

```
clc;clear;close all; format short g;
```

Task 2 - Root Solving

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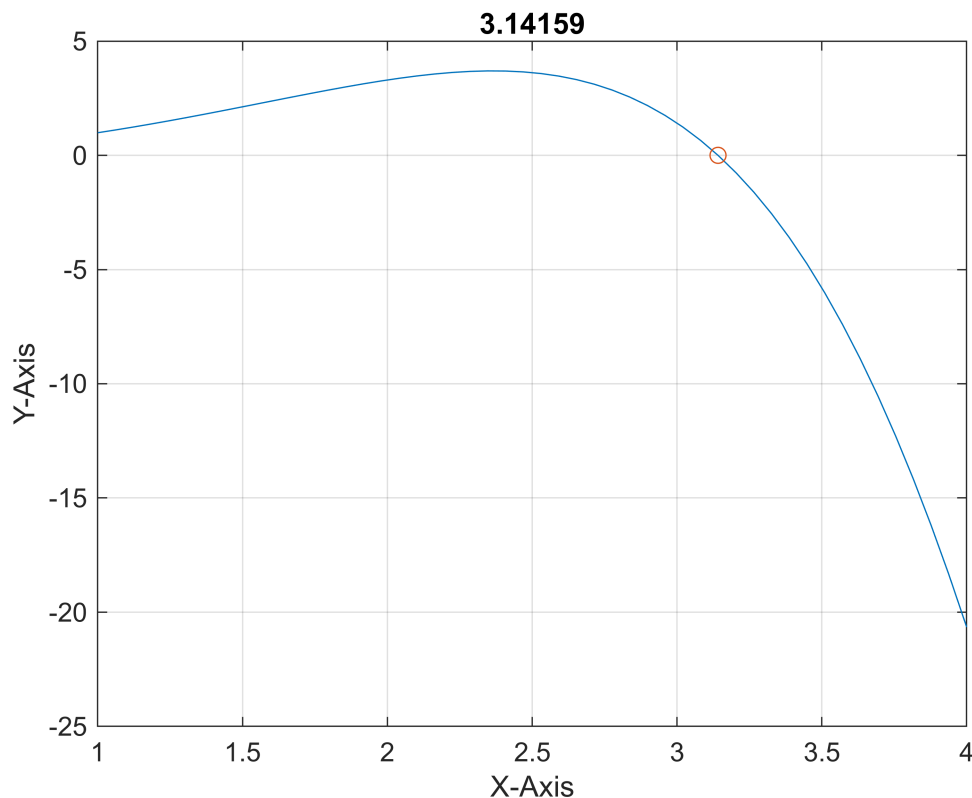
1 Bisection $f(x) = \sin(x)\sinh(x)$; Limits = [1 4] [3].....	1
1 Bisection $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$ [3].....	2
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1. Bisection Method

1 Bisection $f(x) = \sin(x)\sinh(x)$; Limits = [1 4] [3]

```
Root = BisecSolvePlot('@(x)sin(x).*sinh(x)',1,4,50,1e-6);
```

