

Mechanics Assignment

by Lee Wellington

Submission date: 15-May-2020 12:11PM (UTC+0100)

Submission ID: 127753075

File name: 15632_Lee_Wellington_Mechanics_Assignment_177944_1137048343.docx (3.49M)

Word count: 1770

Character count: 9086

Thermofluids and Applied Mechanics (Applied Mechanics element)

Lee Wellington – LW094662

Assignment 1 – Complex Static and Dynamic Systems.

Table of Contents

Table of Contents	1
List of Figures & Tables	2
Question 1	3-7
Question 2	8-11
Question 3	12-13
Question 4	14-18
Question 5	18-21
References	21
Bibliography	22

List of Figures & Tables

Figure 1 – Additional Mass Added to Wheel	3
Figure 2 – Static Imbalance Vs Dynamic Imbalance.....	4
Figure 3 – Diagram of Balancing Rig.....	4
Figure 4 – Final Arrangement to Give Static & Dynamic Balance	7
Figure 5 – Main Elements of Epicyclic Gearbox.....	8
Figure 6 – Example of a Complex Epicyclic Gear Train.....	10
Figure 7 – Diagram of Gear Train	11
Figure 8 – Diagram of Lifting Device	12
Figure 9 - High Pressure Pipelines.....	14
Figure 10 – Thick Walled Pressure Vessel.....	14
Figure 11 – Diagram of Steel Pipe.....	15
Figure 12 – Radial Stress Variation Profile	17
Figure 13 – Circumferential Stress Variation Profile	17
Figure 14 – Diagram of Shaft	18
Table 1 – MRL Table.....	5
Table 2 – Gear Ratios When Sun is Fixed	9
Table 3 – Gear Ratios When Ring is Fixed	9
Table 4 – Gear Ratios When All Gears Are Free to Rotate	10
Table 5 – Task 2 Gear Ratios.....	11
Table 6 – Radial and Circumferential Stresses	16

Question 1

Task A

Balancing is an important factor when considering the operation of a rotational device as it can eliminate unwanted characteristics such as vibration and noise. These detrimental attributes can affect the efficiency and safe operation of a system and in some severe cases even cause premature failure due to fatigue. From converting Newton's second law of motion ($F = ma$) into angular form ($F = m\omega^2 r$) it is evident that the centrifugal force produced through rotation is directly proportional to the angular velocity squared. Meaning the effects of an unbalanced force will be more severe the faster a system rotates. To reduce the effects of imbalance it is advantageous to counteract any 'out of balance' forces by deliberately adding additional mass. A simple example of this is when a car wheel is balanced and extra mass is added to a specific point on the wheel rim as shown in figure 1.



Figure 1 – Additional Mass Added to Wheel



(Plombco, 2020)

The two main types of balancing that can be achieved in rotational motion;

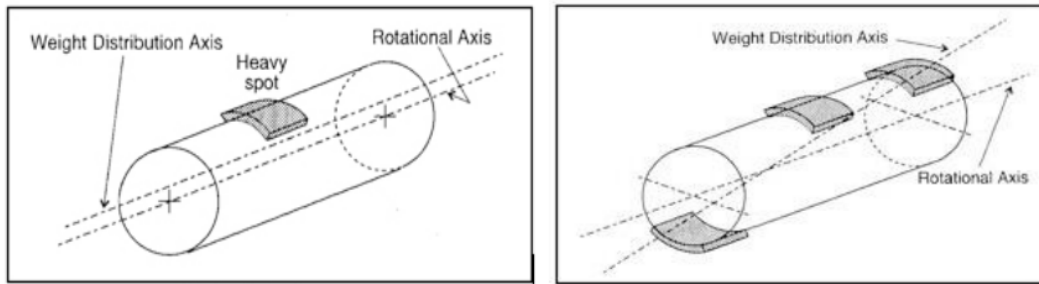
1. Static Balancing

Static balancing occurs when all masses acting on a rotating shaft are positioned on the same plane. To achieve complete balance in this scenario the sum of the centrifugal forces must equal zero ($\sum m\omega^2 r = 0$). When completely balanced the centre of mass will be positioned in the axis of rotation.

2. Dynamic Balancing

Dynamic balancing occurs when the masses acting on a rotating shaft are positioned at different planes along the shaft. This non-coplanar scenario requires the sum of the centrifugal forces to equal zero ($\sum m\omega^2 r = 0$) in addition to the sum of the moments also equalling zero ($\sum m\omega^2 r * L = 0$).

Figure 2 – Static Imbalance Vs Dynamic Imbalance



(XYO BALANCER.COM, 2011)

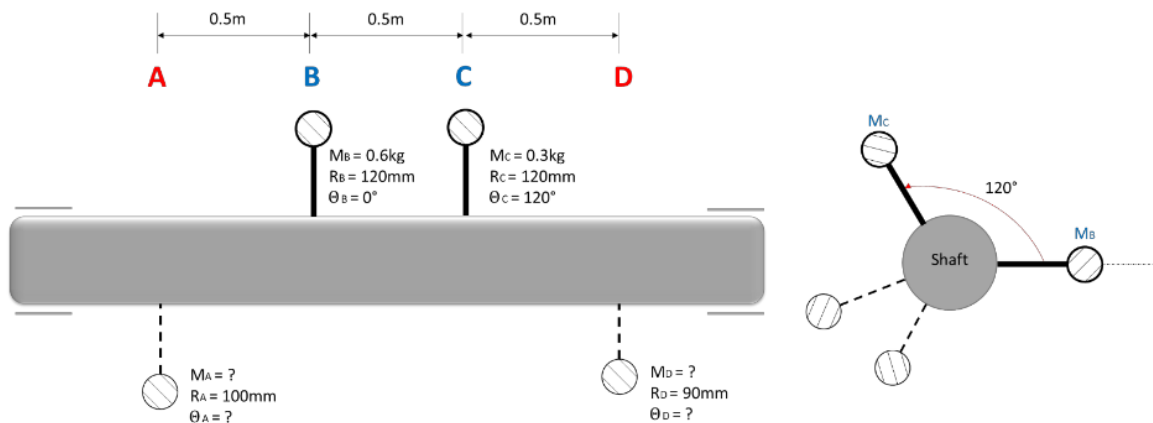
Balancing is of particular importance when designing high speed rotating or reciprocating components for the following machinery;

