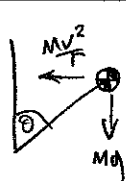
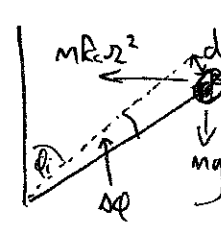




EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BEng Motorcycle Engineering	Level:	6
Module:	Advanced Motorcycle Dynamics		
Author:	Owen Williams	Page:	

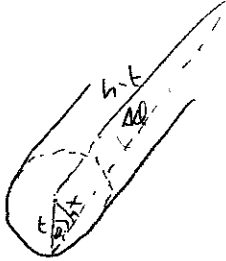
Q	Question	Marks
1a	 $\tan \theta_i = \frac{Mv^2}{r \times g} \quad \theta_i = \tan^{-1} \left(\frac{v^2}{rg} \right)$ $\theta_i = \tan^{-1} \left(\frac{(50 \times 5.8)^2}{25g} \right) = \underline{38.2^\circ}$	(3)
b	$G_m = I \omega^2 \cos \theta_i = (2 \times 0.44) \times \left(\frac{13.9}{0.3} \right) \times \left(\frac{13.9}{25} \right) \times \frac{38.2^\circ}{\cos 38.2^\circ}$ $G_m = \underline{17.8 \text{ Nm}}$	(4)
c	 $\text{Torque} = F \times d = d \sqrt{(Mv^2/r)^2 + (Mg)^2}$ $\text{Torque (gyro)} = G_m = I \omega^2 \cos \theta_i$ $d = \frac{I \omega^2 \cos \theta_i}{\sqrt{(Mg)^2 + (Mv^2/r)^2}}$ $d = \frac{17.8}{\sqrt{(285g)^2 + (285 \times 25 \times (\frac{13.9}{25})^2)}} = \frac{17.8}{3554.2} = \underline{5 \text{ mm}}$	(5)



d from previous diagram we get

$$\Delta\theta = \sin^{-1}\left(\frac{d}{h}\right) = \sin^{-1} \frac{5 \times 10^{-3}}{0.65} = 0.44^\circ \quad (3)$$

e



$$\sin \theta_i = \frac{x}{t} \quad \sin \Delta\theta = \frac{x}{h-t}$$

$$t \sin \theta_i = (h-t) \sin \Delta\theta$$

$$\sin \Delta\theta = \frac{t \sin \theta_i}{(h-t)}$$

$\therefore$ height of CoG will reduce $\Delta\theta$ as h increases by looking at above equation. Also true for h in equation 'Part d' so agree mat.

Total 20

2a

$$G_m = I \omega \dot{\theta} \quad (\text{from } G_m = I \omega \Delta \cos \theta_i)$$

$$\text{So we have } G_m = 0.65 \times \frac{40}{0.3} \times 1.8 = 156 \text{ Nm} \quad (3)$$

b Due to this accident you would expect to see the steering quickly turn to the right against the lock-stops

$$G_m = 0.44 \times \frac{40}{0.3} \dot{\delta} \quad (G_m = I_2 \omega \dot{\delta})$$

$$\text{So } \dot{\delta} = 2.66 \text{ rad/sec.}$$

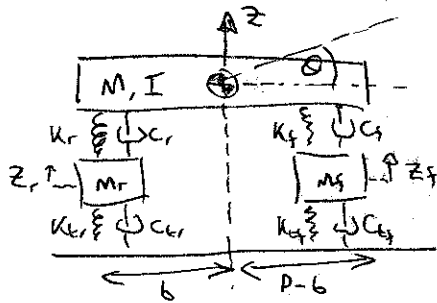
This is assuming no further rider input at the bars (3)



2c	Assuming the case that this torque is applied in the sense of 'turning the bars to the left', this torque could create a roll motion equal and opposite to the highside accident	(3)
d	In this case we would need $G_m = I_{fw} \dot{\delta} \cos 21.5$ to give us $\dot{\delta}' = \frac{156}{0.65 \times (\frac{40}{0.3})} = 1.8 \text{ rad s}^{-1}$ $\dot{\delta} = 1.8 \times \cos 21.5 = 1.67$. we can see that this is equal to the initial roll rate due to the accident situation when considered vertically but reduces slightly due to the rake angle of the bike.	(5)
e	Describe low speed 'Bicycle model'	(3)
f	Describe high speed 'gyroscopic model'	(3)
Total		20



3a This 4DoF system is represented as:



Half Car Model
Both Springs and Dampers

(5)

b Resolving forces for the undamped system.

$$M\ddot{z} + (k_f + k_r)z + (P-b)k_f\theta - b k_r\theta - k_f z_f - k_r z_r = 0$$

$$\Rightarrow M\ddot{z} + (k_f + k_r)z + ((P-b)k_f - b k_r)\theta - k_f z_f - k_r z_r = 0$$