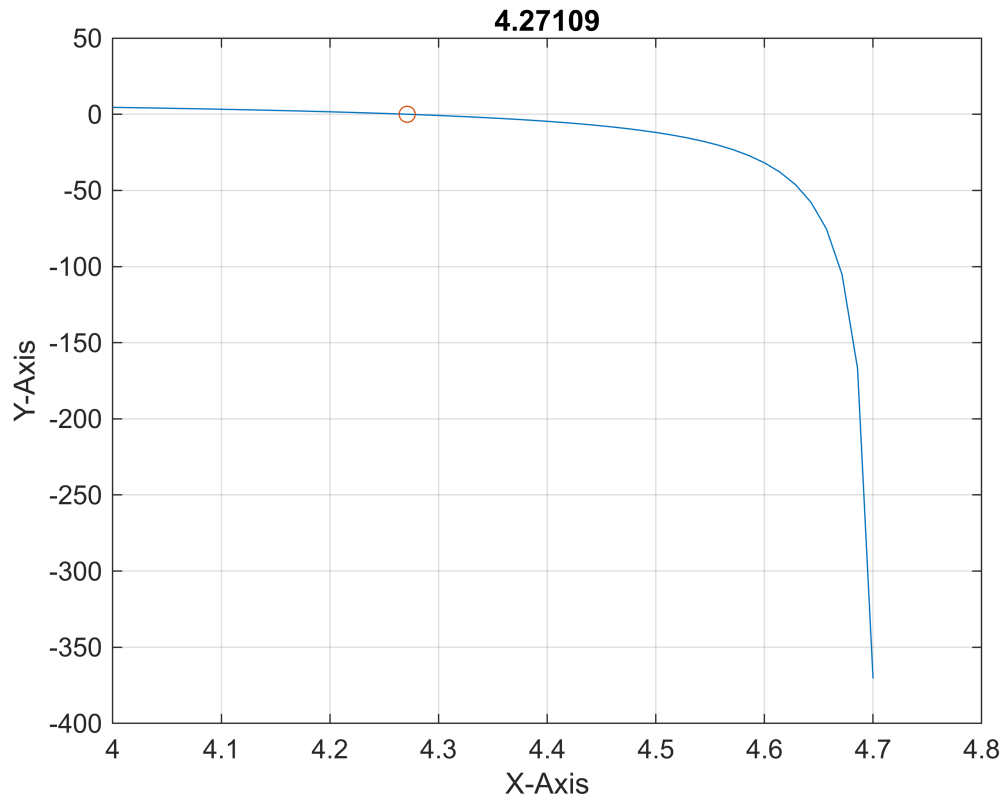
Equation: $@(x)\sin(x).\sinh(x)$
 ITERATION 22
 Root Estimate (7d.p) = 3.1415923



1 Bisection $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$ [3]

```
Root = BisecSolvePlot('@(x)sqrt(10.^(2)-x.^(2))-x.*tan(x)',4,4.7,50,1e-6);
```

Equation: $@(x)\sqrt{10.^(2)-x.^(2)}-x.\tan(x)$
ITERATION 20
Root Estimate (7d.p) = 4.2710950



2. Newton's Method

2 Newtons $f(x) = e^x - (1 + x + (x^2/2))$ [3]

```
syms x  
c = diff(exp(x)-1-x-((x^2)/2));  
d = diff(10*(log10(x))-x);  
syms x
```

```
% For Equation 1  
f = @(x) exp(x)-(1+x+((x.^2)/2));  
df = matlabFunction(c);  
[rootf1,Y] = newton(5,f,df,40,1e-6);
```

Iteration: 1,
Error (10 d.p.): 0.9122272122,
Root Estimate (10 d.p.): 4.0877727878

Iteration: 2,
Error (10 d.p.): 0.8467523165,
Root Estimate (10 d.p.): 3.2410204713

Iteration: 3,
Error (10 d.p.): 0.7536393235,
Root Estimate (10 d.p.): 2.4873811478

Iteration: 4,
Error (10 d.p.): 0.6378592972,
Root Estimate (10 d.p.): 1.8495218507

Iteration: 5,
Error (10 d.p.): 0.5123353333,
Root Estimate (10 d.p.): 1.3371865174

Iteration: 6,
Error (10 d.p.): 0.3922797127,
Root Estimate (10 d.p.): 0.9449068047

Iteration: 7,
Error (10 d.p.): 0.2887557108,
Root Estimate (10 d.p.): 0.6561510939

Iteration: 8,
Error (10 d.p.): 0.2062676685,
Root Estimate (10 d.p.): 0.4498834254

Iteration: 9,
Error (10 d.p.): 0.1441774354,
Root Estimate (10 d.p.): 0.3057059899

Iteration: 10,
Error (10 d.p.): 0.0992545308,
Root Estimate (10 d.p.): 0.2064514591