- Engines
- Pumps
- Fans
- Compressors

Task B

An experimental balancing rig consists of four discs A,B,C and D mounted on a rotating shaft as shown in figure 3.

Figure 3 – Diagram of Balancing Rig



From extracting key information, the following MRL table was created.

Table 1 – MRL Table

Plane	Mass (kg)	Radius (m)	MR (kgm)	L (m)	MRL (kgm ²)	θ
A	M_A	0.1	$0.1 M_A$	0	0	θ_A
B	0.6	0.12	0.072	0.5	0.036	0
C	0.3	0.12	0.036	1.0	0.036	120
D	M_D	0.09	$0.09 M_D$	1.5	$0.135 M_D$	θ_D

To determine the magnitude and angular position of masses A and D (relative to mass B) the following calculations were performed.

Step 1 – Balance the Moments

Horizontal Components ($\rightarrow$)

$$0.036 \cos(0) + 0.036 \cos(120) + 0.135 M_D \cos(\theta_D) = 0$$

$$0.036 + (-0.018) + 0.135 M_D \cos(\theta_D) = 0$$

$$0.135 M_D \cos(\theta_D) = -0.018$$

$$M_D \cos(\theta_D) = \frac{-0.018}{0.135}$$

$$M_D \cos(\theta_D) = -0.13333$$

1

Vertical Components ($\uparrow$)

$$0.036 \sin(0) + 0.036 \sin(120) + 0.135 M_D \sin(\theta_D) = 0$$

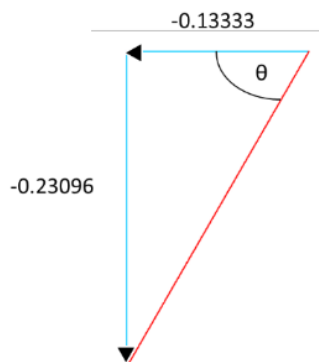
$$0 + 0.03118 + 0.135 M_D \sin(\theta_D) = 0$$

$$0.135 M_D \sin(\theta_D) = -0.03118$$

$$M_D \sin(\theta_D) = \frac{-0.03118}{0.135}$$

$$M_D \sin(\theta_D) = -0.23096$$

2



$$\tan(\theta) = \frac{O}{A}$$

$$\therefore \theta = \tan^{-1}\left(\frac{-0.23096}{-0.13333}\right)$$

$$\theta = 60^\circ$$

$$\therefore \theta_D = 60 + 180$$

$$\theta_D = 240^\circ$$

From substituting $\theta_D = 240^\circ$ into equation 2.

$$MD \sin(240) = -0.23096$$

$$MD(-0.86603) = -0.23096$$

$$MD = \frac{-0.23096}{-0.86603}$$

$$MD = 0.267 \text{ kg (3dp)}$$

Step 2 – Balance the Forces

Horizontal Components ($\rightarrow$)

$$0.1MA \cos(\theta_A) + 0.072 \cos(0) + 0.036 \cos(120) + 0.02403 \cos(240) = 0$$

$$0.1MA \cos(\theta_A) + 0.072 + (-0.018) + (-0.01202) = 0$$

$$0.1MA \cos(\theta_A) = -0.04198$$

$$MA \cos(\theta_A) = \frac{-0.04198}{0.1}$$

$$MA \cos(\theta_A) = -0.4198$$

1

Vertical Components ($\uparrow$)

$$0.1MA \sin(\theta_A) + 0.072 \sin(0) + 0.036 \sin(120) + 0.02403 \sin(240) = 0$$

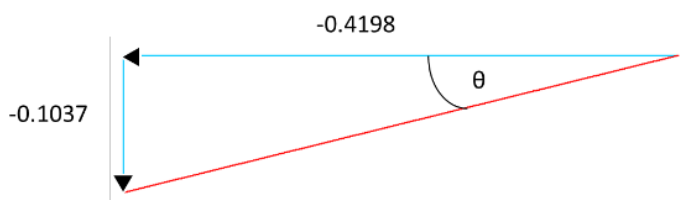
$$0.1MA \sin(\theta_A) + 0 + 0.03118 + (-0.02081) = 0$$

$$0.1MA \sin(\theta_A) = -0.01037$$

$$MA \sin(\theta_A) = \frac{-0.01037}{0.1}$$

$$MA \sin(\theta_A) = -0.1037$$

2



$$\tan(\theta) = \frac{O}{A}$$

$$\therefore \theta = \tan^{-1}\left(\frac{-0.1037}{-0.4198}\right)$$

$$\theta = 13.88^\circ (2dp)$$

$$\therefore \theta_A = 13.88 + 180$$

$$\theta_A = 193.88^\circ$$

From substituting $\theta_A = 193.88^\circ$ into equation 2.

