon due to company's not sharing much information.

Excel is extremely restricting in the way it works and requires a much more in depth way of inserting formulas to get the required outcome, an easier way of making the model may have been more beneficial in Matlab and would save a great amount of time instead of having to plot substantial amounts of data in excel that Matlab would have done in a line of coding however without appropriate training in this subject would prove more difficult and perhaps too time consuming.

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