Iteration: 11,
Error (10 d.p.): 0.0676172018,
Root Estimate (10 d.p.): 0.1388342573

Iteration: 12,
Error (10 d.p.): 0.0457377741,
Root Estimate (10 d.p.): 0.0930964832

Iteration: 13,
Error (10 d.p.): 0.0307899300,
Root Estimate (10 d.p.): 0.0623065532

Iteration: 14,
Error (10 d.p.): 0.0206605692,
Root Estimate (10 d.p.): 0.0416459840

Iteration: 15,
Error (10 d.p.): 0.0138336839,
Root Estimate (10 d.p.): 0.0278123001

Iteration: 16,
Error (10 d.p.): 0.0092492402,
Root Estimate (10 d.p.): 0.0185630599

Iteration: 17,
Error (10 d.p.): 0.0061781029,
Root Estimate (10 d.p.): 0.0123849570

Iteration: 18,
Error (10 d.p.): 0.0041240547,
Root Estimate (10 d.p.): 0.0082609022

Iteration: 19,
Error (10 d.p.): 0.0027517374,
Root Estimate (10 d.p.): 0.0055091648

Iteration: 20,
Error (10 d.p.): 0.0018355449,
Root Estimate (10 d.p.): 0.0036736199

Iteration: 21,
Error (10 d.p.): 0.0012241650,
Root Estimate (10 d.p.): 0.0024494549

Iteration: 22,
Error (10 d.p.): 0.0008163182,
Root Estimate (10 d.p.): 0.0016331367

Iteration: 23,
Error (10 d.p.): 0.0005443048,
Root Estimate (10 d.p.): 0.0010888320

Iteration: 24,
Error (10 d.p.): 0.0003629111,
Root Estimate (10 d.p.): 0.0007259209

Iteration: 25,
Error (10 d.p.): 0.0002419592,
Root Estimate (10 d.p.): 0.0004839617

Iteration: 26,
Error (10 d.p.): 0.0001613140,
Root Estimate (10 d.p.): 0.0003226477

Iteration: 27,
Error (10 d.p.): 0.0001075452,
Root Estimate (10 d.p.): 0.0002151025

Iteration: 28,
Error (10 d.p.): 0.0000716917,
Root Estimate (10 d.p.): 0.0001434109

Iteration: 29,
Error (10 d.p.): 0.0000478039,
Root Estimate (10 d.p.): 0.0000956070

Iteration: 30,
Error (10 d.p.): 0.0000318699,
Root Estimate (10 d.p.): 0.0000637371

Iteration: 31,
Error (10 d.p.): 0.0000212069,
Root Estimate (10 d.p.): 0.0000425302

Iteration: 32,
Error (10 d.p.): 0.0000142396,
Root Estimate (10 d.p.): 0.0000282906

Iteration: 33,
Error (10 d.p.): 0.0000094326,
Root Estimate (10 d.p.): 0.0000188581