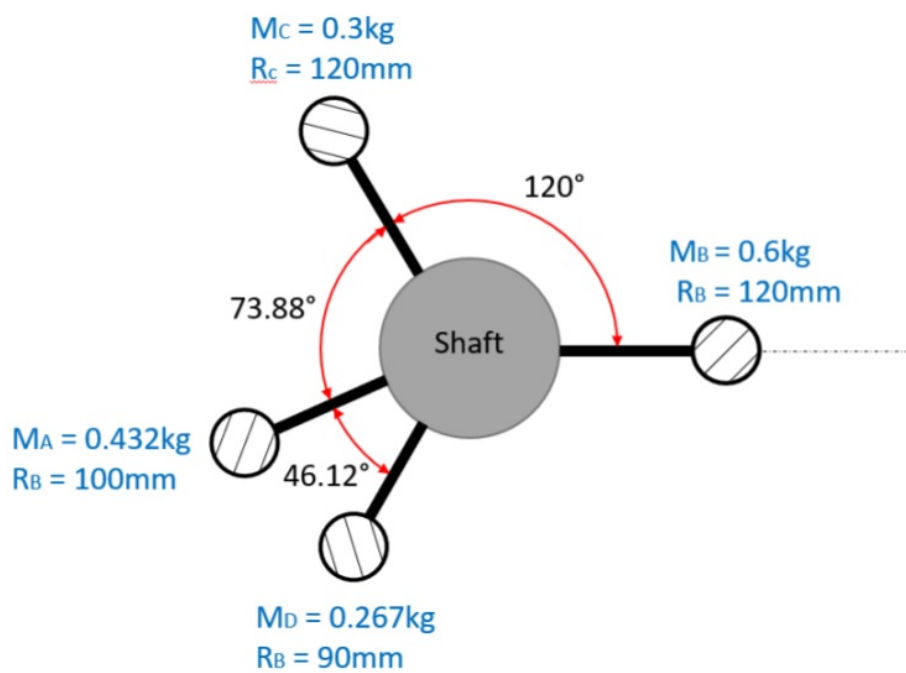
$$M_A \sin(193.88) = -0.1037$$

$$M_A(-0.23989) = -0.1037$$

$$M_A = \frac{-0.1037}{-0.23989}$$

$$M_A = 0.432\text{kg (3dp)}$$

Figure 4 – Final Arrangement to Give Static & Dynamic Balance

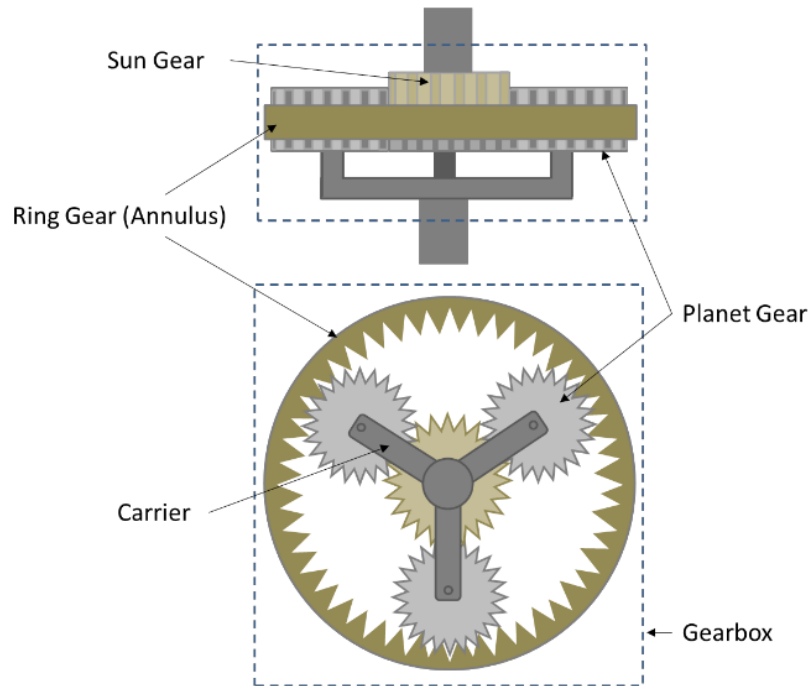


Question 2

Task 1A

A typical epicyclic gearbox consists of the following main elements shown in figure 5.

Figure 5 – Main Elements of Epicyclic Gearbox



Task 1B

Sun Gear – As the name suggests the sun gear is the central gear that meshes with one or more planet gears.

Ring (Annulus) Gear – The ring gear which also referred to as the annulus is an outer ring with internal gears that mesh with one or more planet gears.

Planet Gears – Planet gears are typically held by a carrier and fit between the ring gear and the sun gear. Planet gears can rotate about their axis independently whilst orbiting about the sun gear.

Carrier – The carrier primarily keeps the planet gears evenly spaced apart. It can also be used as a form of input or output if connect to a main shaft.

Task 1C

i). When the arm on a standard epicyclic gearbox is fixed there is no orbiting motion therefore the gear ratio can be derived from simply dividing the number of teeth on the ring gear by the number of teeth on the sun gear.

$$\text{Gear Ratio} = -\left(\frac{\text{Number of Teeth on Ring Gear (R)}}{\text{Number of Teeth on Sun Gear (S)}}\right)$$
$$G.R = -\frac{R}{S}$$

**** negative sign indicates change in direction ****

Therefore, if the sun gear was the input and the ring gear was the output.

$$\text{Sun Gear (Input - Anticlockwise) : Ring Gear (Output - Clockwise)}$$
$$-(R/S) : 1$$

ii). When the sun is fixed the following expressions can be derived to identify the gear ratio achieved.

