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Complex Static and Dynamic Systems

BSC (HONS) MECHANICAL ENGINEERING

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3 Introduction

This report is split into 5 different areas of complex and dynamic systems. The report is constructed by some descriptive tasks and some which require calculations.

Task 1 looks at static and dynamic balancing and the differences between the two scenarios. It is followed by a balancing question which requires the static and dynamic balance to be known.

Task 2 is based on the epicyclical gearbox, task 1 of task 2 is a series of questions involving the main elements, applications and the expressions obtained when an epicyclic gear box is under different scenarios. This is followed by a standard compound epicyclic gear box question.

Task 3 is based on a motor and gearbox scenario where the vertical acceleration is unknown.

Task 4 looks at thick and thin cylinders and the difference between both. It also includes a standard question on thick cylinders.

Task 5 is a question that looks at compound torques on a shaft where the angle of rotation and max stresses are unknown.

4 Question 1.

4.1 Question a)

Balancing of rotating bodies is required to avoid vibration. In mechanical systems such as steam turbines and generators vibrations could result in complete failure of the system. Another example of where balancing is required is found in a car wheel, unbalanced wheels can result in a noisy and uncomfortable experience. Balancing in simple terms involves moving the centre of gravity to the centre of the rotation. (K.Roberston, 2018)

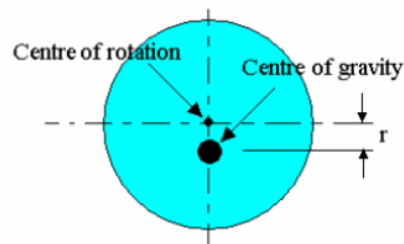


Figure 1 Simple balancing example

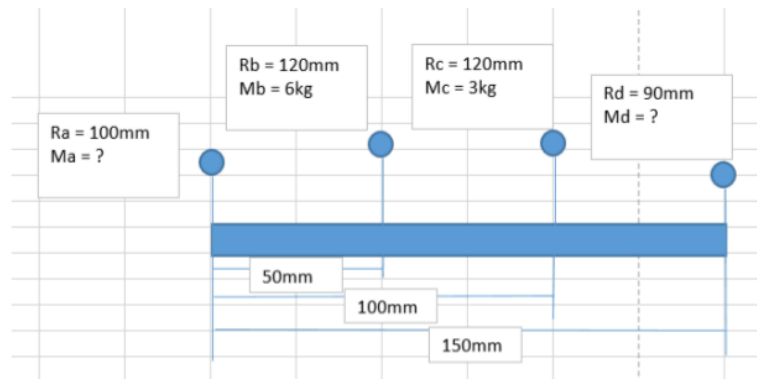
For a body to be completely balancing it requires two things:

Static balancing – this occurs when there is no resultant centrifugal force and the centre of gravity is on the axis of rotation.

Dynamic balancing – This occurs when there is no resulting turning moment on the axis.



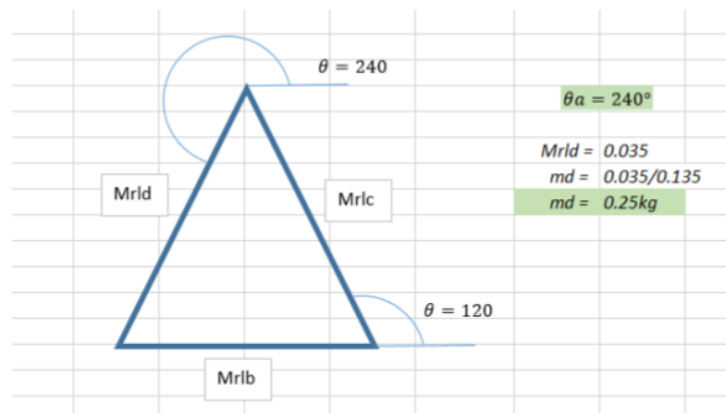
4.2 Question b)