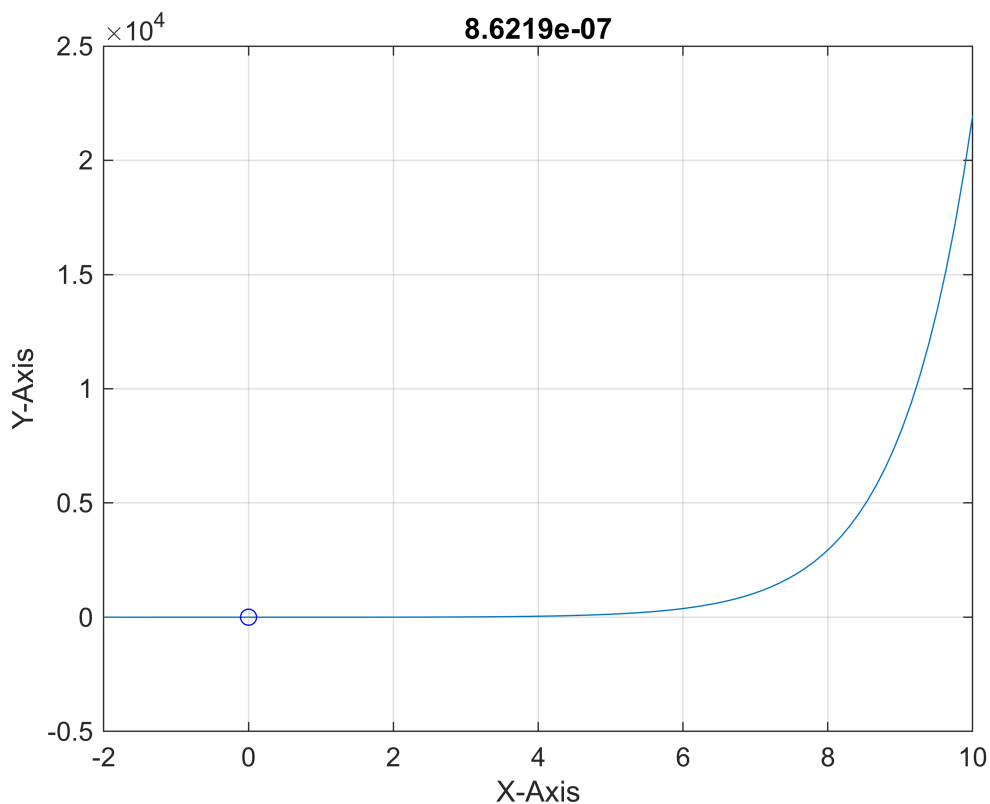
Iteration: 34,
Error (10 d.p.): 0.0000074924,
Root Estimate (10 d.p.): 0.0000113657

Iteration: 35,
Error (10 d.p.): 0.0000034378,
Root Estimate (10 d.p.): 0.0000079279

Iteration: 36,
Error (10 d.p.): 0.0000070657,
Root Estimate (10 d.p.): 0.000008622

Iteration: 37,
Error (10 d.p.): 0.0000000000,
Root Estimate (10 d.p.): 0.000008622

```
x = linspace(-2,10,100);  
f1 = exp(x)-(1+x+((x.^2)./2));  
figure(); plot(x,f1); grid on; hold on;  
plot(rootf1,Y,'bo'); hold off; title(num2str(rootf1));  
xlabel 'X-Axis'; ylabel 'Y-Axis';
```



```
fprintf('Does not achieve tolerance in 20 iterations so increased to %i \r \r' , 40)
```

Does not achieve tolerance in 20 iterations so increased to 40

2 Newtons $f(x) = 10\log(x) - x$ [3]

```
% For Equation 2
```

```
f = @(x) 10*log10(x)-x;  
df = matlabFunction(d);  
[rootf2,Y] = newton(1,f,df,20,1e-6);
```

```
Iteration: 1,  
Error (10 d.p.): 0.2991374534,  
Root Estimate (10 d.p.): 1.2991374534
```