Table 2 – Gear Ratios When Sun is Fixed

	Ring Gear	Planet Gear	Sun Gear (fixed)	Carrier (arm)
Rotate whole train +1 revolution	+1	+1	+1	+1
Fix arm and rotate fixed gear -1 revolution	1.(S/R)	1.(S/P)	-1	0
Add lines 1 and 2	1+(S/R)	1+(S/P)	0	1

iii). When the ring is fixed the following expressions can be derived to identify the gear ratio achieved.

Table 3 – Gear Ratios When Ring is Fixed

	Ring Gear (fixed)	Planet Gear	Sun Gear	Carrier (arm)
Rotate whole train +1 revolution	+1	+1	+1	+1
Fix arm and rotate fixed gear -1 revolution	-1	-1(R/P)	1.(R/S)	0
Add lines 1 and 2	0	1-(R/P)	1+(R/S)	1

Task 1D

When all gears are free to rotate the following expressions can be derived to identify the gear ratio achieved.

Table 4 – Gear Ratios When All Gears Are Free to Rotate

	Ring Gear	Planet Gear	Sun Gear	Carrier (arm)
Rotate whole train +1 revolution	+m	+m	+m	+m
Fix arm and rotate fixed gear -1 revolution	-n.(S/R)	-n.(S/P)	+n	0
Add lines 1 and 2	m-n.(S/R)	m-n.(S/P)	m+n	m

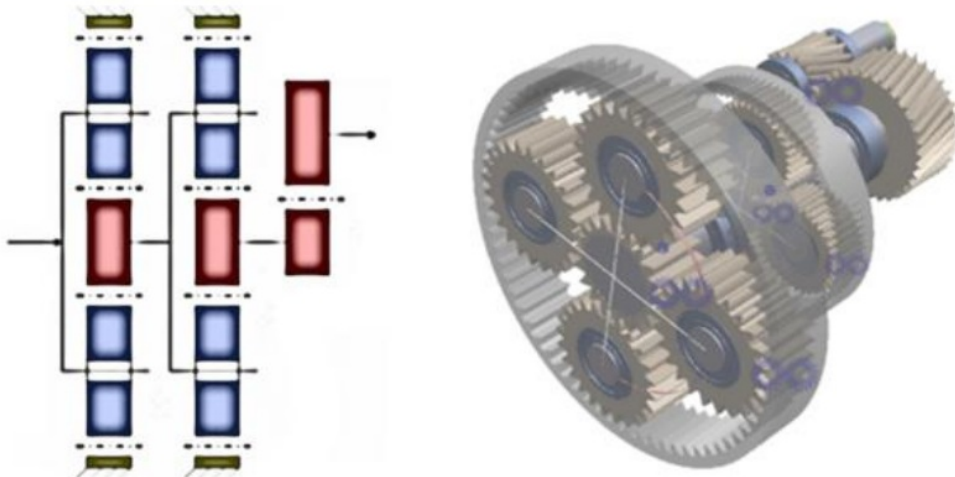
Task 1E

Epicyclic gearboxes are typically ‘used in various automobiles running under Automatic Transmission’ (Mechanical Students, 2020) as they have the ability to provide high ratios in a much more confined space in comparison to standard gear trains. As previously demonstrated the planetary arrangement enables various gear ratio combinations to be achieved depending on the chosen input, output and constraints. Due to the multiple gears involved, epicyclic gear trains provide a much more balanced transmission that can bear high loads.

Task 1F

An example of a complex epicyclic gear train can be seen below in figure 6.

Figure 6 – Example of a Complex Epicyclic Gear Train



As shown in figure 6 the gear train uses compound gearing to create multiple planetary stages. From having 2 gears mounted on the same shaft the output of the 1st planetary stage can become the input for secondary stage. This enables very large gear ratios to be generated using relatively small gears. If the same gear ratios were to be produced using standard gearing, then much larger gearbox with large gears would be required.