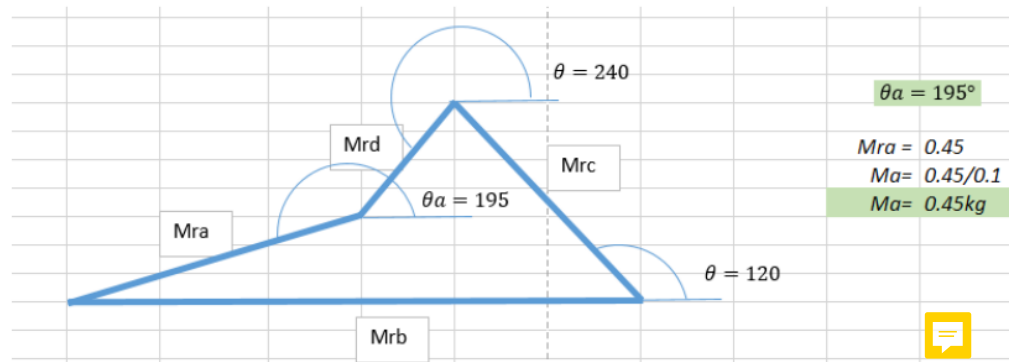
Mrl Table

Plane	Mass (kg)	Radius (m)	Mr (kg/m)	l (m)	mrl (kg/m ²)	θ°
A	Ma	0.1	0.1Ma	0	0	195
B	6	0.12	0.72	0.05	0.036	0
C	3	0.12	0.36	0.1	0.036	120
D	Md	0.09	0.09Md	0.15	0.0135	240

Mrl Polygon (Scale 1cm = 0.01Kg/m²)



Mr (Forces) Polygon (Scale 1cm = 0.1kg/m²)



5 Question Two

5.1 Task 1

5.1.1 Question a)

Below is a typical epicyclic gear box with all four main elements identified:

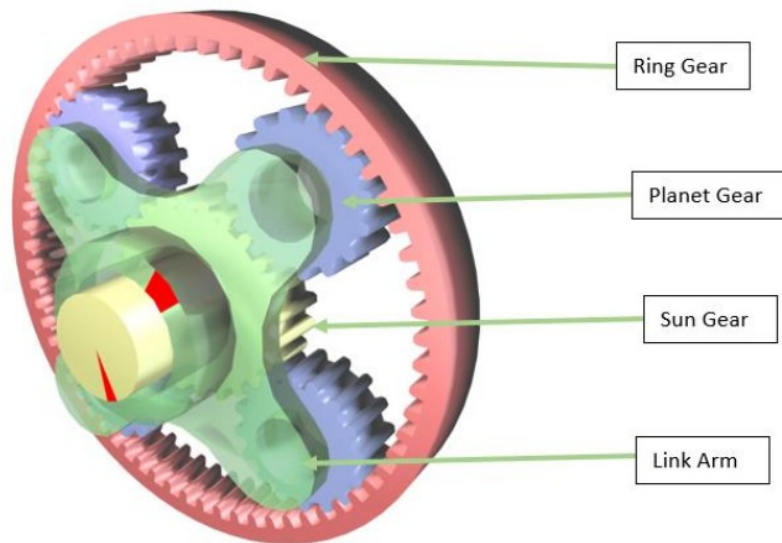


Figure 2 - Typical epicyclic gearbox

5.1.2 Question b)

A compound epicyclic gear train consists of three toothed wheel components. The sun gear, planet gear and ring gear as shown above. The axis of the sun and planet gear are connected by the link arm. The planet wheel meshes with the sun wheel and also the annular ring wheel. The planetary gears rotate around their own axis and are also carried around the sun by the link arm. When the sun arm is fixed the annular ring gear provides the drive, but when the annular ring gear is fixed the sun provides the drive. In both cases the link arm acts as a follower for the planetary gears. (T.Stilling, 2010)

5.1.3 Question c)

- i) In the case of a standard epicyclic gear train where the sun gear, planet gear and ring gear are free to rotate but the link arm is fixed the following solution can be expressed.
Please see diagram for component annotation.

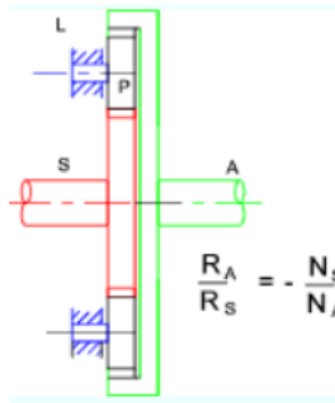


Figure 3 - Fixed link arm diagram

Action	Rotation (Clockwise = +ve) from action			
	Nx = Number of Teeth			
	S	L	P	A
Fix L and rotate Sun + 1 Rev CW	1	0	-Ns/Np	-Ns/Na

- i) In the case of a standard epicyclic gear train where the link arm, planet gear and ring gear are free to rotate but the sun gear is fixed the following solution can be expressed.
Please see diagram below for component annotation.