Rotationally

$$I\ddot{\theta} + ((P-b)k_f - b k_r)z + ((P-b)^2 k_f + b^2 k_r)\theta - (P-b)k_f z_f + b k_r z_r = 0$$

$$\text{for } M_f \quad M_f \ddot{z}_f - k_f z - ((P-b)k_f \theta) + (k_f + k_{t_f})z_f = 0$$

$$\text{for } M_r \quad M_r \ddot{z}_r - k_r z + b k_r \theta + (k_r + k_{t_r})z_r = 0$$

This can be combined to give:

$$\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} + \begin{bmatrix} k_f + k_r & (P-b)k_f - b k_r & -k_f & -k_r \\ (P-b)k_f - b k_r & (P-b)^2 k_f + b^2 k_r & -(P-b)k_f & b k_r \\ -k_f & -(P-b)k_f & k_f + k_{t_f} & 0 \\ -k_r & b k_r & 0 & k_r + k_{t_r} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_f \\ z_r \end{bmatrix} = 0$$

Damping can be considered by a simple $k \rightarrow c$ substitute to give

$$MA + CV + KX = 0$$

in matrix form

10



C

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2x = 0$$

The characteristic equation is

$$m^2 + 2\omega\zeta m + \omega^2 = 0$$

$$\text{So } m = \omega(-\zeta \pm \sqrt{\zeta^2 - 1})$$

for $\zeta = 1$ $m = -\omega$ (equal roots)

$$\text{So } x = (A + Bt)e^{-\omega t}$$

when $t=0$, $x=5$ so

$$5 = e^0(A + B \cdot 0) \Rightarrow A = 5$$

Differentiating $x = (A + Bt)e^{-\omega t}$ (Product rule)

$$\dot{x} = -\omega e^{-\omega t}(A + Bt) + e^{-\omega t} \cdot B$$

Now when $t=0$, $\dot{x}=0$, so

$$0 = -\omega e^0(A + B \cdot 0) + e^0 \cdot B$$

$$B = \omega A = 5\omega$$

Substitute $A=5$ and $B=5\omega$ back in to get

$$x = e^{-\omega t}(5 + 5\omega t)$$

$$x = 5e^{-\omega t}(1 + \omega t)$$

(5)

Total 20



- 4a The capsize motion is in essence a roll motion influenced by the rider action on the bars. The motion is always unstable, similar to an upturned pendulum. modelled as

$$\theta = \theta_0 e^{\frac{t}{\tau}} \quad \text{where } \tau \text{ is the time constant.}$$

(3)

- b Wobble is the oscillation of the front assembly about the steer axis that can become unstable at low to mid speed. Frequency rises as the trail of the motorcycle increases and as the front frame motion decreases.

$$\text{we get: } I_A \ddot{\delta} + C \dot{\delta} + F_s a_n = M_f g b \sin \epsilon + N_f a_n \delta \epsilon$$

these are small

(5)

$$\text{So } I_A \ddot{\delta} + C \dot{\delta} + F_s a_n = 0$$

- c Weave is an oscillation of the whole motorcycle, but mainly driven by the effect at the rear frame of the motorcycle depending on the CoG, wheel motion, ground trail and cornering stiffness. It is given by

$$I_A \ddot{\psi} = -C \dot{\psi} - F_s l$$

(5)

- d The combined model for weave and wobble as described becomes

$$M \begin{bmatrix} \ddot{y} \\ \ddot{\theta}_r \\ \ddot{\theta}_f \end{bmatrix} + C \begin{bmatrix} \dot{y} \\ \dot{\theta}_r \\ \dot{\theta}_f \end{bmatrix} + K \begin{bmatrix} y \\ \theta_r \\ \theta_f \end{bmatrix} = 0 \quad \text{where}$$

$$M = \begin{bmatrix} m_r + m_f & -m_r l & m_f b_f \\ -m_r l & m_r l^2 I_r & 0 \\ m_f b_f & 0 & m_f b_f^2 I_f \end{bmatrix} \quad C = \frac{1}{v} \begin{bmatrix} k_{x_f} + k_{x_r} & -k_{x_r} l & -k_{x_f} a_n \\ -k_{x_r} l & k_{x_r} l^2 + c_v & -c_v \\ -k_{x_f} a_n & -c_v & k_{x_f} a_n^2 + c_v \end{bmatrix}$$

(7)

$$K = \begin{bmatrix} 0 & -k_{x_r} \cos \epsilon & -k_{x_f} \cos \epsilon \\ 0 & k_{x_r} l \cos \epsilon & 0 \\ 0 & 0 & k_{x_r} a_n \cos \epsilon \end{bmatrix}$$

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