Task 2

Figure 7 – Diagram of Gear Train

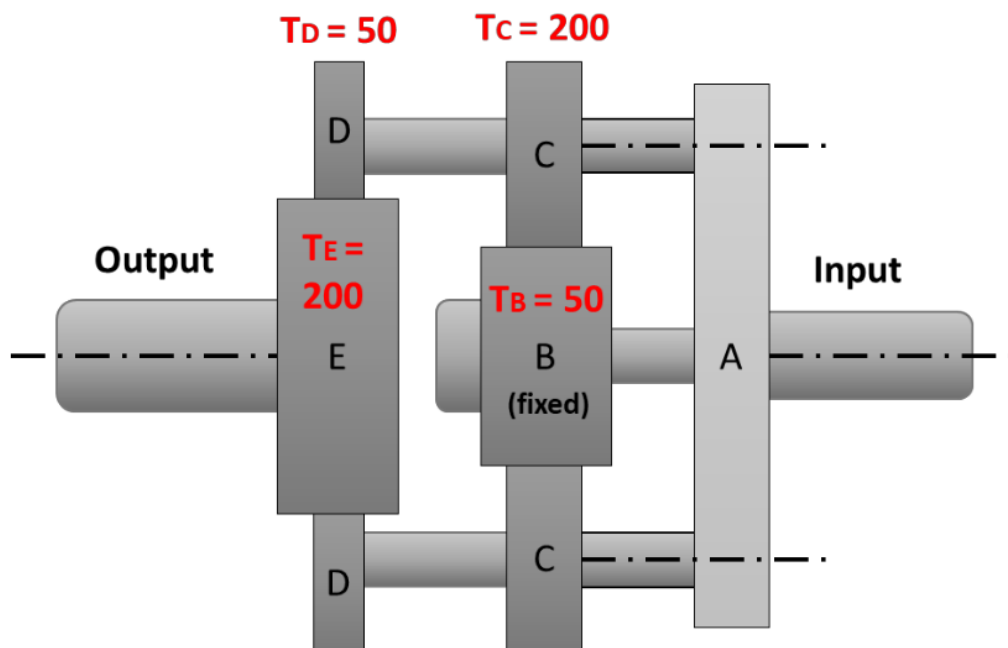


Table 5 – Task 2 Gear Ratios

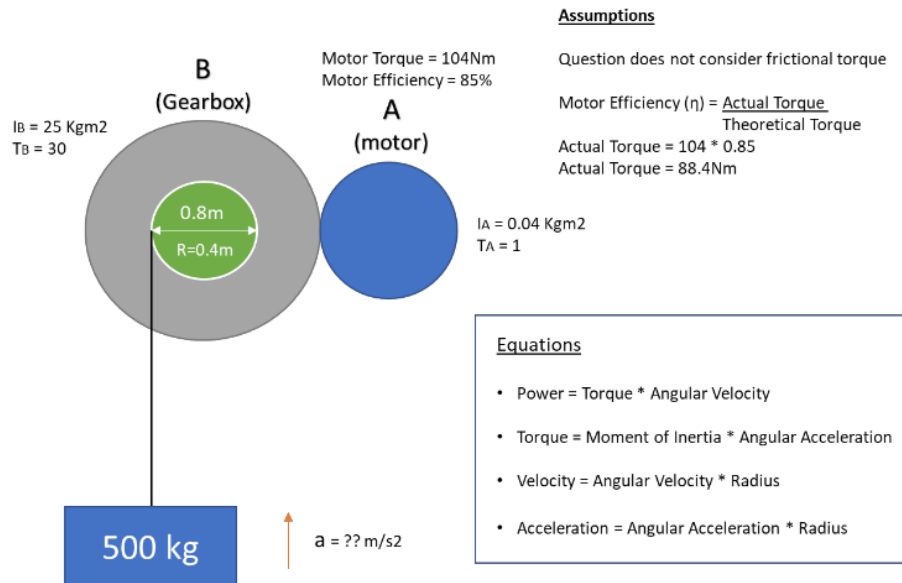
Line No.	Action	A	B	C/D	E
1	Rotate Whole Train +1 Rev	+1	+1	+1	+1
2	Fix Arm & Rotate Fixed Gear -1 Rev	0	-1	$+1 * \left(\frac{50}{200}\right)$	$-1 * \left(\frac{50}{200}\right) \left(\frac{50}{200}\right)$
3	Add Lines 1 & 2	1	0	1.25	0.9375

Gearbox Ratio = 0.9375 : 1 or 15 : 16



Question 3

Figure 8 – Diagram of Lifting Device



To calculate the vertical acceleration of the load (neglecting mass of cable) the following calculations were performed.

$$T\omega_A = ((I_A * \alpha_A) * \omega_A) + ((I_B * \alpha_B) * \omega_B) + (m(g + a) * r_B * \omega_B)$$

$$\alpha_B = \frac{a}{r} \rightarrow \alpha_B = \frac{a}{0.4} \quad \text{Therefore} \quad \alpha_A = \left(\frac{a}{0.4}\right) * \left(\frac{30}{1}\right)$$

$$88.4 * \omega_A = (0.04 * \left(\frac{a}{0.4} * \frac{30}{1}\right) * \omega_A) + (25 * \left(\frac{a}{0.4}\right) * \omega_B) + (500 * (9.81 + a) * 0.4 * \omega_B)$$

Now

$$\frac{\omega_B}{\omega_A} = \frac{T_A}{T_B} \rightarrow \frac{\omega_B}{\omega_A} = \frac{1}{30} \quad \text{Therefore} \quad \omega_B = \frac{1}{30} * \omega_A$$

$$88.4 * \cancel{\omega_A} = (0.04 * \left(\frac{a}{0.4} * \frac{30}{1}\right) * \cancel{\omega_A}) + (25 * \left(\frac{a}{0.4}\right) * \left(\frac{1}{30}\right) * \cancel{\omega_A}) + (500 * (9.81 + a) * 0.4 * \left(\frac{1}{30}\right) * \cancel{\omega_A})$$

$$88.4 = (0.04 * \left(\frac{a}{0.4} * \frac{30}{1}\right)) + (25 * \left(\frac{a}{0.4}\right) * \left(\frac{1}{30}\right)) + (500 * (9.81 + a) * 0.4 * \left(\frac{1}{30}\right))$$

$$88.4 = (0.04 * 75a) + (25 * (2.5a * \frac{1}{30})) + (500 * (9.81 + a) * 0.4 * (\frac{1}{30}))$$

$$88.4 = 3a + 2.083a + 65.4 + 6.667a$$

$$88.4 - 65.4 = 11.75a$$

$$\frac{23}{11.75} = a$$

$$a = 1.96 \text{ m/s}^2 \text{ (2dp)}$$



Question 4

Task A

To determine the difference between a thin and thick cylinder the wall thickness must be considered. For a thin cylinder '*the wall thickness is less than $1/20^{\text{th}}$ of tube or cylinder diameter*' (The Engineering Toolbox, 2020). Subsequently for thick cylinders '*the wall thickness exceeds $1/20^{\text{th}}$ of the diameter*' (Freestudy, 2020). Examples of thick cylinders are;