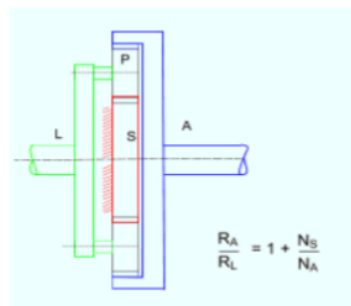
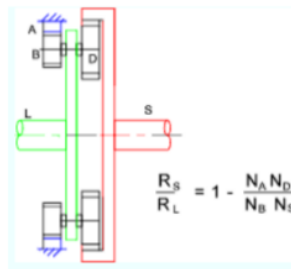


Figure 4 Fixed sun gear example

Action	Rotation (Clockwise = +ve) from action			
	Nx = Number of Teeth			
	S	L	P	A
Turn whole gear through +1 CW rev	1	1	1	1
Fix arm (L) and rotate S CCW -1 rev	-1	0	N_s/N_p	N_s/N_a
Add two motions	0	1	$1 + (N_s/N_p)$	$1 + (N_s/N_a)$

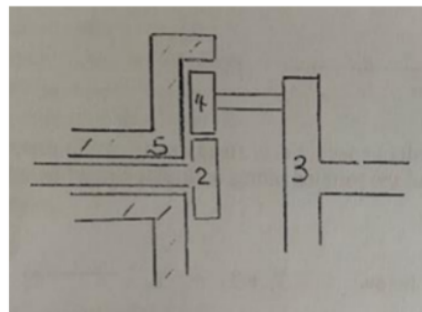
- i) In the case of a standard epicyclic gear train where the link arm, planet gear and sun gear are free to rotate but the ring gear is fixed the following solution can be expressed. Please see diagram below for component annotation.



Action	Rotation (Clockwise = +ve) from action			
	Nx = Number of Teeth			
	S	L	B-D	A
Turn whole gear through +1 CW rev	1	1	1	1
Fix arm (L) and rotate S CCW -1 rev	$1 - (N_A/N_B) \times (N_D/N_S)$	0	$-N_A/N_B$	-
Add two motions	$-(N_A/N_B) \times (N_D/N_S)$	1	$1 - (N_A/N_B)$	0

5.1.4 Question d)

For an epicyclical gear box arrangement where the sun, planet and ring gears are free to rotate. The following solution can be expressed. See diagram from component annotation.



Action	Rotation (Clockwise = +ve) from action			
	Nx = Number of Teeth			
	2	3	4	5
Rotate whole train +m rev	+m	+m	+m	+m
Unlock arm and rotate sun +m rev	+n	0	$-n(T_2/T_4)$	$-n(T_2/T_5)$
Add two motions	$m+n$	m	$m(T_2/T_4)n$	$m(T_2/T_5)n$

5.1.5 Question e)

Advantages of using an epicyclic or planetary gear have many advantages compared to a more traditional gear box. Epicyclic gear boxes or gear trains are able to secure higher gear ratios in a compact and confined space. Therefore, where space is a constraint an epicyclic gear is preferred. Also for a gear ratio similar to that of an epicyclic gear train in a standard gear box would weigh sufficiently more. The power transmitted is also much more efficient than that of a standard gear box around 97%. Planetary gear systems also are more stable and cause less noise to operate, the service life is also greater than a standard gear boxes that experiences a similar load. Due to having more than one planet gear the load is shared through each, therefore giving higher torque transmission and lower inertia. (D.Hiard, 2019)

Planetary gears are used where weight and speed are restricting factors but large amounts of speed reduction and torque are required. Typical applications include tractors, construction equipment and automatic transmissions. (K.Lowis, 2018)

5.1.6 Question f)

Complex epicyclic gear trains can be found in an automatic car transmission. This is where more epicyclic gear trains are placed in series within the same ring gear housing. This is where the output shaft of the first stage becomes the input shaft to the next and so on. It can also be reverses output to input. This can provide a large or small gear ratio depending on the scenario.