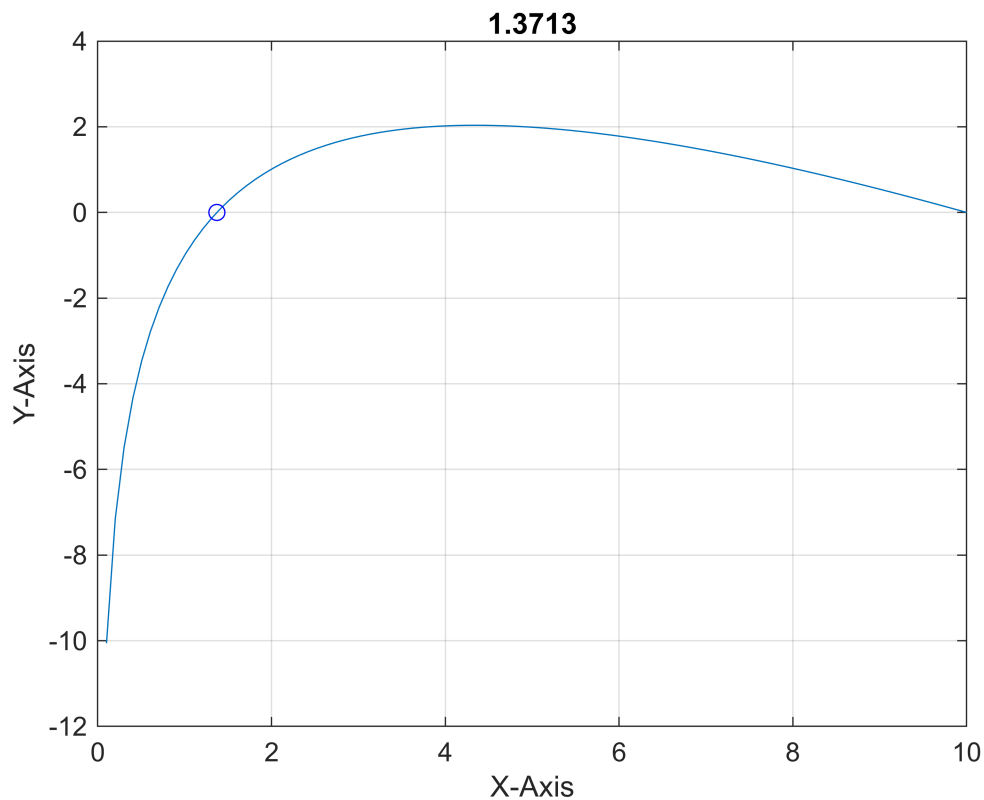
```
Iteration: 2,  
Error (10 d.p.): 0.0693940454,  
Root Estimate (10 d.p.): 1.3685314988
```

```
Iteration: 3,  
Error (10 d.p.): 0.0027530258,  
Root Estimate (10 d.p.): 1.3712845246
```

```
Iteration: 4,  
Error (10 d.p.): 0.0000040496,  
Root Estimate (10 d.p.): 1.3712885742
```

```
Iteration: 5,  
Error (10 d.p.): 0.0000000000,  
Root Estimate (10 d.p.): 1.3712885742
```

```
x = linspace(0,10,100);  
f1 = 10*log10(x)-x;  
figure(); plot(x,f1); grid on; hold on;  
plot(rootf2,Y,'bo'); hold off; title(num2str(rootf2));  
xlabel 'X-Axis'; ylabel 'Y-Axis';
```



3. Newton's Method – Intersection of two functions

3 Newtons Intersect [6]

```
syms x
p = diff(100.*exp(-x)-5.*sin((pi.*x)./2));
q = diff(5.*sin((pi.*x)./2));
syms x
```

```
f = @(x) 100.*exp(-x)-5.*sin((pi.*x)./2);
df = matlabFunction(p);
[Intersection,Y] = newton(4,f,df,20,1e-6);
```

```
Iteration: 1,
Error (10 d.p.): 0.1891028115,
Root Estimate (10 d.p.): 4.1891028115
```

```
Iteration: 2,
Error (10 d.p.): 0.0058190495,
Root Estimate (10 d.p.): 4.1949218610
```

```
Iteration: 3,
Error (10 d.p.): 0.0000097112,
Root Estimate (10 d.p.): 4.1949315722
```