Figure 9 - High Pressure Pipelines



(Oli Quip Limited, 2020)

Figure 10 – Thick Walled Pressure Vessel



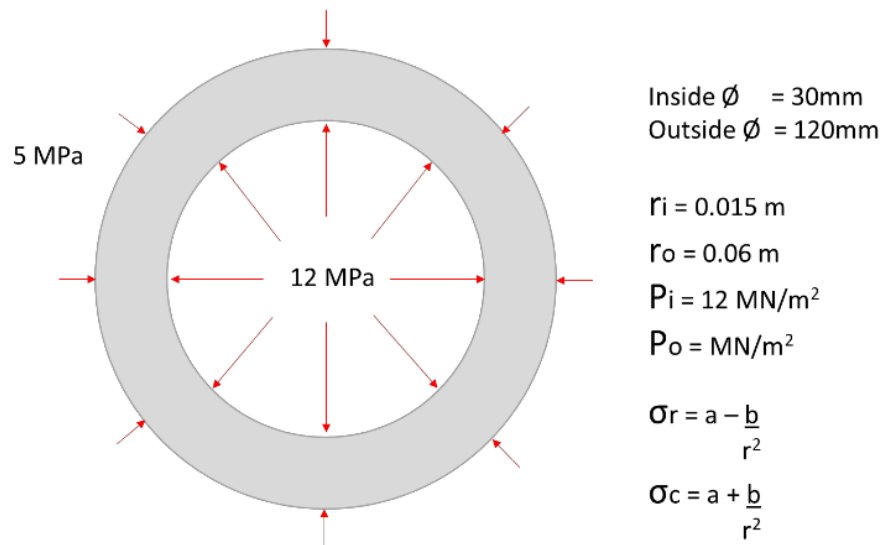
(CN Asia, 2020)

Typically, the radial stresses on a thin walled cylinder in comparison to the longitudinal and circumferential stresses are negligible therefore they are neglected. However, when designing thick cylinders, the radial stresses are more significant therefore they must be considered. Additionally, the stresses are deemed constant across the thickness for thin cylinders however for thick cylinders they are considered variable.



Task B

Figure 11 – Diagram of Steel Pipe



Apply Boundary Conditions

When;

$$\begin{aligned} r = 0.015\text{ m} & \quad \sigma_r = -12\text{ MN/m}^2 \\ r = 0.06\text{ m} & \quad \sigma_r = -5\text{ MN/m}^2 \end{aligned}$$

Therefore;

$$\begin{aligned} -12 \times 10^6 &= a - \frac{b}{0.015^2} & \text{--- (1)} \\ -5 \times 10^6 &= a - \frac{b}{0.06^2} & \text{--- (2)} \end{aligned}$$

From equation 2

$$-5 \times 10^6 = a - 277.778b$$

$$a = -5 \times 10^6 + 277.778b$$

Substitute into equation 1

$$-12 \times 10^6 = -5 \times 10^6 + 277.778b - 4444.444b$$

$$-12 \times 10^6 + 5 \times 10^6 = -4166.666b$$

$$\frac{-7 \times 10^6}{-4166.666} = b$$

$$b = 1680.00 \text{ (2dp)}$$

Therefore;

$$-5 \times 10^6 = a - \frac{1680}{0.06^2}$$

$$a = -5 \times 10^6 + 466666.667$$

$$a = -4533333.333$$

To calculate the circumferential stresses

$$\sigma_{ci} = -4533333.333 + \frac{1680}{0.015^2}$$

$$\sigma_{ci} = 2.933 \text{ MN/m}^2 \text{ (3dp)}$$

$$\sigma_{co} = -4533333.333 + \frac{1680}{0.06^2}$$

$$\sigma_{co} = -4.067 \text{ MN/m}^2 \text{ (3dp)}$$

(i)

Table 6 – Radial and Circumferential Stresses

	σ_r (MPa)	σ_c (MPa)
0.015	-12	2.933
0.06	-5	-4.067



(ii)