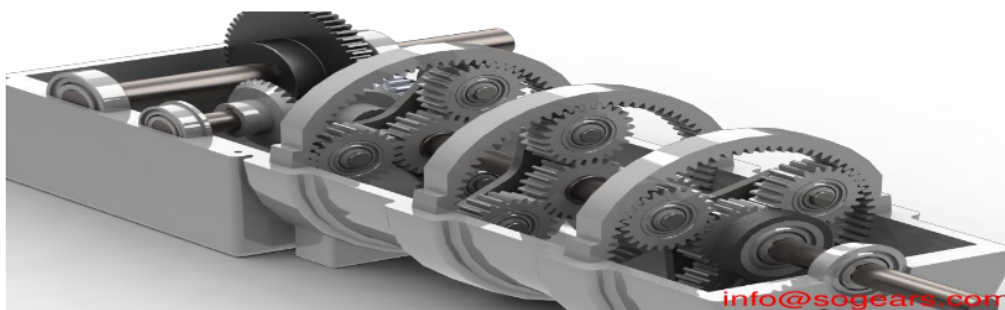
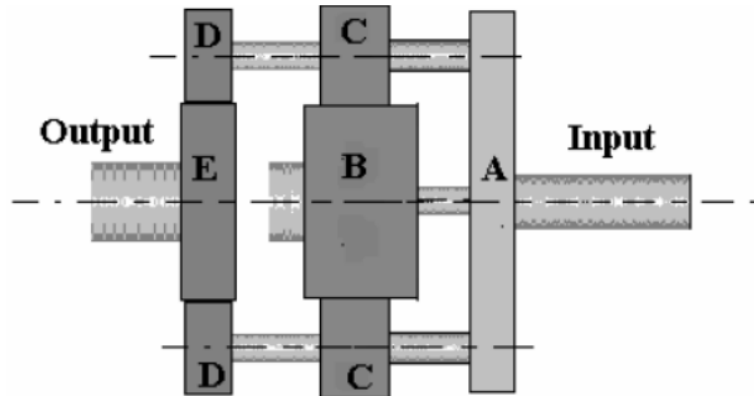


Figure 5 - Complex epicyclic gear box in series

5.2 Task 2



$$TC = 200$$

$$TB = 50$$

$$TD = 50$$

$$TE = 200$$

B = Fixed C/D = Rigidly Connected

	A	B	C/D	E
Rotate whole gear train +1 rev	+1	+1	+1	+1
Fix A and rotate -1 Rev	0	-1	$+1\left(\frac{50}{200}\right)$	$-1\left(\frac{50 \times 50}{200 \times 200}\right)$
Add line 1 and 2	1	0	1.25	0.94

The ratio of the gearbox is 0.94:1 or 15:16



6 Question 3

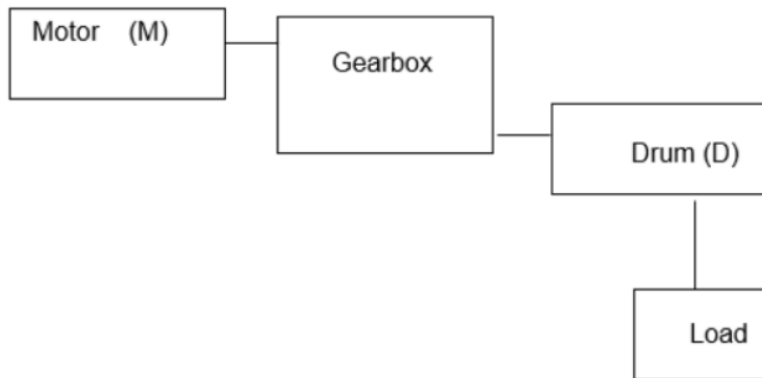


Figure 6 - Question Three Diagram

$$T = I \alpha \propto$$

$$T = 104Nm$$

$$\text{Motor Torque} = 104Nm$$

$$\text{Efficiency} = 85\%$$

$$I = 25Kgm^3$$

$$\frac{T}{I} = \alpha = \frac{104}{25} = 4.16$$

$$\alpha_b = 4.16 \text{ Rads/s}^2$$

$$a = \alpha \times r$$

$$a = 4.16 \times 0.4$$

$$a = 1.6m/s^2$$

$$\alpha_A = 4.16 \times \frac{30}{1} = 124.8 \text{ Rads/s}^2$$

$$\text{Centripetal Acceleration} = F/M$$

Where:

$$\frac{V^2}{r} = \frac{124.8^2}{400} = 38.9m^2$$

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Motor torque at 85% efficiency = 85% of 104Nm = 88.4Nm

$$\alpha = \frac{T}{I} = \frac{88.4}{25} = 3.536 \text{ Rads/s}$$

$$a = \alpha \times r$$

$$3.5 \times 0.4 = 1.414 \text{ m/s}^2$$



7 Question 4

7.1 Question a)

A cylinder regarded as a thin walled is when the thickness is less than 1/20th of the diameter. When the wall is greater than this it is regarded as a thick wall cylinder. A thick cylinder has stress in the radial direction as well as a circumferential stress and longitudinal stress whereas a thin cylinder does but as it so small it is neglected. Radial stress only becomes relevant when the wall thickness exceeds 1/20th of the diameter.

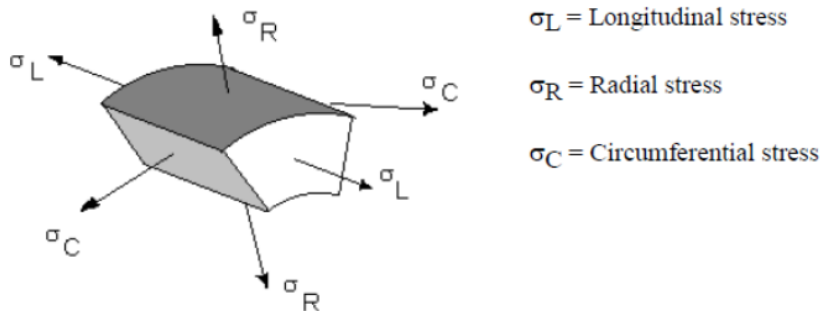


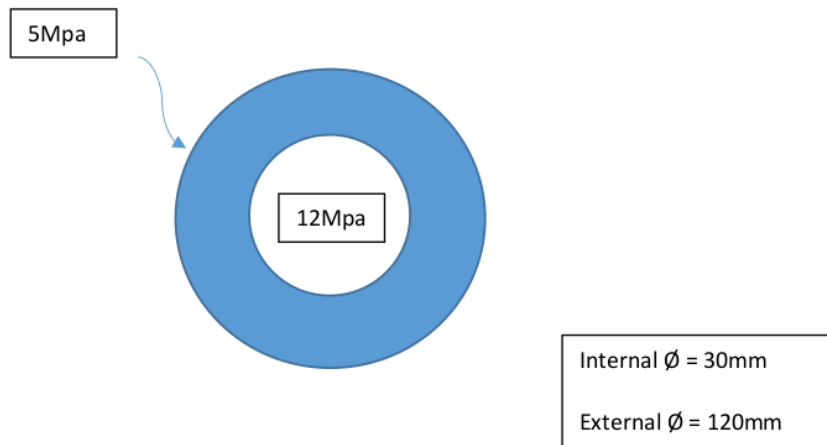
Figure 7 - Thick cylinder stresses

Thick cylinders undergo a process called Autofrettage, a process of pre-stressing the cylinder before actually creating a cylinder. This is done to increase the pressure of the bearing capacity of the cylinder. This technique is used to reverse the effect of the residual stress in the cylinder which therefore increases the pressure bearing capacity by nullifying the residual stress. (J.Tucker, 2015)

Examples of typical uses for thick walled cylinders are in chemical and petroleum, military and nuclear industries where high pressure and high temperatures are continuously cycling through the cylinders. (E.Nikitri, 2014)



Question b)