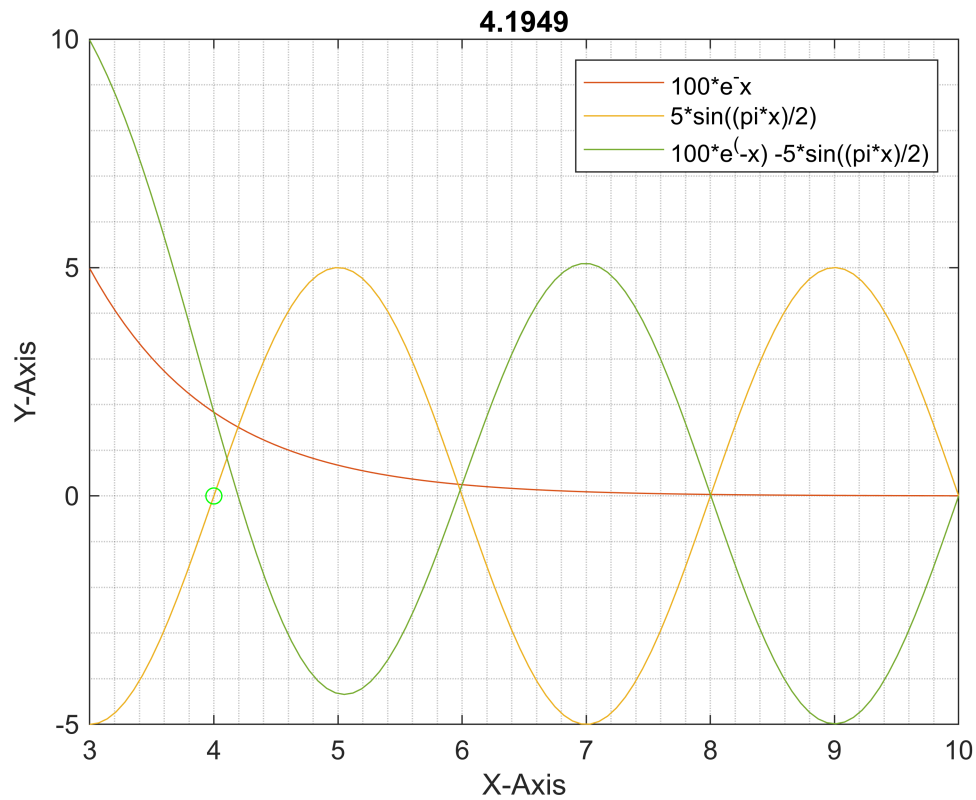
```
Iteration: 4,
```

Error (10 d.p.): 0.0000000000,
Root Estimate (10 d.p.): 4.1949315722

```
x = linspace(3,10,100);  
f = 5.*sin((pi.*x)./2);  
figure(); plot(x,f); xlim([3 10]); hold on;  
title(num2str(Intersection));  
  
f = @(x) 5.*sin((pi.*x)./2);  
df = matlabFunction(q);  
[Intersection,Y] = newton(4,f,df,20,1e-6);
```

Iteration: 1,
Error (10 d.p.): 0.0000000000,
Root Estimate (10 d.p.): 4.0000000000

```
x = linspace(3,10,100);  
f = 100.*exp(-x);  
plot(x,f);  
  
x = linspace(3,10,100);  
f = 5.*sin((pi.*x)./2);  
plot(x,f); plot(Intersection,Y,'og');  
  
x = linspace(3,10,100);  
f = 100.*exp(-x)-5.*sin((pi.*x)./2);  
plot(x,f); hold off; grid minor;  
xlabel 'X-Axis'; ylabel 'Y-Axis';  
legend('','100*e^-x','5*sin((pi*x)/2)','','100*e^(-x) -5*sin((pi*x)/2)')
```



```
f = @(x) 100.*exp(-x)-5.*sin((pi.*x)./2);
df = matlabFunction(p);

f = @(x) 5.*sin((pi.*x)./2);
df = matlabFunction(q);
fprintf('For Green circle on graph \r \r')
```

For Green circle on graph

```
[Intersection,Y] = newton(4,f,df,20,1e-10);
```

```
Iteration: 1,
Error (10 d.p.): 0.0000000000,
Root Estimate (10 d.p.): 4.0000000000
```

```
fprintf('1 Iteration(s) required \r \r')
```