Figure 12 – Radial Stress Variation Profile

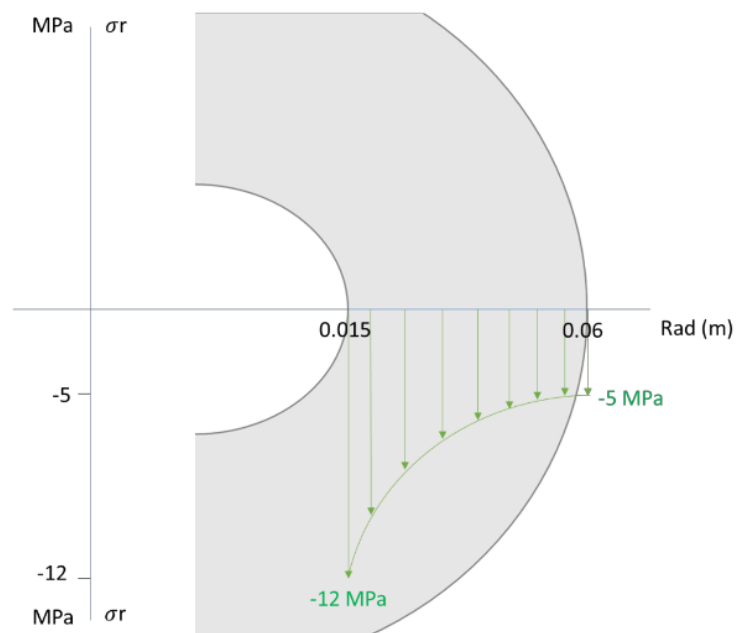
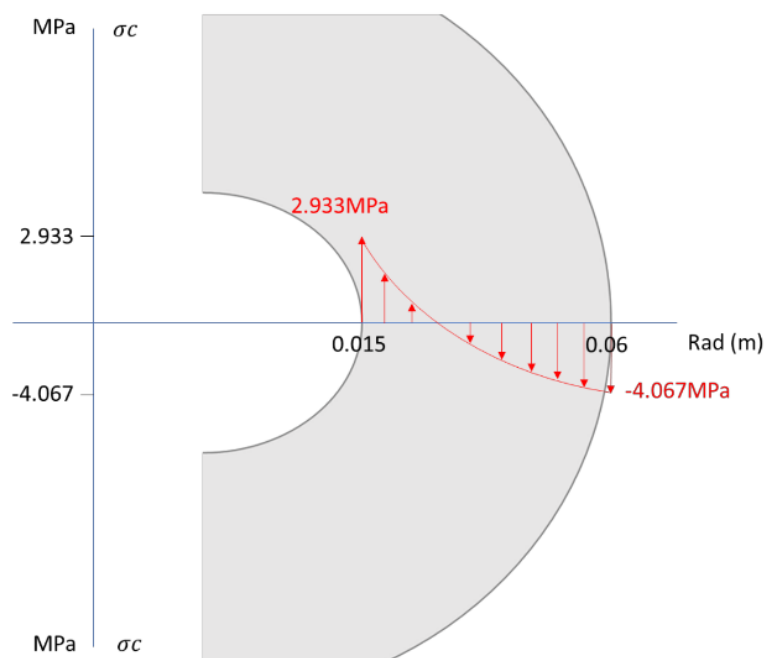
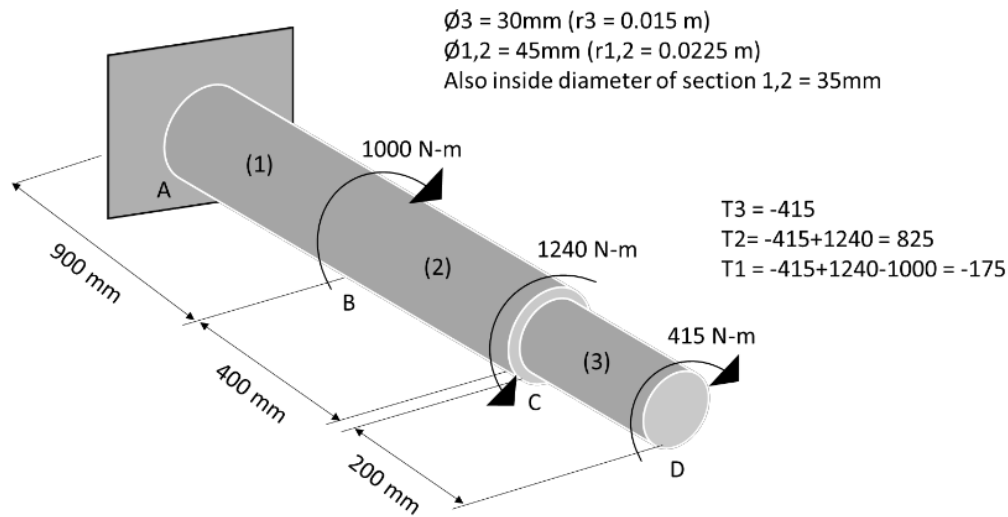


Figure 13 – Circumferential Stress Variation Profile



Question 5

Figure 14 – Diagram of Shaft



From;

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

The maximum stress in each section can be calculated using;

$$\tau = \frac{Tr}{J}$$

Section 3

$$\tau_3 = \frac{T_3 r_3}{J_3}$$

$$J_3 = \frac{\pi D^4}{32}$$

$$\tau_3 = \frac{415 * 0.015}{7.95 \times 10^{-8}}$$

$$J_3 = \frac{\pi * 0.03^4}{32}$$

$$\tau_3 = 78.302 \text{ MN/m}^2 \text{ (3dp)}$$

$$J_3 = 7.95 \times 10^{-8} \text{ m}^4 \text{ (2dp)}$$

Section 2

$$\tau_2 = \frac{T_2 r_2}{J_2}$$

$$J_2 = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

$$\tau_2 = \frac{825 * 0.0225}{2.55 \times 10^{-7}}$$

$$\tau_2 = 72.794 \text{ MN/m}^2 (3dp)$$

$$J_2 = \frac{\pi * 0.045^4}{32} - \frac{\pi * 0.035^4}{32}$$

$$J_2 = 2.55 \times 10^{-7} \text{ m}^4 (2dp)$$

Section 1

$$\tau_1 = \frac{T_1 r_1}{J_1}$$

$$J_2 = J_1$$

$$\tau_1 = \frac{175 * 0.0225}{2.55 \times 10^{-7}}$$

$$\tau_1 = 15.441 \text{ MN/m}^2 (3dp)$$

i) Maximum stress occurs in section 3 (78.302 MN/m²)