When $r = 0.015 \theta = -12Mpa$

When $r = 0.06 \theta = -5Mpa$

Equation 1 $-12 \times 10^6 = \frac{a+b}{(0.015)^2}$

Equation 2 $-5 \times 10^6 = \frac{a+b}{(0.06)^2}$

From equation 2

$$-5 \times 10^6 = a - 277.778b$$

$$a = -5 \times 10^6 + 277.778b$$

Sub into equation 1

$$-12 \times 10^6 = -5 \times 10^6 - 277.778b + 4444.444b$$

$$-12 \times 10^6 + 5 \times 10^6 = 4166.666b$$

$$\frac{-7 \times 10^6}{4166.666} = b$$

$$b = -1680.00$$

$\therefore$

$$a = -5 \times 10^6 - 277.7(-1680)$$

$$a = -4.5 \times 10^6$$

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To calculate circumferential stress:

$$\theta c = a - \frac{b}{r^2} \text{ When } r = 0.015$$

$$-4.5 \times 10^6 = \frac{(-1680)}{(0.015)^2} = 2.9 \times 10^6$$

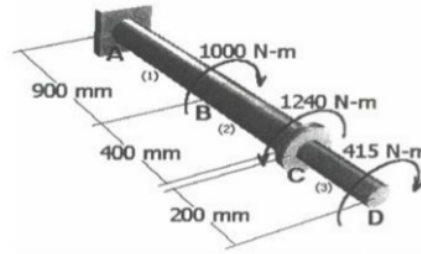
$$\theta c = a - \frac{b}{r^2} \text{ When } r = 0.06$$

$$-4.5 \times 10^6 = \frac{(-1680)}{(0.06)^2} = -4 \times 10^6$$

Boundary Conditions are shown below:

Radius (m)	θr (N/m ²)	θc (N/m ²)
0.015	-12 × 10 ⁶	2.9 × 10 ⁶
0.06	-5 × 10 ⁶	-4 × 10 ⁶

8 Question 5



$$G = 75 \text{ Mpa}$$

$$T_1 = -415 \text{ Nm}$$

$$T_2 = -415 \text{ Nm} + 1240 \text{ Nm} = 825 \text{ Nm}$$

$$T_3 = -415 \text{ Nm} + 1240 \text{ Nm} - 1000 \text{ Nm} = -175 \text{ Nm}$$

i) To calculate the maximum shear stress in the shaft:

$$\frac{T}{J} = \frac{J}{R} = \frac{G\theta}{L}$$

$$\therefore \tau = \frac{Tr}{J}$$

Section 1

$$\tau_1 = \frac{T_1 r_1}{J_1}$$

$$J = \frac{\pi \times 0.03^4}{32} = 7.95 \times 10^{-8}$$

$$\tau_1 = \frac{415 \times 0.015}{7.95 \times 10^{-8}} = 78.3 \text{ Mn/m}^2$$

Section 2

$$\tau_2 = \frac{T_2 r_2}{J_2}$$

$$J = \frac{\pi \times 0.045^4}{32} - \frac{\pi \times 0.035^4}{32} = 2.55 \times 10^{-7}$$

$$\tau_1 = \frac{825 \times 0.0225}{2.55 \times 10^{-7}} = 72.79 \text{ Mn/m}^2$$

Section 3

$$\tau_1 = \frac{175 \times 0.0225}{2.55 \times 10^{-7}} = 15.44 \text{ Mn/m}^2$$

The maximum shear stress of the shaft can be found in section 1 at 78.3 Mn/m^2

ii) The angle of rotation of each section:

Section 1

$$\theta_1 = \frac{-425 \times 0.2}{(7.95 \times 10^{-8})(75 \times 10^9)} = 0.014 \text{ Radians}$$

$$\frac{0.014}{2\pi} \times 360 = 0.80 \text{ degrees}$$

Section 2

$$\theta_2 = \frac{825 \times 0.4}{(2.55 \times 10^{-8})(75 \times 10^9)} = 0.017 \text{ Radians}$$

$$\frac{0.017}{2\pi} \times 360 = 0.97 \text{ degrees}$$

Section 2

$$\theta_3 = \frac{-175 \times 0.9}{(2.55 \times 10^{-8})(75 \times 10^9)} = 0.0082 \text{ Radians}$$

$$\frac{0.0082}{2\pi} \times 360 = 0.47 \text{ degrees}$$

iii) The angle of rotation of D relative to A

$$-0.014 + 0.017 - 0.0082 = 0.0052 \text{ Radians}$$

$$\frac{0.0052}{2\pi} \times 360 = 2.94 \text{ degrees}$$



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