

### Exercise 1:

A researcher is interested in differences between blondes, brunettes and redheads in terms of introversion/extroversion. She selects random samples from a college campus, gives each subject a test of social introversion and comes up with the results given

| Blondes | Brunettes | Redheads |
|---------|-----------|----------|
| 5       | 3         | 2        |
| 10      | 5         | 1        |
| 6       | 2         | 7        |
| 2       | 4         | 2        |
| 5       | 3         | 2        |
| 3       | 5         | 3        |

#### 1. State your hypotheses

$H_0$ : There is no difference in introversion levels between different groups

$H_1$ : There are differences between introversion levels in the groups

#### 2. Select $\alpha$ , your significance level

$\alpha = 0.05$

#### 3. Calculate $F$

Using the single-factor ANOVA test in Excel,  $F = 1.82$

#### 4. Find $F_{crit}$

Using the single-factor ANOVA test in Excel,  $F_{crit} = 3.68$

#### 5. Decide whether or not to reject the null hypothesis

$F$  is smaller than  $F_{crit}$  so we cannot reject the null hypothesis.

## Exercise 2

A company is thinking of replacing their current flavour of drink with two new flavours. They do a taste test with each of the new flavours and their current flavour and get their testers to rate the tastes out of ten. These are their results

| New flavour 1 | New flavour 2 | Current flavour |
|---------------|---------------|-----------------|
| 8             | 2             | 7               |
| 9             | 3             | 6               |
| 10            | 4             | 5               |
| 10            | 2             | 8               |
| 6             | 1             | 10              |
| 8             | 3             | 6               |
| 9             | 2             | 7               |

$$\hat{\omega}^2 = \frac{SS_B - (K - 1)MS_W}{SS_T + MS_W}$$

$SS_B$  = Sum of squares between groups

$SS_T$  = Sum of squares for the total

$MS_W$  = Mean square within groups

$K$  = number of groups

1. State your hypotheses

$H_0$ : There is no difference in response levels between the flavours

$H_1$ : There are differences in response levels between the flavours

2. Select  $\alpha$ , your significance level

$\alpha = 0.05$

3. Calculate  $F$

Using the single-factor ANOVA test in Excel,  $F = 38.4$

4. Find  $F_{crit}$

Using the single-factor ANOVA test in Excel,  $F_{crit} = 3.55$

5. Decide whether or not to reject the null hypothesis

$F$  is larger than  $F_{crit}$  so we must reject the null hypothesis.

6. If you reject the null hypothesis, calculate the strength of association

$$\hat{\omega}^2 = \frac{SS_B - (K - 1)MS_W}{SS_T + MS_W} = \frac{142.57 - (3 - 1)1.86}{176 + 1.86} = \frac{142.57 - 3.72}{177.86} = \frac{138.85}{177.86} = 0.78 = 78\%$$

### Exercise 3

Suppose we have three groups

Groups 1 and 2 have been given a treatment of some sort

Group 3 is the control group

Conduct an ANOVA test to identify if there are significant differences between groups

If such differences exist, carry out *post hoc* tests to identify which treatments made a significant difference

| Group 1 | Group 2 | Group 3 |
|---------|---------|---------|
| 22      | 6       | 8       |
| 16      | 10      | 6       |
| 17      | 13      | 4       |
| 18      | 13      | 5       |
|         | 8       | 2       |
|         | 4       |         |

1. State your hypotheses

$H_0$ : There is no differences between the groups

$H_1$ : There are differences between the groups

2. Select  $\alpha$ , your significance level

$\alpha = 0.05$

3. Calculate  $F$

Using the single-factor ANOVA test in Excel,  $F = 22.23$

4. Find  $F_{crit}$

Using the single-factor ANOVA test in Excel,  $F_{crit} = 3.89$

5. Decide whether or not to reject the null hypothesis

$F$  is larger than  $F_{crit}$  so we must reject the null hypothesis.

6. If you reject the null hypothesis, calculate the strength of association

$$\hat{\omega}^2 = \frac{SS_B - (K - 1)MS_W}{SS_T + MS_W} = \frac{402.98 - (3 - 1)9.06}{511.73 + 9.06} = \frac{402.98 - 18.12}{520.79} = \frac{384.86}{520.79} = 0.74 = 74\%$$

7. If you reject the null hypothesis, perform *post hoc* tests to identify those pairs of means that are significantly different.

Use the Scheffe post-hoc test for each pair of groups. Calculate  $C_{obt}$  for each pair and compare it with  $C_{crit}$  (which only needs calculating once).

$$C_{crit} = \sqrt{(K - 1)(F_{crit})} = \sqrt{(3 - 1)3.89} = \sqrt{7.78} = 2.79$$

$$\hat{\omega}^2 = \frac{SS_B - (K - 1)MS_W}{SS_T + MS_W}$$

$SS_B$  = Sum of squares between groups

$SS_T$  = Sum of squares for the total

$MS_W$  = Mean square within groups

$K$  = number of groups

$$C_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_W \left( \frac{1}{N_1} + \frac{1}{N_2} \right)}}$$

$\bar{X}_1, \bar{X}_2$  are the means of the two groups being compared

$N_1, N_2$  are the  $N$ s of the two groups

$MS_W$  is the within-group mean square from your ANOVA

$$C_{crit} = \sqrt{(K - 1)(F_{crit})}$$

$K$  is the number of groups

$F_{crit}$  is the critical value of the  $F$  test from your ANOVA

Group 1 v. Group 2

$$C_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_W \left( \frac{1}{N_1} + \frac{1}{N_2} \right)}} = \frac{18.25 - 9}{\sqrt{9.06 \left( \frac{1}{4} + \frac{1}{6} \right)}} = \frac{9.25}{\sqrt{9.06(0.42)}} = \frac{9.25}{\sqrt{3.81}} = \frac{9.25}{1.95} = 4.74$$

$C_{obt} > C_{crit}$  so there is a significant difference between groups 1 and 2

Group 1 v. Group 3

$$C_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_W \left( \frac{1}{N_1} + \frac{1}{N_2} \right)}} = \frac{18.25 - 5}{\sqrt{9.06 \left( \frac{1}{4} + \frac{1}{5} \right)}} = \frac{13.25}{\sqrt{9.06(0.45)}} = \frac{13.25}{\sqrt{4.08}} = \frac{13.25}{2.02} = 6.59$$

$C_{obt} > C_{crit}$  so there is a significant difference between groups 1 and 3

Group 2 v. Group 3

$$C_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_W \left( \frac{1}{N_1} + \frac{1}{N_2} \right)}} = \frac{9 - 5}{\sqrt{9.06 \left( \frac{1}{5} + \frac{1}{6} \right)}} = \frac{4}{\sqrt{9.06(0.37)}} = \frac{4}{\sqrt{3.35}} = \frac{4}{1.83} = 2.18$$

$C_{obt} < C_{crit}$  so there is no significant difference between groups 2 and 3