ii) Calculate angle of twist of each section using;

$$\theta = \frac{TL}{JG}$$

Section 3

$$\theta_3 = \frac{T_3 L_3}{J_3 G}$$

$$\theta_3 = \frac{-415 * 0.2}{7.95 \times 10^{-8} * 75 \times 10^9}$$

$$\theta_3 = -0.01392 \text{ Rads (5dp)}$$

Section 2

$$\theta_2 = \frac{T_2 L_2}{J_2 G}$$

$$\theta_2 = \frac{825 * 0.4}{2.55 \times 10^{-7} * 75 \times 10^9}$$

$$\theta_2 = 0.01725 \text{ Rads (5dp)}$$

Section 1

$$\theta_1 = \frac{T_1 L_1}{J_1 G}$$

$$\theta_1 = \frac{-175 * 0.9}{2.55 \times 10^{-7} * 75 \times 10^9}$$

$$\theta_1 = -0.00824 \text{ Rads (5dp)}$$

iii) Angle of rotation of D relative to A

$$\theta_D \rightarrow \theta_A = -0.00824 + 0.01725 + (-0.01392)$$

$$\theta_D \rightarrow \theta_A = -0.00491 \text{ Rads}$$

Converted to degrees

$$\theta_D \rightarrow \theta_A = \frac{-0.00491}{2\pi} * 360$$

$$\theta_D \rightarrow \theta_A = -0.28^\circ (2dp)$$



References

- CN Asia, 2020. *Thick Walled Pressure Vessels*. [Online]
Available at: <http://www.cnasia.com/products/thick-wall-pressure-vessels/>
[Accessed 04th May 2020].
- Freestudy, 2020. *Thin Walled Vessels and Thick Walled Cylinders*. [Online]
Available at: <http://www.freestudy.co.uk/statics/complex/t1.pdf>
[Accessed 03rd May 2020].
- Mechanical Students, 2020. *Epicyclic Gearbox: Components, Working Principle, Advantages, Disadvantages, and Applications*. [Online]
Available at: <https://mechanicalstudents.com/epicyclic-gearbox/>
[Accessed 2nd May 2020].
- Oli Quip Limited, 2020. *High Pressure Seamless Pipe*. [Online]
Available at: <http://www.oilquip.co.uk/highpressureseamlesspipe.html>
[Accessed 04th May 2020].
- Plombco, 2020. *Wheel Balancing Weights*. [Online]
Available at: <https://www.plombco.com/>
[Accessed 20th April 2020].
- Power Transmission World, 2014. *Powerful analysis of wind turbine gearboxes*. [Online]
Available at: <http://www.powertransmissionworld.com/powerful-analysis-of-wind-turbine-gearboxes/>
[Accessed 3rd May 2020].
- The Engineering Toolbox, 2020. *Stress in Thin-Walled Tubes or Cylinders*. [Online]
Available at: https://www.engineeringtoolbox.com/stress-thin-walled-tube-d_948.html
[Accessed 03rd May 2020].
- XYO BALANCER.COM, 2011. *Vibration in Rotating Equipment*. [Online]
Available at: <http://www.xyobalancer.com/xyo-balancer-blog/vibration-in-rotating-equipment>
[Accessed 21st April 2020].

Bibliography

- CN Asia, 2020. *Thick Walled Pressure Vessels*. [Online]
Available at: <http://www.cnasia.com/products/thick-wall-pressure-vessels/>
[Accessed 04th May 2020].
- Davies, C., 2019. Course Handout - Epicyclic Gear Trains. In: s.l.:s.n.
- Davies, C., 2020. Course Handout - Balancing of Rotating Machinery. In: s.l.:s.n.
- Freestudy, 2020. *Thin Walled Vessels and Thick Walled Cylinders*. [Online]
Available at: <http://www.freestudy.co.uk/statics/complex/t1.pdf>
[Accessed 03rd May 2020].
- Gear Solutions, 2008. *Epicyclic Gearing - A Handbook*. [Online]
Available at: <http://gearsolutions.com/features/epicyclic-gearing-a-handbook/>
[Accessed 28th April 2020].
- Mechanical Students, 2020. *Epicyclic Gearbox: Components, Working Principle, Advantages, Disadvantages, and Applications*. [Online]
Available at: <https://mechanicalstudents.com/epicyclic-gearbox/>
[Accessed 2nd May 2020].
- Oli Quip Limited, 2020. *High Pressure Seamless Pipe*. [Online]
Available at: <http://www.oilquip.co.uk/highpressureseamlesspipe.html>
[Accessed 04th May 2020].
- Plombco, 2020. *Wheel Balancing Weights*. [Online]
Available at: <https://www.plombco.com/>
[Accessed 20th April 2020].
- Power Transmission World, 2014. *Powerful analysis of wind turbine gearboxes*. [Online]
Available at: <http://www.powertransmissionworld.com/powerful-analysis-of-wind-turbine-gearboxes/>
[Accessed 3rd May 2020].
- The Engineering Toolbox, 2020. *Stress in Thin-Walled Tubes or Cylinders*. [Online]
Available at: https://www.engineeringtoolbox.com/stress-thin-walled-tube-d_948.html
[Accessed 03rd May 2020].
- XYO BALANCER.COM, 2011. *Vibration in Rotating Equipment*. [Online]
Available at: <http://www.xyobalancer.com/xyo-balancer-blog/vibration-in-rotating-equipment>
[Accessed 21st April 2020].