1 Iteration(s) required

4. Bracketing (Multiple Roots)

4 Bracketing Bisection [6]

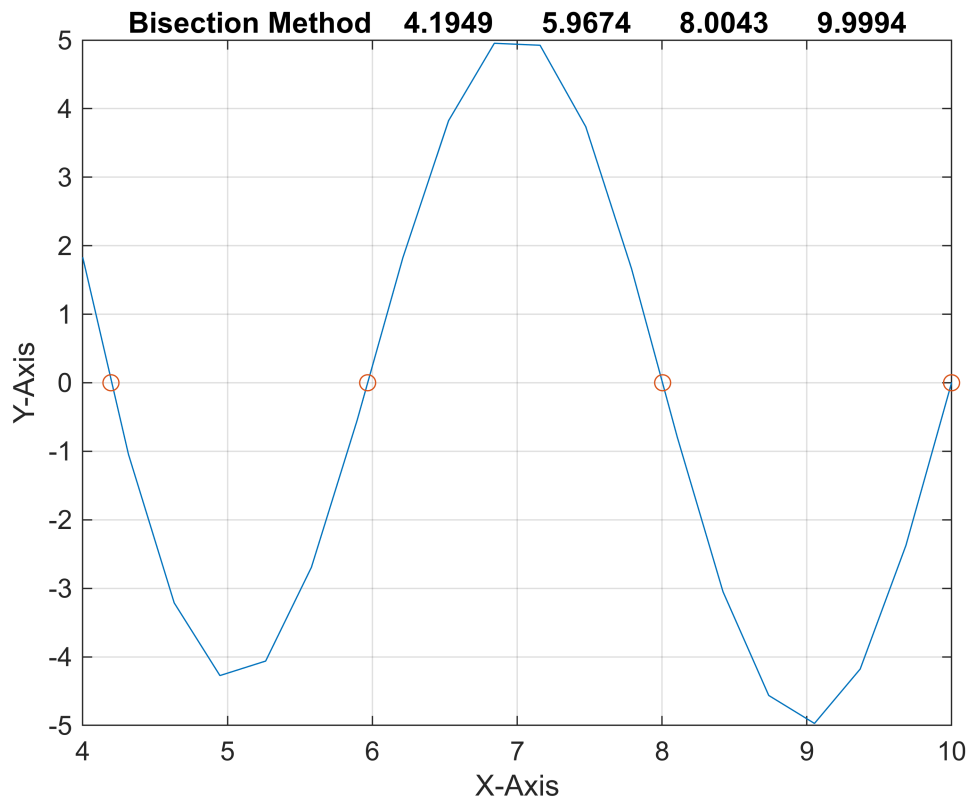
```
[Root,Iteration] = BisecSolve2('@(x)100.*exp(-x)-5.*sin((pi.*x)./2)',...
    4,10,20,1e-6,100);
```

Equation: $@(x)100 \cdot \exp(-x) - 5 \cdot \sin(\pi \cdot x / 2)$
 ITERATION 16
 Root Estimate (6 d.p.) = 4.194932

ITERATION 16
 Root Estimate (6 d.p.) = 5.967379

ITERATION 16
 Root Estimate (6 d.p.) = 8.004253

ITERATION 16
 Root Estimate (6 d.p.) = 9.999422



4 Bracketing Newton [6]

```
f = @(x) 100.*exp(-x)-5.*sin((pi.*x)./2);
df = matlabFunction(p);
[Intersection,Y] = newton2(4,f,df,20,1e-6,10,100);
```

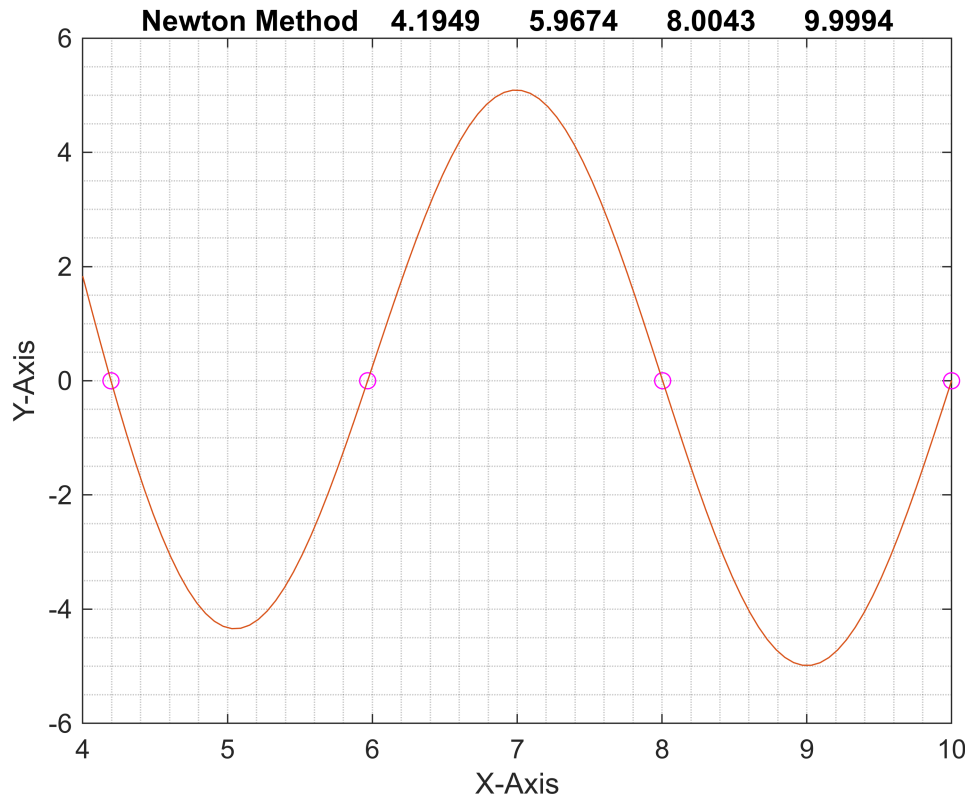
Root Estimates (6 d.p.): 4.194932

Root Estimates (6 d.p.): 5.967379

Root Estimates (6 d.p.): 8.004253

Root Estimates (6 d.p.): 9.999422

```
figure(); plot(Intersection,Y,'om'); hold on;
x = linspace(4,10,100); xlim([4 10]);
f = 100.*exp(-x)-5.*sin((pi.*x)./2);
plot(x,f); hold off; grid minor;
title(['Newton Method ',num2str(Intersection)]);
xlabel 'X-Axis'; ylabel 'Y-Axis';
```



Functions

```
function Root = BisecSolvePlot(EquationString,a,b,MaxCount,Tol)
f = str2func(EquationString);
fprintf('\n Equation: %s \r',EquationString)
Root = bisect(f,a,b,MaxCount,Tol);
x_val = linspace(a,b,MaxCount);
y_val(1,1:MaxCount) = NaN;
for i=1:MaxCount
y_val(1,i) = feval(f,x_val(i));
end
yRoot = feval(f,Root);
figure();plot(x_val,y_val,'',Root,yRoot,'o')
title(Root); grid on;
xlabel 'X-Axis'; ylabel 'Y-Axis';
```

end

```
function [Root,Iteration] = BisecSolve2(EquationString,a,b,MaxCount,Tol,n)
f = str2func(EquationString);
fprintf('\n Equation: %s \r',EquationString)
[Root,Iteration] = bisect2(f,a,b,MaxCount,Tol,n);
x_val = linspace(a,b,MaxCount);
y_val(1,1:MaxCount) = NaN;
for i=1:MaxCount
y_val(1,i) = feval(f,x_val(i));
end
yRoot = feval(f,Root);
figure();plot(x_val,y_val,'',Root,yRoot,'o')
title(['Bisection Method ',num2str(Root)]); grid on;
xlabel 'X-Axis'; ylabel 'Y-Axis';
end
```